THE ELEMENTS

OF

ARITHMETIC.

BY

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SIXTH EDITION, REVISED AND ENLARGED.

Calcutta:

PUBLISHED BY THACKER, SPINK & CO.

1896.

CALCUTTA:

PRINTED BY SANYAL & CO., AT THE BHARAT MIHIR PRESS,
26, SCOTT'S LANE.

APPENDICES A AND B, AND THE ANSWERS TO THE APPENDICES
PRINTED BY A. C. SEN, AT THE SAKHA PRESS,
33, MUSALMANPARA LANE.

PREFACE.

This little book is intended to help Indian students in learning the principles and practice of Arithmetic. It contains all that is usually given in works on Arithmetic, together with such additional matter as has been deemed necessary to meet the wants of those for whom it is prepared. The explanations are given in a form such as would facilitate a clear and rational understanding of the principles; and the examples appended to each chapter would, it is hoped, furnish ample exercise for the student.

NARIKELDANGA, CALCUTTA,
November 1879.

G. D. B.

PREFACE TO THE FOURTH EDITION.

In this Edition, two Appendixes have been added, one containing a large number of additional examples, and the other, the University Examination Papers. For Appendix I and for the answers to the examples in both the Appendixes, I am indebted to Babu Jadav Chandra Basu, a Teacher of Mathematics in the Metropolitan Institution.

NARIKELDANGA, CALCUTTA, *April 1888.* G. D. B.

PREFACE TO THE SIXTH EDITION.

In this Edition, a few additions and alterations have been made here and there, and a new Appendix marked A containing a collection of well graduated examples has been inserted before Appendixes I and II which have now been marked as Appendixes B and C. For the Appendix A, I am wholly indebted to Babu Haran Chandra Banerji, M. A., B. L., Professor of Mathematics in the Ripon College, and to Babu Kedar Nath Basu, B. A., a Senior Teacher of that Institution.

NARIKELDANGA, CALCUTTA, *December 1895.* G. D. B.

CONTENTS.

ERRATA.

Page 13,	last line,	*for*	CL.)	*read*	CIↄ.
Page 86,	line 5,	*for*	$\frac{1}{6}$	*read*	$\frac{1}{8}$.
Page 86,	line 29,	*for*	$\frac{3+3}{4+7}$	*read*	$\frac{3+8}{4+8}$.
Page 111,	line 7,	*for*	$\cdot\dot{8}$	*read*	$\cdot 1\dot{8}$.
Page 132,	line 17,	*for*	21 lbs.	*read*	2 lbs.
Page 224,	line 22,	*for*	$\frac{1}{3}$	*read*	$\frac{1}{6}$.
Page 250,	last line,	*for*	1580	*read*	1530.
Page 251,	line 28,	*for*	109820000000	*read*	109826000000.

APPENDIX A.

Page 6, line 6, *for* 5 *read* 4.

Page 6, line 7, *for* than the fourth *read* than the second.

Page 6, line 8, *for* fifth *read* third.

Page 11, line 34, *for* 18413 *read* 18411.

Page 14, line 3, *add* '7 miles in circumference' *after* 'field'.

Page 14, line 4, *for* at *read* in.

Page 14, line 5, *for* 4, 12, 16, *read* 3, 4, 5 respectively.

Page 18, line 17, *for* ÷ *read* +.

Page 22, last line, *for* $\frac{3}{8}-\frac{5}{6}$ *read* $\frac{5}{6}-\frac{3}{8}$.

Page 23, line 2, *for* $\frac{1}{3}-(1+\frac{1}{12})$ *read* $\frac{1}{3}+(1-\frac{1}{12})$.

Page 28, line 19, *for* ·42 *read* $\cdot\dot{4}\dot{2}$.

Page 28, line 20, *for* − *read* +.

Page 29, line 15, *for* 391 393' *read* 3·91 393.

Page 37, line 20, *for* − *redd* +.

Page 40, line 4, *for* £118 *read* £29.

Page 49, line 15, *for* 3 : 2 *read* 31 : 4.

THE ELEMENTS OF ARITHMETIC.

INTRODUCTION.

Article 1. Definition. Arithmetic is the science of *numbers*.

2. Defs.. A Unit or **Unity** means *one*.

A unit, or any part of unity, or any collection of units or parts of unity or both, is called a **Number**.

Thus, *one*, *one-fourth*, *two*, *three-fourths*, *four and a half* are numbers.

3. Defs. A number consisting wholly of entire units is called an **Integer** or a **Whole Number**.

A number not consisting wholly of entire units is called a **Fraction**.

Thus *one*, *two*, *three*, are integers; *half*, *three-fourths*, *two and a half*, are fractions.

4. Defs. A number consisting of units or parts of units of no particular kind is called an **Abstract Number**.

A number consisting of units or parts of units of some particular kind is called a **Concrete Number**.

Thus, *one*, *two*, *half*, considered simply, are abstract numbers; but in the instances, *one rupee*, *two yards*, *half a seer*, *one*, *two*, and *half*, are concrete numbers.

5. From Arts. 3 and 4 it will be seen, that every number must be either an integer or a fraction; and that each of these again must be either abstract or concrete. Thus, numbers may be divided into the following four classes:—

I. Abstract Integers.

II. Abstract Fractions.

III. Concrete Integers.

IV. Concrete Fractions.

6. Def. Zero, Cipher, or **Nought,** means *nothing* or no number.

It indicates the absence of number.

Being no number, it can neither be integral nor fractional, neither abstract nor concrete, in the ordinary sense of the terms. But in one sense, it may be said, in each case, to be of the same kind as the number whose absence it indicates.

Sometimes for convenience of expression, zero is said to be a number. Thus we say that zero is the smallest number in Arithmetic. The meaning of such a proposition is this : zero being nothing, is less than any number that we can take, and is, therefore, the least of all numbers.

7. Def. Infinity means an infinitely large number, or a number larger than any number that we can take.

It may, therefore, for convenience of expression, be considered as the largest possible number.

8. Hence, *all numbers*, in Arithmetic, *range between zero and infinity*.

9. To facilitate operations, numbers are expressed by certain figures or symbols ; thus,

one, *two*, *three*, *four*, *&c.*, are expressed by the figures,
1, 2, 3, 4, &c.

Defs. The art of expressing by *figures* any number which is given in *words* is called **Notation.**

The art of expressing in *words* any number which is given in *figures* is called **Numeration.**

10. When there are any two given numbers, amongst the different numbers which may be produced by operating with them in different ways, we must be able to know what particular number results—

(1) when they are taken together ;

(2) when one of them is taken from the other ;

(3) when one of them is taken a number of times equal to the other ; and

(4) when it is counted how often one of them is contained in the other.

The operations for finding the results in the above four cases are defined below.

11. **Def. Addition** is the method of finding what number, called the **Sum,** results from the taking together of two or more given numbers called the **Summands.**

The sign +, read **Plus,** placed between two numbers, indicates that they are to be added together. Thus 1 + 2 means that *one* is to be added to *two.*

12. **Def. Subtraction** is the method of finding what number, called the **Difference** or **Remainder,** results from the taking of a smaller number, called the **Subtrahend,** from a greater, called the **Minuend.**

The sign −, read **Minus,** placed between two numbers, indicates that the latter is to be subtracted from the former. Thus 2 − 1 means that *one* is to be subtracted from *two.*

13. **Def. Multiplication** is the method of finding what number, called the **Product,** results from the taking of one given number, called the **Multiplicand,** another given number of times. This last mentioned given number is called the **Multiplier.** The Multiplicand and the Multiplier are both sometimes called the **Factors** of the product.

The sign ×, read **Into,** placed between two numbers, indicates that the former is to be multiplied by the latter. Thus 2 × 3 means that *two* is to be multiplied by *three.*

14. **Def. Division** is the method of finding what number of times one number, called the **Divisor,** is contained in another, called the **Dividend.** The first mentioned number is called the **Quotient.**

The sign ÷, read **By** or **Divided by,** placed between two numbers, indicates that the former is to be divided by the latter. Thus 4 ÷ 2 means that *four* is to be divided by *two.* So also one number written above another with a line between them, indicates that the former is to be divided by the latter. Thus $\frac{4}{2}$ means the same thing as 4 ÷ 2.

15. The four operations defined above, Addition, Subtraction, Multiplication and Division, form the basis of all other Arithmetical operations, as will be seen in the sequel. These four are accordingly called the **Fundamental Operations** in Arithmetic. They will be considered with reference to the four classes of numbers in four separate Chapters. The other operations will be defined in their proper places as we proceed.

16. **Defs.** Two or more numbers expressed in figures and connected by one or more signs of operation, form what is called an **Expression.**

Thus $1+2$ is an expression.

When an expression is equal to another number or expression, the sign $=$, read **Equals**, is placed between them to indicate this equality, and the whole expression consisting of the two equal numbers with the sign of equality placed between them, is called an **Equation**.

Thus $1+2=3$ is an equation.

The signs $\because$ and $\therefore$ mean **Because** and **Therefore** respectively.

CHAPTER I.

THE FUNDAMENTAL OPERATIONS WITH ABSTRACT INTEGERS. MEASURES AND MULTIPLES.

SECTION I. NOTATION AND NUMERATION.

17. As every integer consists wholly of entire units, we see that *one* is the least integer; the next integer *two* is formed by adding *one* to *one*; the next, *three*, by adding *one* to *two*; and so on: and generally, *every succeeding integer is formed by adding unity to the preceding.*

Thus we get integers from *one* to *one hundred* and upwards without limit.

18. To express integers by symbols, the following are the three possible modes that we can adopt:—

Ist. We can take a symbol for unity, and express every other number by repetition of this symbol.

2nd. We can take a separate symbol for every separate number.

3rd. We can take separate symbols for some of the integers and express others by combinations of these.

The first and second modes are obviously *complete*, that is, by them we can express *all* numbers; but they are also obviously *inconvenient*, when we have to express large numbers; for the first mode requires much space, and the second, the use of a large number of distinct symbols. Accordingly, these two modes have never been adopted, and we have only the third mode left for us.

But adopting the third mode, there still remains the question,—How many different symbols shall we take, and what mode combining them shall we adopt?

The Greeks answered this question in one way, and invented their system of Notation. This is a complicated system, and so it has become obsolete.

The Romans answered it in another way, and invented their system of Notation. This too is not quite simple; but it is not so complicated as the Greek system, and is still retained in use occasionally, as in indicating the hour marks on dials of clocks and watches.

The Hindus answered the question in a third way, and invented their system of Notation. This system was borrowed from them by the Arabs, and from these last, by the nations of Europe, and is on

that account sometimes called the Arabic system of Notation. It is now the *Common System of Notation* in almost all civilized countries. It is the best system of Notation that has been invented. We proceed to describe it in the next Article.

19. The Common System of Notation.

In this system, the numbers *one. two, three, four, five, six, seven, eight, nine* and *zero*, are expressed by the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, which are called **Digits**; and all other numbers are expressed by combinations of these figures according to the following Rule of *Convention* :—

Rule. A figure in the foremost place towards the right has its simple value, a figure removed *one* place towards the left has its value increased *ten-fold*, a figure removed *two* places towards the left has its value increased ten times ten fold or a *hundred-fold*, and so on, the value of a digit increasing *ten-fold* at each step of removal towards the left; and zero may stand in any of these places except the last on the left, to show the absence of significant digits in those places, and to indicate the proper places for the digits to the left.

Def. This increased value which a figure has in consequence of its *position*, is called its **Local Value**, as distinguished from the value which it has when standing alone, and which is called its **Intrinsic Value.**

Let us now see if this system is *complete* and *convenient.*

20. This system of Notation is *complete.* For we can express every integer in this system. Thus :—

One, two, three, four, five, six, seven, eight, nine, are expressed by 1, 2, 3 4 5 6 7 8 9. *Ten* is expressed by 10; for 0 standing in the foremost place on the right, 1 is removed one place towards the left, and means ten times 1 or ten, and there is nothing more besides. *Eleven* is expressed by 11; for the 1 on the right means 1 unit, and the other 1 means ten times 1 or ten units, and the whole therefore means ten and one or eleven. Similarly, *twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen,* and *nineteen* are expressed by 12, 13, 14, 15, 16, 17, 18, and 19.

Twenty being two tens, will be expressed by 20.

So numbers from *twenty-one* to *ninety-nine* will be expressed by combinations of two digits, thus: 21, 22,......29, 30, 31.........99.

Ninety-nine is the largest number that can be expressed by two digits. The next number *one hundred* will be expressed by 100; for the 1 standing to the left of two zeros has its value increased a hundredfold, and there is nothing more besides.

Thus, numbers from *one* to *one hundred* will be expressed as follows :

one	1	twentysix	26	fiftyone	51	seventysix	76
two	2	twentyseven	27	fiftytwo	52	seventyseven	77
three	3	twentyeight	28	fiftythree	53	seventyeight	78
four	4	twentynine	29	fiftyfour	54	seventynine	79
five	5	thirty	30	fiftyfive	55	eighty	80
six	6	thirtyone	31	fiftysix	56	eightyone	81
seven	7	thirtytwo	32	fiftyseven	57	eightytwo	82
eight	8	thirtythree	33	fiftyeight	58	eightythree	83
nine	9	thirtyfour	34	fiftynine	59	eightyfour	84
ten	10	thirtyfive	35	sixty	60	eightyfive	85
eleven	11	thirtysix	36	sixtyone	61	eightysix	86
twelve	12	thirtyseven	37	sixtytwo	62	eightyseven	87
thirteen	13	thirtyeight	38	sixtythree	63	eightyeight	88
fourteen	14	thirtynine	39	sixtyfour	64	eightynine	89
fifteen	15	forty	40	sixtyfive	65	ninety	90
sixteen	16	fortyone	41	sixtysix	66	ninetyone	91
seventeen	17	fortytwo	42	sixtyseven	67	ninetytwo	92
eighteen	18	fortythree	43	sixtyeight	68	ninetythree	93
nineteen	19	fortyfour	44	sixtynine	69	ninetyfour	94
twenty	20	fortyfive	45	seventy	70	ninetyfive	95
twentyone	21	fortysix	46	seventyone	71	ninetysix	96
twentytwo	22	fortyseven	47	seventytwo	72	ninetyseven	97
twentythree	23	fortyeight	48	seventythree	73	ninetyeight	98
twentyfour	24	fortynine	49	seventyfour	74	ninetynine	99
twentyfive	25	fifty	50	seventyfive	75	one hundred	100

One hundred and one will be represented by 101 ; for the first 1 on the right is 1 unit, and the last 1 on the left, being removed two places towards the left, is 1 hundred, and there is nothing more. Similarly numbers from *one hundred and two to one hundred and nine* will be represented by 102, 103, 104, 105, 106, 107, 108, and 109.

One hundred and ten will be reprensented by 110 ; for the last 1 on the left is 1 hundred, the next is 1 ten, and there is nothing more. *One hundred and eleven* will be represented by 111, as the last 1 on the left means one hundred, the next to its right, 1 ten, and the next after it, 1 unit.

Similarly, numbers from *one hundred and twelve to nine hundred and ninety-nine* will be represented by combinations of three digits, thus : 112, 113,..., 120, 121,......199, 200, 201,...999.

Nine hundred and ninety-nine is the largest number that can be expressed by three digits. The next number *one thousand* will be represented by 1000. And so on.

Thus all integers can be expressed in this system.

21. The Common System of Notation is also highly *convenient* as will appear from the following considerations.

In ordinary language, the first nine numbers are named *one*, *two* &c., *nine*; the next is named *ten*; the next nine numbers are named *eleven*, *twelve*, *thirteen*, &c., *nineteen*, *i.e.* with slight modifications, *one* and *ten*, *two* and *ten*, *three* and *ten* &c., *nine* and *ten*.

The next number is named *twenty* or *two tens*, and those after it are named *twenty-one*, *twenty-two*, &c., *thirty*, *thirty-one*, &c., *ninety-nine*, *i.e.*, *two tens and one*, *two tens and two*, &c., *three tens*, *three tens and one*, &c., *nine tens and nine*. The next number is *one hundred* and those after it are named *one hundred and one*, *one hundred and two*, &c., *nine hundred and ninety-nine*. And so on.

Thus in common language an integer is named by naming separately and explicitly the number of *units*, the number of *tens*, the number of *hundreds*, the number of *thousands*, &c., that it contains, in the order commencing with the highest, none of these numbers being greater than nine, and some of them being sometimes wanting.

And we have seen that in the Common System of Notation, an integer is expressed by expressing separately and explicitly the number of *units*, the number of *tens*, the number of *hundreds*, the number of *thousands*, &c., that it contains, in the order commencing with the highest, and proceeding from the left to the right, none of these numbers being greater than nine, and some of them being sometimes zeros.

Hence we see that the mode of *expressing* a number in the Common System of Notation is just the same as the mode of *naming* it in common language, the number ten forming the basis of both; and we can pass from the name to the symbolical expression, and *vice versa*, without any difficulty, taking care only, in Notation to fill up the vacant places with zeros, and in Numeration to observe the indications of zeros.

It is this easy convertibility of the names of numbers to their symbolical expressions and *vice versa*, that makes this system of Notation so peculiarly convenient.

22. From Arts. 19-21 we see that when a number is expressed by figures, the place of the foremost figure on the right is the units' place, that of the second to the left is the tens' place, that of the third, the hundreds' place, and so on, as will be seen in the following Table called the Numeration Table :—

&c.	Trillions.	Hundreds of Thousands of Billions.	Tens of Thousands of Billions.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Thousands of Millions.	Tens of Thousands of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
&c.	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
					Billions.						Millions.						Units.		

The period of 6 places from the 19th to the 24th consists of *trillions*; the next period of 6 places consists of *quadrillions*, the next period, *quintillions* ; &c.

Hence a number expressed in figures can be analyzed into its constituent units, tens, hundreds, &c., by separating its digits, and putting for each its proper local value.

Thus for example, let it be required to analyze 302064 into its units, tens, hundreds, &c.

We have $302064 = 3$ hundreds of thousands + no tens of thousands + 2 thousands + no hundreds + 6 tens + 4 units
$= 300000 + 2000 + 60 + 4.$

23. The Indian Numeration Table in common use runs thus :—

&c.	Hundreds of Crores.	Tens of Crores.	Crores.	Tens of Lacs.	Lacs.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
&c.	10	9	8	7	6	5	4	3	2	1

24. The following are the Rules for Notation and Numeration :

Rule I. In Notation, write down in their proper places digits corresponding to the numbers expressing the units, tens, hundreds, &c., in the given number, commencing with the highest place and fill up the vacant places with ciphers.

Rule II. In Numeration, after dividing by commas the figures composing the given number into periods of 3 figures each, commencing with the foremost figure on the right, read out the periods with their proper denominations, commencing from the left.

Example 1. Write down in figures, two thousand and fifty-millions, six hundred and thirteen thousand, five hundred and nine.

Here the first or the highest number *two* expressing thousands of millions, belongs to the 10th place : the 9th place is vacant, there being no hundreds of millions in the number ; there being fifty millions, *i. e.*, *five* tens of millions, the *five* belongs to the 8th place : the 7th place is vacant there being no millions over and above the fifty millions ; the next number *six* expressing hundreds of thousands, belongs to the 6th place ; there being thirteen thousands, *i. e.*, *one* ten of thousands and *three* thousands, the *one* belongs to the 5th place and the *three* to the 4th ; the 3rd place will be filled by the *five* of five hundred ; the 2nd place is vacant ; and the 1st place is filled by *nine*. The whole number will therefore be expressed thus,—2050613509.

Ex. 2. Write in words the number 320506190.

Dividing the figures into periods we have 320, 506, 190 ; and we see that the highest digit 3 belongs to the 9th or the hundreds of millions' place, the next, 2, to the tens of millions' place &c. ; the number will therefore be read thus :—Three hundred and twenty millions, five hundred and six thousand, one hundred and ninety.

25. The figure 0 annexed to the *right* of any integer expressed in figures increases the value of every constituent digit tenfold by pushing it one place towards the left, and thus increases the entire number tenfold. But annexed to the *left* of an integer, it has no effect.

Thus 150 is ten times 15.

26. We have seen that *ten* forms the basis of our ordinary mode of reckoning numbers. This is the case in almost every language, and was in all probability the fact which led to the invention of the Common System of Notation. But there arises the question, How to *account for* this fact ? In answer to this, the following *theory* has been advanced, of which there is no reason to doubt the correctness.

In primitive society, in the infancy of Arithmetic, the *ten* fingers must have formed the readiest counters. But with the fingers, *one* man can count only up to *ten*; and to count more, he must have some means of indicating that his ten fingers have all been counted over. The readiest means available for such purpose would appear to have been the raising of the fingers of a *second* man, one each time that the *first* man counted over all his ten. In this way, eleven, twelve &c., will be counted by *one* raised finger of the second man representing that all the ten fingers of the first have been once raised, and *one*, *two* &c. raised fingers of the first. *Twenty* will be counted when the first man has counted over all his ten fingers again, and the second man has accordingly raised *two* fingers to indicate this. And so on. It was from this primitive mode of counting, that the almost universal practice of reckoning numbers by periods of ten seems to have originated.

This theory receives some cofirmation also from the fact that the word *digit*, used to designate the elementary figures, also means a *finger*.

We may observe that the *digits*, including zero, in the successive places used to express a number in the Common System of Notation, correspond to the *numbers of the fingers* of the successive persons, raised to count the same number in the above mode. Thus, in 10,

the 0 in the 1st place corresponds to *no* raised finger of the 1st man
and 1.........2nd..............................1...........................2nd......

So, in 23,

the 3 in the 1st...............................3.........fingers.........1st.......
and 2.........2nd..............................2...........................2nd......

And so in other cases.

The inquiring student is referred for further information on the subject of Notation to the Article on Arithmetic by Dr. Peacock in the Encyclopædia Metropolitana.

27. The Roman System of Notation.

In this system, the following are the different symbols used, *viz.*,

I,	V,	X,	L,	C,	D or IƆ,	M or CIƆ,
for 1,	5,	10,	50,	100,	500,	1000,

respectively. Other numbers are expressed by combinations of these according to the following Rules :—

1st. A character followed by one or more characters of equal or less value indicates a number equal to the sum of the values of all the characters. A character preceded by another of less value denotes a number equal to the difference of their values.

Thus $II = 1 + 1 = 2$; $IV = 5 - 1 = 4$.

2nd. Every Ↄ annexed to IↃ increases the value of the latter tenfold. Every C prefixed and Ↄ annexed to CIↃ increases the value of the latter tenfold.

Thus IↃↃ = 5000, CCIↃↃ = 10,000.

3rd. A line drawn over a character increases its value thousand-fold.

Thus $\overline{I} = 1000$.

The representation of one, two, three, by I, II, III, *i. e.*, by one, two, three strokes, is natural enough. The representation of ten, which forms a turning point in our computation, by X or a cross with one stroke crossing another, and of five, the half of ten, by the upper half of X, seems also to be equally natural.

To the beginner the following Table may be of use.

I,	II,	III,	IV,	V,	VI,	VII,	VIII,	IX,	X,
1,	2,	3,	4,	5,	6,	7,	8,	9,	10,
XI,	XII,	XIII,	XIV,	XV,	XVI,	XVII,	XVIII,	XIX,	XX,
11,	12,	13,	14,	15,	16,	17,	18,	19,	20,

XXX,	XL,	L,	LX,	LXX,	LXXX,	XC,	C,
30,	40,	50,	60,	70,	80,	90,	100.

Examples I.

1. Express in figures the following :—

(1) Ten ; twelve ; fifteen ; nineteen ; twenty-eight ; forty-four ; fifty-six ; sixty-one ; eighty-four ; ninety-two.

(2) One hundred and one ; one hundred and ten ; one hundred and fifty-four ; three hundred ; four hundred and five ; five hundred and sixty ; seven hundred and seventy-four.

(3) One thousand and one ; two thousand and fifty-one ; three thousand two hundred and sixty-three ; four thousand ; five thousand five hundred ; six thousand seven hundred and eighty.

(4) One hundred thousand and one ; two hundred thousand and three hundred ; three hundred and six thousand seven hundred and nine ; four hundred and fifty-six thousand and four ; five hundred and sixty-seven thousand, four hundred and thirty-two.

(5) Two millions and one ; three millions and twenty-nine ; four millions, five hundred and sixty ; five millions, six hundred thousand, and seventy-four ; six millions, seven hundred and fifty-four thousand, three hundred and twenty-one.

(6) Three billions; four billions and five; five billions, six thousand, seven hundred and eight; seven billions, nine hundred and thirteen millions, five hundred and seventy-nine thousand, one hundred and thirty-five.

(7) Nineteen trillions; twcnty trillions and tweenty-four; thirty-one trillions, five hurdred and fifty-six thousand seven hundred and nine millions, eight hundred and twanty seven thousand five hundred and twenty.

(8) One lac and one; two lacs three thousand and three; five lacs sixty-one thousand seven hundred and twenty; fifteen lacs thirty thousand six hundred and twelve.

(9) Two crores and two; three crores five lacs seven thousand and nine; five crores sixty-four lacs thirty-two thousand one hundred and seventy-eight.

(10) Two hundred and sixteen crores fifty lacs sixteen thousand seven hundred and eighteen.

2. Express in words the following :—

(1) 18; 20; 37; 58; 69; 85; 97.

(2) 203; 340; 456; 690; 708; 991.

(3) 1009; 2029; 3690; 4862.

(4) 102030; 230450; 300004; 745621.

(5) 123456789; 987654321; 102030405.

(6) 2468101214; 248163264128.

(7) 501002003300400; 369121518212427 30.

(8) 2305843008139952128; 137438691328.

3. Express in words according to the Indian Numeration Table the numbers in the preceding Examples (4) and (5).

4. Analyze by separating into units, tens, &c., the numbers in the preceding Examples (1), (2) and (3).

5. Express in Roman numerals the following :—

(1) 25; 33; 46; 87; 99.

(2) 101; 220; 314; 516; 999.

(3) 1001; 1856; 1864.

6. Express in figures the following :—

(1) XXVII; XXXIV; XXXXV; XLVI.

(2) XCIX; CCCI; MXL; DCL.

(3) MDCCCLVI; CLↃDLXXXII; MIX.

SECTION II. SIMPLE ADDITION.

28. Def. The addition of abstract integers or of concrete integers of the same denomination is called **Simple Addition.**

29. The following Table called the Addition Table should be committed to memory by the beginner.

ADDITION TABLE.

1 and	2 and	3 and	4 and	5 and	6 and	7 and	8 aud	9 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10
2 ... 3	2 ... 4	2 ... 5	2 ... 6	2 ... 7	2 ... 8	2 ... 9	2 ...01	2 ... 11
3 ... 4	3 ... 5	3 ... 6	3 ... 7	3 . . 8	3 ... 9	3 ...10	3 ...11	3 ... 12
4 ... 5	4 ... 6	4 ... 7	4 ... 8	4 ... 9	4 . .10	4 ...11	4 ...12	4 ... 13
5 ... 6	5 ... 7	5 ... 8	5 ... 9	5 ...01	5 ...11	5 ...12	5 ...13	5 ... 14
6 ... 7	6 ... 8	6 ... 9	6 ...10	6 ...11	6 ...12	6 ...13	6 ...14	6 ... 15
7 . . 8	7 ... 9	7 ...10	7 ...11	7 ...12	7 ...13	7 ..14	7 ...15	7 ... 16
8 ... 9	8 . .10	8 ...11	8 ...12	8 ...13	8 ..14	8 ...15	8 ...16	8 ... 17
9 ...10	9 ...11	9 ...12	9 ...13	9 . .14	9 ...15	9 ...16	9 ...17	6 ... 18

30. Rule for Addition. Write down in figures the several summands one under the other, so that units may be under units, tens under tens, &c., and draw a line below the last.

Add up the digits in the column of units and place the figure in the units' place of the sum, below the column of units ; carry the number composed of the other digits, if any, add it along with the digits in the tens' column, and place the figure in the lowest place of this sum below the tens' column ; and carrying the number composed of the other digits, if any, to the next column, add it along with the digits in that column, and proceed as before. Repeat this process to the last column, and put the last sum in full. The entire number thus obtained is the sum required.

Ex. Add together 1099, 588, 689, and 2409.

By the Rule we have,

$$\begin{array}{r} 1099 \\ 588 \\ 689 \\ 2409 \\ \hline 4785 \end{array}$$

Reason for the Rule. To add numbers together is to add up successively the numbers of units, the numbers of tens, &c., that they contain ; and this is done by adding together the digits in their units' places, those in their tens' places, &c., and placing the sums under those corresponding places, taking care, when any of these sums contains a number of a denomination higher than that of the digits added, to carry such number and add it to the sum of the

digits of the next place. Thus in the Example above, the several summands contain $9+8+9+9$ units, $9+8+8+0$ tens, $0+5+6+4$ hundreds, and $1+2$ thousands, and the entire sum required will be the sum of these units, together with the sum of these tens, the sum of these hundreds, and the sum of these thousands. Now $9+8+9+9$ units $=35$ units, *i.e.*, 5 units and 3 tens; 5 therefore is the digit that belongs to the units' place of the sum required, and we accordingly put 5 in the units' place. The 3 tens which the sum of the figures in the units' place gives, must now be added to the $9+8+8+0$ tens, thus giving $3+9+8+8+0$ or 28 tens *i. e.* 8 tens and 20 tens or 2 hundreds; we accordingly put 8 in the tens' place, and carry 2 to be added to $0+5+6+4$ the sum of the digits in the hundreds' column. We thus get $2+0+5+6+4$ or 17 hundreds, *i.e.*, 7 hundreds and 10 hundreds or 1 thousand; 7 therefore is to be put in the hundreds' place, and 1 is to be carried and added to $1+2$ thousands. We thus get $1+1+2$ or 4 thousands, and 4 is accordingly placed in the thousands' place. The whole sum, therefore, is 4 thousands $+7$ hundreds $+8$ tens $+5$ units, *i.e.*, 4785.

Worked out at full length, the process will stand thus :—

$$\begin{array}{lrl}
 & 1099 & =1000+0+90+9 \\
 & 588 & =500+80+8 \\
 & 689 & =600+80+9 \\
 & 2409 & =2000+400+0+9 \\
\text{The sum} & & =3000+1500+250+35 \\
 & & =3000+1000+500+200+50+30+5 \\
 & & =4000+700+80+5 \\
 & & =4785
\end{array}$$

31. Proof. The correctness of the result in Addition may be tested thus :—

Add the several summands omitting one of them; to the sum thus obtained add the number omitted; and then if this last sum is the same as the sum of all the numbers, the result is in all probability correct.

The reason for this is obvious.

Ex. Add together 359, 1267, 486, 29.

Operation	Proof
359	359
1267	1267
486	486
29	29
2141	1782
	359
	2141

Examples II.

1. Add

(1)	(2)	(3)	(4)	(5)	(6)
1	11	21	11	13	123
2	12	22	21	35	321
3	13	23	31	57	456
4	14	24	41	79	654
5	15	25	51	911	789
6	16	26	61	113	987
7	17	27	71	335	
8	18	28	81	577	
9	19	29	91	799	

(7)	(8)	(9)	(10)	(11)	(12)
135	147	999	6789	1357	1234
531	258	777	9876	7531	567
790	741	555	6798	3175	89
97	852	333	6978	1537	1011
248	159	111	9678	5173	121
842	951	222	6879	7135	31

(13)	(14)	(15)
15720	1000001	727421
9134	5200025	483853
720	8630368	959496
1516	1974791	416151
9208	1250521	752726
872	9600069	986383
	7000007	1303

(16)	(17)	(18)
12345	123456789	121144
6789	987654321	169196
1011	432159876	225256
12131	123456789	289324
4151	341258967	361400
61718	412359678	441484
404		

2. Add together—Six billions, five hundred and nienty-five millions, twenty-one thousand eight hundred and eighty-nine; two hundred and twenty thousand six hundred millions, one thousand and twenty-eight; fifty-six millions and fifty-six; one hundred and ninety-eight millions, one hundred and ninety-eight; sixty thousand and fifty.

3. Add together:—Two crores ten lacs fifty thousand and five; sixty-six lacs eleven thousand and seven; twenty-eight lacs seven thousand and five; fifty crores sixty lacs and seventy.

4. Add together:—123, 246, 4812, and 9624; also 456, 912, 1824, and 3648.

5. Find the sum of 65793, 752810, 446602, and 3979; also of 89210, 3579, 1012 and 201623.

6. Add together the sums of 35, 55, and 75; 44, 64, and 84; 12, 24, and 36; and 29, 39, and 49.

7. Add together the values of $1+2+3+4$, $11+12+13+14$, $21+22+23+24$, and $31+32+33+34$; also of $2+4+6$, $1+3+5$, $8+10+12$, $7+9+11$, and $2+4+8$.

8. Find the sum of 19 repeated 9 times; of 21 repeated 11 times; of 32 repeated 8 times; of 64 repeated 8 times; and of 40 repeated 9 times. Find also the sum of these sums.

SECTION III. SIMPLE SUBTRACTION.

32. Defs. The Subtraction of abstract integers or of concrete integers of the same denomination is called **Simple Subtraction**.

When a number is subtracted from another, then from the remainder so obtained, then from the next remainder, and so on, as long as the operation can be carried on, the entire process is called **Continued Subtraction.** It shews how often one number can be taken from another, or, in other words, how often it is contained in that other.

33. Def. The sign $\overline{\qquad\qquad}$ is called a **Vinculum**, and the signs ( ) and { } are called **Brackets**. When numbers are placed under the former, or are enclosed within the latter, it is to be understood that the operations indicated within the sign are to be performed first, and the resulting number is then to be affected by the operations indicated outside.

Thus, $7-\overline{5-2}$ or $7-(5-2)$ means that 2 is to be first subtracted from 5, and then the result 3 is to be subtracted from 7.

34. Proposition I. If the same number be added to, or subtracted from, both the minuend and the subtrahend the difference remains unchanged.

Thus, $7-5=2$; and $7+2-(5+2)=9-7=2$ also;
and so also $7-3-(5-3)=4-2=2$.

Prop. II. Subtrahend + remainder = minuend.

Prop. III. Subtrahend = minuend − remainder.

For the minuend being made up of the subtrahend and the remainder, if the remainder is taken from it, there remains the subtrahend.

Prop. IV. If the same number be added to and subtracted from any number, the latter remains unchanged.

Prop. V. When an expression consists of several numbers some of which are to be added together and the others are to be subtracted from those first mentioned, the value of the expression is equal to the difference between the sum of the former and that of the latter.

Thus $3+6-4-1+5-7$ = diff. between $3+6+5$ and $4+1+7$

= ....14 and 12

= 2.

35. The following Table, called the Subtraction Table, should be committed to memory by the beginner.

SUBTRACTION TABLE.

1 from	2 from	3 from	4 from	5 from	6 from	7 from	8 from	9 from
1 = 0	2 = 0	3 = 0	4 = 0	5 = 0	6 = 0	7 = 0	8 = 0	9 = 0
2... 1	3... 1	4... 1	5... 1	6... 1	7... 1	8... 1	9... 1	10... 1
3... 2	4... 2	5... 2	6... 2	7... 2	8... 2	9... 2	10... 2	11... 2
4... 3	5... 3	6... 3	7... 3	8... 3	9... 3	10... 3	11... 3	12... 3
5... 4	6... 4	7... 4	8... 4	9... 4	10... 4	11... 4	12... 4	13... 4
6... 5	7... 5	8... 5	9... 5	10... 5	11... 5	12... 5	13... 5	14... 5
7... 6	8... 6	9... 6	10... 6	11... 6	12... 6	13... 6	14... 6	15... 6
8... 7	9... 7	10... 7	11... 7	12... 7	13... 7	14... 7	15... 7	16... 7
9... 8	10... 8	11... 8	12... 8	13... 8	14... 8	15... 8	16... 8	17... 8
10... 9	11... 9	12... 9	13... 9	14... 9	15... 9	16... 9	17... 9	18... 9

36. Rule for Subtraction. Place the smaller number under the greater, so that units may be under units, tens under tens, &c., and draw a line below.

Subtract each figure of the subtrahend, commencing with the units' figure, from the corresponding figure of the minuend, and write the difference below it. When any digit in the subtrahend is greater than the corresponding digit in the minuend, add 10 to the latter and then subtract the lower digit from the sum so obtained, and put down the difference below, taking care to carry 1 and add it to the next figure in the subtrahend before subtracting it from the corresponding figure in the minuend. The entire number thus obtained is the difference required.

Ex. 1. Subtract 938 from 6183.

By the Rule we have

$$\begin{array}{r} 6183 \\ \underline{938} \\ 5245 \end{array}$$

Reason for the Rule. To subtract one number from another is to subtract successively the number of units, the number of tens, &c., that the former contains, from the number of units, the number of tens, &c., respectively in the latter ; and this is done by subtracting the digits in the successive places in the subtrahend from the corresponding digits of the minuend, and placing the differences under those corresponding places. If any digit in the subtrahend is greater than the corresponding digit in the minuend, to make the subtraction of that digit possible, we add to the digit in the minuend 10 of its own denomination, *i. e.*, 1 of the *next higher* denomination, and to keep the difference unchanged, we add 1 of this *next higher* denomination to the subtrahend, (Art. 34, Prop. I), *i. e.*, add 1 to the next digit in the subtrahend, before subtracting it from the digit above it. Thus in the Example above, 3 units being less than 8 units, we add 10 units, *i. e.*, 1 ten to 3 making it 13, and taking 8 units from 13 units, we have 5 units left ; we therefore put 5 in the units' place of the difference required. And having added 1 ten to the minuend, to keep the difference unchanged, we must add 1 ten to the subtrahend ; accordingly we carry 1 and add it to the 3 tens, thus getting 4 tens, which taken from 8 tens, leave 4 tens ; we therefore put 4 in the tens' place. Again, 1, *i. e.*, 1 hundred being less than 9 or 9 hundreds, we add 10 hundreds or 1 thousand to it, thus getting 11 hundreds, and taking 9 hundreds from this, we have 2 hundreds for the difference; so 2 is put in the hundreds' place. And having added 1 thousand to the minuend, we must add it also to the subtrahend; and taking this 1 thousand from 6 thousands, we have 5 thousands for the difference, and so we put 5 in the thousands' place. The whole difference, therefore, is 5 thousands + 2 hundreds + 4 tens + 5 units, *i. e.*, 5245.

Worked out at full length, the process will stand thus :—

$$\begin{array}{lllllll}
\text{Diff. of } 6183 & & & & & & \\
\quad\text{and } 938 & & & & & & \\
=\text{diff. of } 6000 + 100 + 80 + 3 & & & & & & \\
\quad\text{and } \qquad 900 + 30 + 8 & & & & & & \\
=\text{diff. of } 6000+ & (1000+100)+ & 80 & + & (10+3) \\
\quad\text{and } 1000+ & 900 + (10+30) & & + & 8 \\
=\text{diff. of } 6000+ & 1100 + & 80 & + & 13 \\
\quad\text{and } 1000+ & 900 + & 40 & + & 8 \\
\qquad 5000+ & 200 + & 40 & + & 5 \\
\qquad 5245 & & & &
\end{array}$$

Ex. 2. Find by Continued Subtraction how often 5 can be taken from 17.

```
17
 5
12...............rem. after 5 is taken once ;
 5
 7...................................twice ;
 5
 2...................................thrice ;
```

and 5 being greater than the last remainder 2, cannot be taken from it any more.

Thus 5 can be taken 3 times from 17, and there remains the number 2 besides.

Ex. 3. What number must be added to 12 to produce 19?

Since by Art. 34, Prop. II,

the diff. between two numbers + the smaller = the greater

$\therefore$ the no. reqd. = the diff. between 19 and 12

= 7

Ex. 4. What number must be subtracted from 21 to give 15?

Since by Art. 34, Prop. III,

the subtrahend = the minuend − the remainder,

$\therefore$ the no. reqd. = 21 − 15

= 6

37. Proof. To test the correctness of the result obtained by Subtraction, add the subtrahend and the difference together; and if the sum equals the minuend, the result may be presumed to be correct.

The reason for this is clear from Art. 34, Prop. II.

Ex. Subtract 325 from 1203.

Operation	Proof
1203	325
325	878
878	1203

Examples III.

1. Subtract the smaller number from the greater in the following Examples:—

(1)		(2)		(3)		(4)		(5)	
	18		27		30		47		67
	12		18		20		36		49

(6)	123 45	(7)	678 6	(8)	101 91	(9)	151 88	(10)	100 99
(11)	202122 23240	(12)	484950 51520	(13)	657687 566778	(14)	192837 28416		
(15)	918273 827364	(16)	900009 98765	(17)	80604020 9070503	(18)	987654321 123456789		
		(19)	20004 10005	(20)	40009005 30008007				

2. (1) By how much is the number five thousand greater than five hundred ?

(2) By how much is the number five lacs greater than five thousand ?

(3) By how much is the number six crores greater than three lacs ?

3. What number must be added

to	78	to make	100 ;	to	890	1000 ;
...	75	...	120 ;	...	725	2009 ;
...	64	...	128 ;	...	99999	111111 ;
...	120	...	240 ;	...	88888	222222 ;
...	400	...	529 ;	...	50005	100000 ?

4. Find the difference between the sum and the difference of a million and a billion, and of a crore and a lac.

5. Find by continued Subtraction the number of times that

(1)	90	is contained in	900.	(6)	1234		9876.
(2)	99	...	999.	(7)	576	...	4000.
(3)	123	...	1234.	(8)	677		5000.
(4)	556	...	2556.	(9)	881		6400.
(5)	2578	...	8752.	(10)	500		2640.

6. Find the last remainder in each of the above Examples.

7. (1) What number together with the sum of 15 and 16 will be equal to the difference between 2 and 200 ?

(2) What number together with the difference between 5 and 15 will be equal to the sum of 6 and 16 ?

8. What number must be subtracted

from	100	to give	67 ;
...	210	...	78 ;
...	320	...	89 ;
...	1029	...	890 ;
...	542	...	120 ;
...	3589	...	1234 ?

SECTION IV. SIMPLE MULTIPLICATION.

38. In Multiplication, one of the given numbers namely, the multiplier *must always be an abstract number.* For it indicates the *number of times* that the multiplicand is to be repeated; and there would be no meaning in saying that a number is repeated a concrete number of times, such as *five rupees* times, or the like, as that would be absurd.

The multiplicand may be abstract or concrete, and the product will be abstract or concrete accordingly.

39. Defs. When the multiplicand is an abstract number, or a concrete number of one denomination only, the Multiplication is called **Simple Multiplication.**

When the product of two numbers is multiplied by a third number, then the product so obtained, by a fourth, and so on, the process is called **Continued Multiplication**, and the last product, the **Continued Product** of all the numbers.

40. Multiplication is a short method of adding any number repeated as a summand any number of times.

Thus 7×4 means 7 taken 4 times, *i. e.* $7+7+7+7$, and is equal to 28.

41. To multiply one abstract number by another is the same thing as to multiply the latter by the former.

Thus 3×4 is the same as 4×3.

$$
\begin{aligned}
\text{For } 3\times 4 &= 3+3+3+3 \\
&= \quad 1+1+1 \\
&\quad +1+1+1 \\
&\quad +1+1+1 \\
&\quad +1+1+1 \\
\textit{i. e.} \quad &= 4 \text{ horizontal rows of 3 ones each,} \\
\text{or} \quad &= 3 \text{ vertical rows of 4 ones each,} \\
\textit{i. e.} \quad &= 4 \text{ ones repeated 3 times} \\
&= 4\times 3.
\end{aligned}
$$

42. Defs. When an integer is the product of two or more other integers, it is called a **Composite Number**, and those other integers are called its **Factors.** When an integer has no such factors, it is called a **Prime Number.**

Thus, 6, 9, 10 are composite numbers being composed of the factors 2 and 3, 3 and 3, 2 and 5, respectively;

and 2, 5, 7 are prime numbers.

The continued product of any number repeated as a factor, is called a **Power** of that number, being called the **First, Second, Third,** &c. **Power**, according as the factor enters once, twice, thrice, &c. The second and the third power of a number are respectively called the **Square** and the **Cube** of that number.

The power of a number is denoted by that number having above it to its right a number indicating how often the former is repeated as a factor, and this last number is called the **Index** of the power.

Thus,

2 is called the	1st power of 2, and is denoted by	2^1 or 2 ;
2×2 or 4	............2nd..	2^2 ;
$2\times2\times2$ or 8	............3rd..	2^3 ;
&c.	&c.	&c.

43. When the multiplier is the product of a series of factors, the product of the multiplicand and the multiplier is obtained by multiplying the former by the first factor of the latter, then the product so obtained by the next factor, and so on to the last factor of the multiplier.

$$\begin{aligned} \text{Thus } 7\times6 &= 7+7 + 7+7 + 7+7 \\ &= (7+7)+(7+7)+(7+7) \\ &= (7\times2)+(7\times2)+(7\times2) \\ &= (7\times2)\times3, \text{ 2 and 3 being the factors of 6.} \end{aligned}$$

44. Prop. I. A number $\times$ zero or zero $\times$ a number $=$ zero. For a number taken *no* number of times, or *nothing* taken any number of times, will only give *nothing* as the product.

Prop II. A number is multiplied by 10, 100, &c., by affixing one, two, &c., ciphers to its right.

The reason is clear from Art. 25.

Prop. III. The product of any number and the sum of any two others is equal to the sum of the products of that number and each of the others.

Thus $5\times(3+4)=5\times3+5\times4$.

For $5\times(3+4)$ means 5 taken as often as there are units in 3 and 4 taken together, *i. e.*, as often as there are units in 3 and also as often as there are units in 4 ;

$$\begin{aligned} \therefore\quad 5\times(3+4) &= (5+5+5)+(5+5+5+5) \\ &= 5 \text{ taken 3 times} + 5 \text{ taken 4 times} \\ &= 5\times3+5\times4. \end{aligned}$$

45. The following Table called the Multiplication Table should be committed to memory.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100
11	22	33	44	55	66	77	88	99	110
12	24	36	48	60	72	84	96	108	120
13	26	39	52	65	78	91	104	117	130
14	28	42	56	70	84	98	112	126	140
15	30	45	60	75	90	105	120	135	150
16	32	48	64	80	96	112	128	144	160
17	34	51	68	85	102	119	136	153	170
18	36	54	72	90	108	126	144	162	180
19	38	57	76	95	114	133	152	171	190
20	40	60	80	100	120	140	160	180	200

	11	12	13	14	15	16	17	18	19	20
11	121	132	143	154	165	176	187	198	209	220
12		144	156	168	180	192	204	216	228	240
13			169	182	195	208	221	234	247	260
14				196	210	224	238	252	266	280
15					225	240	255	270	285	300
16						256	272	288	304	320
17							289	306	323	340
18								324	342	360
19									361	380
20										400

Note.—The Multiplication Table given above is in the form in which it is taught to boys in Bengal in their vernacular.

46. Rule for Multiplication. Write down the multiplier under the muliplicand, so that units may be under units, tens under tens, &c., and draw a line below.

Multiply every figure of the multiplicand beginning with the units' by the units' figure of the multiplier; place the units' figure of the first product under the figure of the multiplier that is being used; carry its tens' figure, if any, add it to the next product, and place the units' figure of that product thus increased in the next place to the left; and carrying the tens' figure, if any, add it to the next product, and proceed as before, down to the last prodct, and write that product, increased as above, in full.

Repeat the same process with the tens,' hundreds', &c. figures of the multiplier, taking care to place the first digit in each case under the figure of the multiplier, that is being used.

Add up the several lines of figures as in Addition, and the sum will be the product required.

Ex. 1. Multiply 5304 by 2039.

By the Rule we have,

```
    5304
    2039
   -----
   47736
  15912
 0000
10608
--------
10814856
```

Reason for the Rule. To multiply one number by another is to multiply the number of units, the number of tens, &c., contained in the former, first by the number of units, then by the number of tens, &c., in the latter, and then to add up these partial products; and this is done by multiplying every figure of the former by each figure of the latter, and adding up the several products, *regard being had to the local values of the figures.*

Thus in the example above, the required product will be obtained by taking 5304 first 9 times, then 30 times, and then 2000 times, and adding up the partial products thus obtained.

Now to obtain 9 times 5304, we take 9 times 4 units, thus getting 36 units, *i. e.*, 3 tens and 6 units, and we ∴ put 6 in the units' place; we then take 9 times 0 tens *i. e.*, 0 tens, which with the 3 tens in hand give 3 tens, and we ∴ put 3 in the tens' place; we next take 9 times 3 hundreds, thus getting 27 hundreds, *i. e.*, 2 thousands and 7 hundreds, and we ∴ put 7 in the hundreds' place; we lastly take 9 times 5 thousands, or 45 thousands, which with the 2 thousands in hand, give 47 thousands, and we ∴ put 7 and 4 in the thousands' and tens of thousands' places respectively. Thus we have 9 times 5304 = 47736.

Next, to obtain 30 times 5304, we take 3 times 5304 or 15912, which is obtained in the same way as 9 times 5304, and then multiply 15912 by 10, thus getting 159120 for the true partial product. (Art. 43.) The cipher at the end of this true partial product is omitted for convenience' sake, and the multiplication by 10 is indicated by removing the digits in the product of 5304 and 3 one step towards the left.

The next line consists of zeros, as it consists of the product of 5304 and 0 hundreds.

Lastly, to obtain 2000 times 5304, we take 2 times 5304 or 10608, and then multiply 10608 by 1000, thus getting 10608000 for the true partial product. We omit the 3 ciphers at the end for the sake of convenience, and indicate the multiplication by 1000 by removing the digits in the product of 5304 and 2 three places to the left.

Worked out at length, the process will stand thus :—

$$\begin{array}{r} 5304 \\ \underline{2039} \\ 47736 \\ 159120 \\ 000000 \\ \underline{10608000} \\ 10814856 \end{array}$$

Or more fully thus :—

Since $2039 = 2 \times 1000 + 0 \times 100 + 3 \times 10 + 9$,

$\therefore$ $5304 \times 2039 = 5304 \times 9 + 5304 \times 3 \times 10 + 5304 \times 0 \times 100 + 5304 \times 2 \times 1000$

$$\begin{aligned} \text{Now } 5304 \times 9 &= 9 \times 5000 + 9 \times 300 + 9 \times 0 + 9 \times 4 \\ &= 45000 + 2700 + 0 + 36 \\ &= 47736 \end{aligned}$$

$$\begin{aligned} \text{Similarly } 5304 \times 3 \times 10 &= 159120 \\ 5304 \times 0 \times 100 &= 000000 \\ 5304 \times 2 \times 1000 &= 10608000 \end{aligned}$$

$\therefore$ the required prod. $= 10814856$

Ex. 2. Find the continued product of 21, 22, and 23.

$$\begin{array}{r} 21 \\ \underline{22} \\ 42 \\ \underline{42} \\ 462 \\ \underline{23} \\ 1386 \\ \underline{924} \\ 10626 \end{array}$$

47. Additional Rules.

Rule I. When a cipher occurs in the midst of other figures in the multiplier, the line of zeros in the series of partial products may be omitted.

Rule II. When the multiplicand or multiplier or both contain ciphers at the end on the right, the product is obtained by first multiplying the numbers stripped of these ciphers, and then affixing to the right of the product thus obtained all the ciphers omitted.

Ex. Multiply 4300 by 2030

By the Rule we have

$$\begin{array}{r} 43 \\ \underline{203} \\ 129 \\ \underline{86} \\ 8729 \end{array}$$

8729 ; $\therefore$ the full product $= 8729000$.

Reason for the Rule. By Art 44, Prop. II,

$$4300=43\times 100, \text{ and } 2030=203\times 10;$$

$$\therefore 4300\times 2030=43\times 100\times 203\times 10=43\times 203\times 100\times 10$$
$$=43\times 203\times 1000 \quad =8729\times 1000$$
$$=8729000.$$

48. Multiplication by Factors.

Rule. When the multiplier is composed of simple factors, multiply the multiplicand by the first factor, then the first product by the second factor, and so on. The last product will be the one required.

The reason for this is clear from Art. 43.

Ex. Multiply 204 by 72.

Since $72=8\times 9$, we have by the Rule

$$\begin{array}{r} 204 \\ 8 \\ \hline 1632 \\ 9 \\ \hline 14688 \end{array} \;\ldots\ldots\; \text{the prod. reqd.}$$

49. Proof. The correctness of the result in Multiplication may be tested by the following method, called **"Casting out the Nines"**:—

From the sum of the digits of the multiplicand subtract 9 as often as it can be subtracted, and set down the last remainder. Do the same thing with the multiplier and the product, and set down the corresponding remainders. Then if the last remainder left after successively subtracting 9 from the product of the remainders corresponding to the multiplier and the multiplicand, be the same as the remainder corresponding to the product, the operation has, very probably, been correctly performed.

Thus, referring to Ex. 1 in Art. 46,
we have the sum of the digits of the multiplicand $=12$,
....multiplier $=14$,
....product $=33$,
and $12-9=3$, the last rem. corresponding to the multiplicand,
$14-9=5$,...multiplier,
and $33-9=24$,
$24-9=15$,
$15-9=6$,...product.
Now $3\times 5=15$,
and $15-9=6$,...

Hence we may say that the operation has been correctly performed.

The comparison of the remainders is usually exhibited thus :—

$$\begin{array}{ccc} & 6 & \\ 3 & \times & 5 \\ & 6 & \end{array}$$

The reason for this will be found in Art. 60.

Examples IV.

1. Multiply—

(1) 123 by 4, 5, 6, 8, and 9.
(2) 456 by 7, 8, 9, 10, and 11.
(3) 789 by 3, 6, 9, 12, and 15.
(4) 123456789 by 5, 10, 15, 20, and 25.
(5) 987654321 by 8, 12, 16, 20, and 24.
(6) 123789 by 456, 654, and 465.
(7) 123456 by 789, 987, and 798.
(8) 1002003 by 405, 504, and 450.
(9) 4050607 by 809, 908, and 890.
(10) 80901001 by 1020, 3040, and 5060.
(11) 1200240048 by 12036 and 63021.
(12) 987654321 by 123456789 and 13579.
(13) 12340000 by 8900, 6700, and 4500.
(14) 5678000 by 91011000 and 121300.
(15) 89101100 by 50600 and 60500.

2. Find by multiplication by factors the values of :—

(1) 123456 × 16.	(2) 567890 × 32.
(3) 567890 × 64.	(4) 987650 × 128.
(5) 69121518 × 81.	(6) 13579 × 240.

3. Find the continued product of

(1) The first nine integers.
(2) 2, 4, 6, 8, 10, and 12.
(3) 3, 6, 9, 12, and 15.
(4) 1, 7, 11, 13, and 19.

4. Find the values of the following :—

(1) $1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2$.

(2) $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3$.

(3) $25^2 \times 15^3$.

5. Multiply one crore by one lac, and fifteen crores by sixteen lacs.

6. Multiply the sum of 1 lac and 1 crore by their difference, and also by twice that difference.

7. Multiply the difference between a lac and a thousand by that between a million and a crore.

SECTION V. SIMPLE DIVISION.

50. In Art. 14, the quotient has been defined to mean the number of times that the divisor is contained in the dividend. It may also mean the magnitude of each part of the dividend when it is divided into the number of parts indicated by the divisor; and this, in fact, is its meaning, according to the sense in which Division is ordinarily understood. Thus, when in common parlance we say, "Divide 8 by 2" the result we want is the *magnitude of each part* of 8 when it is divided into 2 equal parts, and not the *number of times* that 8 contains 2. The *numerical value* of the quotient, however, is the same, whichever meaning is attached to it.

To shew this, let us take as an example $13 \div 4$.

Then, by our Definition, in Art. 14, $13 \div 4$ is 3 with 1 over, *i. e.*, 13 contains 4, 3 times, and there is 1 over ; or in other words, 13 is 3 times 4, together with 1.

But 3 times 4 is the same as 4 times 3 (Art. 41).

Therefore, 13 is 4 times 3 together with 1 ; or in other words, 13 contains 3, 4 times, and there is 1 over ; *i. e.*, 13 divided into 4 equal parts gives 3 as the magnitude of each part, and there is 1 remaining undivided.

Sometimes the quotient can have either meaning, and sometimes only one of the two, as will be seen in the next Article.

51. Prop. I. If the dividend be an abstract number, the divisor must also be an abstract number; and the quotient will be an abstract number, indicating either the number of times that the dividend contains the divisor, or the magnitude of each part of the dividend, when it is divided into the number of parts denoted by the divisor.

Prop. II. If the dividend be a concrete number, the divisor may be either abstract or concrete; and in the former case, the quotient will be a concrete number, indicating the magnitude of each part of the dividend when it is divided into the number of parts denoted by the divisor; and in the latter, it will be an abstract number denoting the number of times that the dividend contains the divisor.

The reason for this will be clear from the following considerations :—

Take as an example, the number *ten* divided by the number *two*.

In 10÷2, the quotient 5 can have either meaning : it may mean, either the number of times that 10 contains 2, or the magnitude of each part of 10 when it is divided into 2 equal parts.

In 10÷2 *rupees*, the operation would be unmeaning and absurd in either sense, as we cannot say that 2 *rupees* are contained in the abstract number 10 any number of times, nor can we divide the abstract number 10 into 2 *rupees* parts.

In 10 *rupees*÷2, the quotient 5 can only mean 5 *rupees*, *i. e.*, the magnitude of each part of the sum of 10 *rupees*, when it is divided into 2 equal parts. It cannot have the other meaning, for we cannot say that 10 *rupees* contain the abstract number 2 any number of times.

In 10 *rupees*÷2 *rupees*, the quotient 5 can only mean the abstract number 5, *i. e.*, the number of times that the sum of 2 *rupees* is contained in the amount 10 *rupees*. It cannot have the other meaning, for there can be no meaning in saying that 10 *rupees* are to be divided into 2 *rupees* parts.

52. Defs. The Division of abstract integers or concrete integers of one denomination only is called **Simple Division.**

In the Division of integers, where the divisor is not contained in the dividend an integral number of times exactly, the number which remains after the former is taken from the latter the greatest possible number of times, is called the **Remainder.**

Thus in 17÷5, since 17 contains 5, 3 times and no more, with 2 over, the result of the operation is the quotient 3 with the remainder 2.

53. Division is a short method of subtracting one number repeatedly from another, to find how often the former is contained in the latter.

Thus, to divide 17 by 5 is to repeat the following operation of Subtraction,—(1) $17-5=12$; (2) $12-5=7$; (3) $7-5=2$; whence we see that 17 contains 5, 3 times with 2 over.

54. Prop. I. Dividend = divisor × quotient + remainder.

Prop. II. The product divided by the multiplicand or the multiplier = the multiplier or the multiplicand.

These evidently follow from the Definitions.

Prop. III. A number is divided by 10, 100, &c., by striking off one, two, &c. figures from its right, the figures struck off constituting the remainder in each case.

Thus take any number 1357.

Then 1357 $= 1350 + 7 = 135 \times 10 + 7$,

and also $= 1300 + 57 = 13 \times 100 + 57$,

.... = &c. ... &c.

Hence $1357 \div 10$ gives the quotient 135 with the remainder 7,

$1357 \div 100$ ....13..........................57,

&c. &c. &c.

Prop. IV. The result of the division of the sum of any two numbers by a third is equal to the sum of the results of the division of those two numbers by the third number.

Thus taking any three numbers 8, 11 and 3,

$$\frac{8+11}{3} = \frac{8}{3} + \frac{11}{3}$$

For, $\frac{8+11}{3}$

or $\frac{19}{3}$ gives the quotient 6, with 1 remaining to be divided by 3.

Now $\frac{8}{3}$ ....2..........2..........................2..........................,

and $\frac{11}{3}$ ....3..........2.. ;

$\therefore \frac{8}{3} + \frac{11}{3}$..........................2 + 3..........2 + 2..,

i. e...........................5..........4.. ;

but $\frac{4}{3}$..........................1..........1.. ;

$\therefore \frac{8}{3} + \frac{11}{3}$..........................5 + 1..........1..,

i. e., the same result as $\frac{8+11}{3}$.

55. Rule for Division. Place the dividend and the divisor thus :—

Divisor) Dividend (

Mark off from the dividend on its left the least number of figures making a number not less than the divisor; ascertain by trial the number of times that the divisor is contained in it, by inquiring how often the figure in the highest place of the divisor is contained in the first figure, or, if that is too small, in the number composed of the first two figures, on the left of the partial dividend; place this number on the right of the dividend, as the first or the highest figure of the quotient; and subtract the product of this number multiplied by the divisor from the portion of the dividend marked off.

To the right of the remainder thus obtained, annex the next figure of the dividend, and if the number thus formed be not less than the divisor, repeat with it the same process that was performed with the part of the dividend first marked off, putting the number now obtained for the quotient, as the next figure of the quotient required. If the above mentioned number be less than the divisor, annex a cipher to the quotient, bring down the next figure of the dividend, and so on, until the number becomes not less than the divisor.

Repeat this process until there is no more figure of the dividend to be annexed to the remainder. The last remainder, if any, will be the remainder after Division.

Ex. Divide 969708 by 482.

By the Rule we have
```
482)969708(2011
    964
    ---
      570
      482
      ---
       888
       482
       ---
       406
```

Reason for the Rule.—To divide one number by another is to ascertain how many units of times not exceeding 9, how many tens of times not exceeding 9, how many hundreds of times not exceeding 9, &c., the divisor is contained in the dividend; or in other words, to ascertain successively the digits in the units', tens', hundreds', &c. places of the quotient beginning with the highest.

Thus in the Example above, the portion of the dividend first marked off, 969 is really 969 *thousands*, and it contains 482, 2 *thousands* of times, with 5 *thousands* over; accordingly we put 2 as the first figure or the figure in the *thousands'* place of the quotient.

Annexing now the next or the *hundreds'* figure, 7, of the dividend, to the difference between 969000 and 2000 times 482, *i. e.*, to 5000, we get 5700 or 57 *hundreds*, which contain 482 no *hundreds* of times; we therefore put 0 as the *hundreds'* figure of the quotient.

Bringing down the next or the *tens'* figure 0, of the dividend, we get 5700 or 570 *tens*, which contain 482, 1 *ten* of times with 88 *tens* over; we accordingly put 1 as the *tens'* figure of the quotient.

Lastly, annexing to the 88 tens or 880, the units' figure 8 of the dividend, we obtain 888 *units*, which contain 482, 1 *unit* of time with 406 over; and we accordingly put 1 in the *units'* place of the quotient.

Thus the complete quotient is 2011, and the final remainder, 406.

Worked out at length, the process will stand thus :—

$$\begin{array}{l} 482\,)\,969000+700+0+8\,(\,2000+0+10+1 \\ \quad\ \ \underline{964000} \\ \qquad\ \ 5000+700=5700 \\ \qquad\qquad\qquad\quad\ \ \underline{4820} \\ \qquad\qquad\qquad\qquad 880+8=888 \\ \qquad\qquad\qquad\qquad\qquad\quad\ \ \underline{482} \\ \qquad\qquad\qquad\qquad\qquad\quad\ \ 406 \end{array}$$

56. Short Division. The mode of operation in Art. 55 is called **Long Division.** When the divisor is a small number not exceeding 20, the operation may be shortened into what is called **Short Division,** by the following Rule :—

Rule. Place the divisor and the dividend thus:—

Divisor) Dividend.

Place the several figures of the quotient under the line drawn below the dividend, performing the several subtractions mentioned in the Rule in Art. 55 *mentally.*

Ex. Divide 205963 by 11.

By the Rule we have $11\,)\,\underline{205963}$

$\qquad\qquad\qquad\qquad 18723 - 10$ rem.

57. Division by Factors.

Rule. When the divisor is composed of factors which are small numbers, divide the dividend by the first factor, then the first quotient by the second factor, then the second quotient by the third factor, and so on, to the last factor. The last quotient will be the one required; and the true remainder will be obtained by multiplying each remainder by all the divisors preceding its own, and adding together these products and the first remainder.

Ex. Divide 123248 by 72.

Since $72=6\times12=6\times4\times3$, we have by the Rule

$$72\begin{cases}3 \\ 4 \\ 6\end{cases}\left|\begin{array}{l}\underline{123248} \\ \underline{41082-2} \\ \underline{10270-2} \\ \quad 1711-4\end{array}\right.$$

The quotient is 1711 and the remainder is $4\times4\times3+2\times3+2=56$.

The reason for the Rule may be shewn thus :—

$$\begin{aligned}123248&=3\times41082+2 \text{ (Art. 54, Prop. I.)}\\&=3\times(4\times10270+2)+2\\&=3\times4\times10270+3\times2+2 \text{ (Art. 44, Prop. III.)}\\&=3\times4\times(6\times1711+4)+3\times2+2\\&=3\times4\times6\times1711+3\times4\times4+3\times2+2\\&=72\times1711+3\times4\times4+3\times2+2.\end{aligned}$$

58. Additional Rules.

Rule I. To divide a number by 10, 100, &c., cut off 1, 2, &c. figures from the right of the dividend, and the resulting number will be the quotient, and the number composed of the figures struck off the remainder.

The reason for this is given in Art. 54, Prop. III.

Rule II. When the divisor ends in ciphers on its right, cut off these and an equal number of figures from the right of the dividend, and perform the division with these stripped off numbers. The quotient obtained will be the quotient required, and the true remainder will be obtained by annexing to the right of the remainder after division the figures of the dividend struck off.

Ex. Divide 126987 by 2300.

By the Rule we have 23,00) 1269,87 (55

$$\begin{array}{r}\underline{115}\\119\\\underline{115}\\487\end{array}$$

The *reason for the Rule* may be shewn thus :—

$$\begin{aligned}126987&=126900+87,\\&=100\times1269+87;\\\text{and }\ 1269&=23\times55+4;\\\therefore\ 126987&=100\times(23\times55+4)+87\\&=100\times23\times55+100\times4+87\\&=2300\times55+400+87\\&=2300\times55+487.\end{aligned}$$

59. Proof. The correctness of the result in Division may be tested thus :—

Multiply the divisor by the quotient, and add the remainder to the product. If the sum equals the dividend, the operation has been correctly performed.

The reason for this is clear from Art. 54, PROP. I.

The several partial products for this multiplication are already to be found in the operation of division, being the several subtrahends from below upward, in that operation.

Ex. Taking the Example in Art. 55, we have,

$$\begin{array}{rl}
482 & \text{......divisor.} \\
2011 & \text{......quotient.} \\
\hline
482 & \\
4820 & \\
9640 & \\
969302 & \\
406 & \text{......remainder.} \\
\hline
969708 & \text{......dividend.}
\end{array}$$

60. We may here give the reason for the Rule of "Casting out the Nines," given in Art. 49, as a mode of testing the accuracy of results in Multiplication. It depends upon the following Proposition :—

Any number divided by 9 leaves the same remainder as the sum of its digits so divided leaves.

To prove this, take any number 28075.

Then,

$$\begin{aligned}
28075 &= 20000+8000+70+5 \\
&= 2\times 10000+8\times 1000+7\times 10+5 \\
&= 2\times(9999+1)+8\times(999+1)+7\times(9+1)+5 \\
&= 2\times 9\times 1111+2+8\times 9\times 111+8+7\times 9\times 1+7+5 \\
&= 9\times(2\times 1111+8\times 111+7\times 1)+2+8+7+5\,;
\end{aligned}$$

$$\begin{aligned}
\therefore 28075\div 9 &= 9\times(2\times 1111+8\times 111+7\times 1)\div 9 \\
&\quad +(2+8+7+5)\div 9 \text{ (Art 54, Prop. IV.)} \\
&= 2\times 1111+8\times 111+7\times 1+(2+8+7+5)\div 9\,;
\end{aligned}$$

and $\therefore$ the remainder left after dividing 28075 by 9 is the same as the remainder left after dividing $2+8+7+5$ by 9.

Now, reverting to the Example in Art. 49, we have,

the multiplicand $5304 = 9\times 589+3$,

where $3=$ rem. left after dividing 5304 or $5+3+4$ by 9; and the multiplier $2039=9\times 226+5$,
where $5=$ rem. left after dividing 2039 or $2+3+9$ by 9;
$\therefore$ the product $=(9\times 589+3)\times(9\times 226+5)$

$$=9\times 226\times(9\times 589+3)+5\times(9\times 589+3)$$
$$=9\times 226\times(9\times 589+3)+9\times 589\times 5+3\times 5\ ;$$

the product $\div 9=\frac{9\times 226\times(9\times 589+3)}{9}+\frac{9\times 589\times 5}{9}+\frac{3\times 5}{9}$

$$=226\times(9\times 589+3)+589\times 5+\frac{3\times 5}{9};$$

i. e., if the product obtained is correct, the remainder left after dividing the product by 9, which is the same as the remainder left after dividing the sum of the digits in the product by 9, equals the remainder left after dividing 3×5 by 9.

If there is any error of 9, or if the digits in the product have their positions changed, this test will not enable us to detect the error in that case.

Examples V.

1. Divide—

(1) 1234 by 2, 3, 4, and 5.

(2) 3456 by 3, 4, 5, and 6.

(3) 5678 by 5, 6, 7, and 8.

(4) 78910 by 7, 8, 9, and 10.

(5) 123456789 by 5, 10, 15, 20, and 25.

(6) 987654321 by 8, 12, 16, 20, and 24.

(7) 123789 by 456, 654, and 465.

(8) 123456 by 789, 987, and 798.

(9) 1002003 by 405, 504, and 450.

(10) 4050607 by 809, 908, and 890.

(11) 80901001 by 1020, 3040, and 5060.

(12) 1200240048 by 12036 and 63021.

(13) 987654321012345 by 123456 and 123456789.

(14) 12340000 by 8900, 6700, and 4500.

(15) 1000000000000 by 1111, 11110, and 111100.

(16) 11111111111 by 1111, 11110, 9999, and 99990.

2. Divide by the method of Short Division, by factors if necessary—

(1) 3456 by 2, 3, 4, 5, and 6.

(2) 13579 by 4, 8, 12, and 16.

(3) 1000000 by 2, 3, 4, 5, 6, and 7.

(4) 11111111 by 8, 9, 10, 11, and 12.

(5) 2222222 by 13, 14, 15, 16, 17, and 18

(6) 33333333 by 19, 20, 21, 22, and 24.

(7) 87654321910 by 32, 33, 34, and 35.

(8) 24681012 by 42, 44, 48, 49, and 51.

3. Divide one crore and one by one lac and one, and then the product of the quotient and the divisor by one thousand and one.

4. Divide the sum of a crore and a million by their difference.

5. Divide the product and the sum of a million and a lac by the difference between a million and a lac and the quotient of a million divided by a lac respectively.

SECTION VI. MEASURES AND MULTIPLES.

61. Defs. **A Measure** or an **Aliquot Part** of an integer is an integer that is contained in the former an integral number of times exactly.

Hence unity is evidently a measure of every integer.

A **Common Measure** of two or more integers is an integer that is contained in each of them an integral number of times exactly.

The **Greatest Common Measure** of two or more integers is the greatest integer that is contained in each of them an integral number of times exactly. The initials G. C. M. are often used for the words *greatest common measure.*

62. Defs. A number exactly divisible by 2 is called an **Even** number.

A number not exactly divisible by 2 is called an **Odd** number.

Integers which have no common measure except unity are said to be **Prime** to each other.

A number is said to be resolved into its **Elementary Factors** when it is resolved into factors which are prime numbers.

63. Defs. A **Multiple** of an integer is an integer that contains the former an integral number of times exactly.

Hence every integer is a multiple of unity.

A **Common Multiple** of two or more integers is an integer that contains each of the former an integral number of times exactly.

The **Least Common Multiple** of two or more integers is the least integer that contains each of the former an integral number of times exactly. The initials L. C. M. are often used for the words *least common multiple.*

64. Prop I. A measure of a number is also a measure of any multiple of that number.

For, the measure is contained an integral number of times exactly in the number, and this again is contained an integral number of times in its multiple; hence the measure must be contained in the multiple an integral number of times exactly.

Prop. II. Every common measure of two numbers will measure their sum and their difference, and also any multiple of either.

Thus, 4 being a common measure of 20 and 28, (for $20=4\times 5$, and $28=4\times 7$),

$$28+20=7 \text{ times } 4+5 \text{ times } 4=(7+5) \text{ times } 4$$
$$=12 \text{ times } 4,$$

and $\therefore$ 4 measures $28+20$.

So, $28-20=7$ times $4-5$ times $4=(7-5)$ times 4

$$=2 \text{ times } 4,$$

and $\therefore$ 4 measures $28-20$.

And $\because$ 4 measure $28+20$ and $28-20$,

$\therefore$ by Prop. I, it will measure any multiple of either.

65. Rule for finding the Greatest Common Measure of Two Numbers. Divide the greater number by the less, then divide the divisor by the remainder, if any, and so on, repeating this process till there is no remainder. The last divisor will be the greatest common measure required.

Ex. Find the G. C. M. of 98 and 70.

By the Rule we have

```
70) 98 (1
    70
    28) 70 (2
        56
        14) 28 (2
            28
```

$\therefore$ 14 is the G. C. M. required.

Reason for the Rule.

(1) The number 14 is a common measure of 98 and 70.

For 14 measures 28,

$\therefore$ 14 measures 2×28, and $\therefore$ $2 \times 28 + 14$ or 70 (Art. 64),

and $\therefore$ 14 measures $70 + 28$ or 98.

Hence 14 is a common measure of 98 and 70.

(2) It is also their greatest common measure.

For every number that measures 70 and 98,

measures $98 - 70$ or 28 (Art. 64),

and $\therefore$ measures 2×28 and $\therefore$ $70 - 2 \times 28$ or 14.

Now $\because$ no number greater than 14 can measure 14,

$\therefore$ 70 and 98 can have no common measure greater than 14;

and as 14 is itself a common measure of 70 and 98,

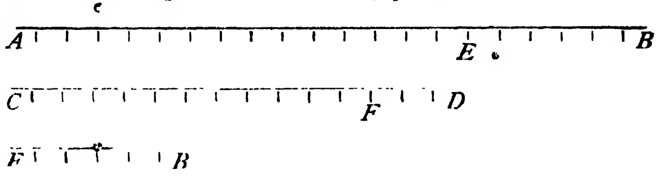
$\therefore$ it is their G. C. M.

The Reason for the Rule for finding the G. C. M. of two numbers may be further explained by the following illustration.

Suppose that we want to find the G. C. M. of the numbers 21 and 15.

Let *AB* and *CD* be two straight lines which are respectively 21 inches and 15 inches in length. Then these may be taken to represent the numbers 21 and 15; and the number representing the number of inches contained in the longest line that measures both *AB* and *CD* will evidently be the G. C. M. of 21 and 15.

To find this longest line we naturally proceed as follows.



We measure the longer line *AB* by the shorter *CD*, and find that it contains *CD* once with a remainder *EB*.

We next measure *CD* by *EB* and find that it contains *EB* twice with a remainder *FD*.

We next measure *EB* by *FD* and find that it contains *FD* twice exactly without a remainder, and we conclude that *FD* is the longest line that can measure both *AB* and *CD*.

For, FD measures EB and therefore CF or $2EB$ and therefore $CF+FD$ or CD ; and as FD measures CD and also EB it must measure $CD+EB$ or $AE+EB$ or AB.

Thus FD measures AB and CD.

Again every line that measures AB and CD must evidently measure $AB-CD$ or EB, and therefore $CD-2EB$ or $CD-CF$ or FD. But no line greater than FD can measure FD.

Therefore FD which is 3 inches in length is the greatest line that can measure AB and CD.

Thus 3 is the G. C. M. of 21 and 15.

Here we have proceeded by successively dividing the longer line by the shorter and then each divisor by the corresponding remainder until there is no remainder. And this is exactly what the Rule directs us to do.

66. Prop. I. Every common measure of two numbers, measures their greatest common measure.

This is clear form the latter part of Art. 65.

Prop. II. Every measure of the greatest common measure of two numbers measures each of those numbers.

67. Rule for finding the Greatest Common Measure of Three or More Numbers. Find the greatest common measure of the first two numbers ; then the greatest common measure of the greatest common measure so found and the third nnmber ; and so on. The last greatest common measure thus obtained will be the one required.

The *Reason for the Rule* is clear.

Call the G. C. M. of the first two dumbers the first G. C. M.

Then ∵ by Art. 66, every common measure of the first G. C. M. and the 3rd number is a common measure of the three numbers,

∴ the greatest common measure of the first G. C. M. and the 3rd number is a common measure of the three numbers.

Again, ∵ every common measure of the three numbers, being a common measure of the 1st and the 2nd, measures the G. C. M. of the 1st and the 2nd, and is therefore a common measure of the first G. C. M. and the 3rd number.

∴ every common measure of the three numbers is a measure of the greatest common measure of the first G. C. M. and the 3rd number.

Now, ∵ a number can have no measure greater than itself, ∴ no common measure of the three numbers can be greater than the greatest common measure of the first G. C. M. and the 3rd number.

And as the greatest common measure of the first G. C. M. and the 3rd number is itself a common measure of the three numbers,

∴ it is their G. C. M.

The same reasoning will hold good in the case of four or more numbers.

Ex. Find the G. C. M. of 32, 40, and 60.

By the Rule we have

```
32) 40 (1              8) 60 (7
    32                    56
     8) 32 (4              4) 8 (2
        32                    8
```

∴ 4 is the G. C. M. required.

68. When any two numbers are divided by their greatest common measure, the quotients are prime to each other.

For, if these quotients have a common factor, the numbers themselves will have that common factor over and above their greatest common measure, which is contrary to the supposition of its being their greatest common measure and is ∴ absurd.

69. Rule for finding the Least Common Multiple of Two Numbers. Divide the product of the numbers by their greatest common measure, and the quotient will be the least common multiple required.

Ex. Find the L. C. M. of 90 and 54

By the Rule we have

```
54) 90 (1                     54
    54                        90
    36) 54 (1          18 ) 4860 (270
        36                  36
        18) 36 (2           126
            36              126
```

∴ 270 is the L. C. M. required.

Reason for the Rule. The least common multiple of any two given numbers being the least number that is divisible by each of

them exactly, will contain as factors *all* the elementary factors of *each* of the given numbers *once*, and *only once*, and *no more* factors besides?

Now the product of any two given numbers being the continued product of *all* their factors, contains as factors all their elementary factors, every elementary factor *common* to these numbers being *twice* repeated, all *all other* elementary factors occurring *only once*. Hence if we divide the product of the numbers by the product of all these common elementary factors, *i. e.*, by the greatest common measure of the given numbers, we shall get their least common multiple.

Thus in the Example above,

$$90=5\times3\times3\times2, \qquad 54=3\times3\times3\times2\ ;$$

$\therefore 90\times54=5\times3\times3\times2\times3\times3\times3\times2$ where $3\times3\times2$ or 18 occurs twice.

Now $\because$ the L. C. M. of 90 and 54 must cotain the factors 5 and 3 and $3\times3\times2$ or 18 and nothing more besides, $\therefore$ the L. C. M. required $=90\times54\div18=270$.

70. Prop. Every common multiple of two numbers is a multiple of their least common multiple.

For, every common multiple of two numbers must contain as factors *all* the elementary factors of *each* of those numbers *at least once*, *i. e.*, must contain as factors *all* the factors of their L. C. M. at least once, and it may contain any other factors besides ;

and $\because$ a number is divisible by the continued product of any number of its factors,

$\therefore$ every common multiple of two numbers is a multiple of their L. C. M.

71. Rule for finding the Least Common Multiple of Three or More Numbers. Find the least common multiple of the first two numbers ; then the least common multiple of the least common multiple so found and the 3rd number ; and so on. The last least common multiple thus obtained will be the one required.

The *Reason For the Rule* is clear.

Call the L. C. M. of the first two numbers the first L. C. M.

Then $\because$ every common multiple of the first L. C. M. and the 3rd number is a common multiple of the three numbers, $\therefore$ the least common multiple of the first L. C. M. and the 3rd number is a common multiple of the three numbers.

Again, ∵ every common multiple of the three numbers, being a common multiple of the 1st and the 2nd, is a multiple of the L. C. M. of the 1st and the 2nd, and is therefore a common multiple of the first L. C. M. and the 3rd number,
∴ every common multiple of the three numbers is a multiple of the least common multiple of the first L. C. M. and the 3rd number.

Now ∵ a number can have no multiple smaller than itself, ∴ no common multiple of the three numbers can be less than the least common multiple of the first L. C. M. and the 3rd number.

And as the least common multiple of the first L. C. M. and the 3rd number is itself a common multiple of the three numbers, ∴ it is their L. C. M.

The same reasoning will hold good in the case of four or more numbers.

Ex. Find the L. C. M. of 32, 48, and 80

By the Rule we have

32) 48 (1	48	
32	32	
16) 32 (2	96	
32	144	
	16) 1536	
	96	∴ 96 is the L. C. M. of 32 and 48.
80) 96 (1	96	
80	80	
16) 80 (5	16) 7680	
80	480.	480 is the L. C. M. reqd.

72. Additional Rule. When the L. C. M. of several numbers is to be found, write down the numbers separated by commas in a line from left to right, leaving out such of them as are measures of any of the others.

Find by inspection the least integer greater than unity that measures any two or more of those remaining numbers; set it down on the left of the series, as a divisor; and put down in a line below each number the quotient after dividing it by the divisor, or the number itself when it is not exactly divisible by the divisor, thus obtaining a second series of numbers.

Treat this series exactly in the same way as the first; and proceed on, until a series is obtained in which the numbers are all prime to each other.

The continued product of all the divisors and all the numbers in the last series will be the L. C. M. required.

Ex. Find the L. C. M. of 9, 12, 15, 16, 20, 24, and 95.

By the Rule we have (leaving out 12 which measures 24),

$$\begin{array}{r|llllll}
 & 9, & 15, & 16, & 20, & 24, & 95 \\
\hline
2 & 9, & 15, & 8, & 10, & 12, & 95 \\
\hline
2 & 9, & 15, & 4, & 5, & 6, & 95 \\
\hline
3 & 9, & 15, & 2, & 5, & 3, & 95 \\
\hline
5 & 3, & 5, & 2, & 5, & 1, & 95 \\
\hline
 & 3, & 1, & 2, & 1, & 1, & 19
\end{array}$$

$\therefore$ the L. C. M. reqd. $=2\times2\times2\times3\times5\times3\times1\times2\times1\times1\times19=13680$.

Reason for the Rule. The L. C. M. of the given numbers must contain as factors *all* the elementary factors of *each* of the given numbers *once* and *only once*, and *no more* factors besides. Hence we can leave out of consideration such of the given numbers as are the measures of any of the others, for all the factors of these omitted numbers are contained in those that they measure. Now the continued product of the numbers that are retained, will contain as factors all their elementary factors, those that are not common to any two numbers occurring only once, and those that are common to two or more numbers being repeated as often as there are numbers which have those common factors. To prevent this repetition of common factors, we divide those numbers that have any common factors by such factors; and the continued product of these divisors or common factors each taken only once, and the ultimate quotients which are prime to each other, must be the L. C. M. required.

Thus in the Example above, 12 being a measure of 24 is left out.

The other numbers resolved into elementary factors stand thus :—

$9=3\times3$, $\quad 15=3\times5$, $\quad 16=2\times2\times2\times2$,
$20=2\times2\times5$, $\quad 24=2\times2\times2\times3$, $\quad 95=5\times19$.

Hence the required L. C. M. must contain as factors the two factors 3 and 3 of 9; the factor 5 only of 15, the other factor 3 having already been taken; the four factors 2, 2, 2, and 2 of 16; no factor of 20, the factors 2, 2, and 5 having already been taken; no factor of 24 for a similar reason; and the factor 19 only of 95, the other factor 5 having already been taken; and it must contain no other factors; *i. e.*,

the L. C. M. reqd. $=3\times3\times5\times2\times2\times2\times2\times19=13680$.

The object of the successive divisions by the common factors 2, 3, 5, is evidently to prevent the recurrence of those factors.

73. In the application of the rule in Art. 72, the following Propositions will be of use.

Prop. I. A number is divisible by 2 if its units' figure is 0 or is divisible by 2.

For, taking any number 370, we have $370 = 10 \times 37 = 2 \times 5 \times 37$ which is evidently divisible by 2.

Again, taking any number 2589, we have

$$2589 = 2580 + 9 = 10 \times 258 + 9,$$

whereof the first part 10×258 containing 10 as a factor is evidently divisible by 2;

so that the whole number is divisible by 2 if its second part, *viz.*, the digit in its units' place, is divisible by 2.

Prop. II. A number is divisible by 3 or 9 if the sum of its digits is divisible by 3 or 9.

For, taking any number 2867, we have

$$\begin{aligned} 2867 &= 2 \times 1000 + 8 \times 100 + 6 \times 10 + 7 \\ &= 2 \times (999 + 1) + 8 \times (99 + 1) + 6 \times (9 + 1) + 7 \\ &= 2 \times 999 + 8 \times 99 + 6 \times 9 + (2 + 8 + 6 + 7) \\ &= 2 \times 9 \times 111 + 8 \times 9 \times 11 + 6 \times 9 \times 1 + (2 + 8 + 6 + 7) \\ &= 9 \times (2 \times 111 + 8 \times 11 + 6 \times 1) + (2 + 8 + 6 + 7), \end{aligned}$$

whereof the first part containing 9 as a factor is evidently divisible by 3 or 9;

so that the whole number is divisible by 3 or 9 if its second part *viz.*, $2 + 8 + 6 + 7$, *i. e.*, the sum of its digits, is divisible by 3 or 9.

Prop. III. A number is divisible by 4 if its last two figures on the right are zeros or compose a number that is divisible by 4.

For, taking any number 28900, we have $28900 = 100 \times 289 = 4 \times 25 \times 289$ which is evidently divisible by 4.

Again, taking any number 78564, we have

$$78564 = 78500 + 64 = 100 \times 785 + 64,$$

whereof the first part is evidently divisible by 4;

so that the whole number is divisible by 4 if its second part, *i. e.*, the number composed of the digits in its tens' and units' places, is divisible by 4.

Prop. IV. A number is divisible by 5 if its units' figure is 0 or 5.

For, taking any number 3760, we have $3760 = 10 \times 376 = 2 \times 5 \times 376$ which is evidently divisible by 5.

Again, taking any number 4695, we have $4695 = 4690 + 5$, whereof the first part is evidently divisible by 5;

so that the whole number is divisible by 5 if its second part, *viz.*, its units' figure, is divisible by 5, *i. e.*, is 5 (that being the only digit that is so divisible).

Prop. V. A number is divisible by 6 if it is divisible by 2 and also by 3.

For $6=2\times 3$, and 2 and 3 are prime to each other.

Prop. VI. A number is divisible by 8 if its last three figures on the right are zeros or compose a number that is divisible by 8.

For, taking any number 687000, we have $687000=1000\times 687 =8\times 125\times 687$ which is evidently divisible by 8.

Again, taking any number 987682, we have

$$987682=987000+682,$$

whereof the first part 987000 is divisible by 8, from what has been just shewn;

so that the whole number is divisible by 8 if its second part, *i.e.*, the number composed of the last three digits on its right, is divisible by 8.

Prop. VII. A number is divisible by 11 if the sums of the figures in its odd and even places differ by 0 or a multiple of 11.

To prove this Proposition, it is necessary first of all to establish the following truth :—

An odd power of ten + unity or an even power of ten − unity is divisible by eleven.

Thus,

$$10^1+1=10+1=11,$$
$$10^2-1=100-1=99=11\times 9,$$
$$10^3+1=1000+1=999+1+1=990+9+2=11\times 90+11$$
$$=11\times(90+1),$$
$$10^4-1=10000-1=9999=11\times 909,$$
$$10^5+1=100000+1=99999+1+1=99990+9+2$$
$$=11\times 9090+11=11\times(9090+1),$$

&c. &c.;

which are all evidently divisible by 11.

Now taking any number 259768, we have

$$\begin{aligned}259768 &=200000+50000+9000+700+60+8\\ &=2\times 100000+5\times 10000+9\times 1000+7\times 100+6\times 10+8\\ &=2\times 10^5+5\times 10^4+9\times 10^3+7\times 10^2+6\times 10^1+8\\ &=2\times 10^5+2-2+5\times 10^4-5+5+9\times 10^3+9-9\\ &\quad+7\times 10^2-7+7+6\times 10^1+6-6+8 \text{ (Art. 34, Prop. IV.)}\\ &=2\times(10^5+1)+5\times(10^4-1)+9\times(10^3+1)\\ &\quad+7\times(10^2-1)+6\times(10+1)+8+7+5-6-9-2,\end{aligned}$$

whereof the numbers within brackets are divisible by 11, as has been already shewn ;

so that the whole number is divisible by 11 if $8+7+5-6-9-2$ is 0 or is divisible by 11, *i. e.*, if the sum of the figures in the odd places − the sum of the figures in the even places = 0 or a multiple of 11.

Prop. VIII. A number is divisible by the product of any factors prime to each other if it is divisible by each of them.

Thus 12 being equal to 3×4, and 3 being prime to 4, a number is divisible by 12 if it is divisible by 3 and 4.

But if the factors are not prime to each other, a number may be divisible by each of them, and yet it may not be divisible by their product.

Thus, taking two numbers 4 and 6 which are not prime to each other but have the factor 2 common to both, we see that the number 12 is divisible by 4 and also by 6, but is not divisible by 4×6. And the reason for this is obvious.

For 12 being $2\times2\times3$, contains each of the factors of 4 and 6 which are prime to each other, namely 2 and 3, once, and besides these, it contains the other factor 2 which is common to both 4 and 6 *only once*, so that though 12 is divisible by each of the two 4 and 6, *when taken alone*, it is not divisible by their *product* which is 4×6 or $2\times2\times2\times3$, that is, which contains their common factor 2 *twice* over and above their factors 2 and 3 which are prime to each other.

74. To ascertain whether any given number is a prime number :—

Rule. Write in a series all the integers from 1 to the given number.

After 2, go on marking every 2nd number with a dot ; then every dotted number is evidently divisible by 2, and the rest are not.

Next take the unmarked number 3, and go on marking every third number after it ; then those that are marked this time are divisible by 3, and the rest are not.

Repeat the same process with every succeeding unmarked number down to the middle number, and if the given number still remains unmarked, it is a prime number.

The *Reason for the Rule* is clear. For if the given number had any factor, it must have been marked when the dotting commenced from that factor.

This method also gives all the prime numbers less than the given number, and shews what numbers are the factors of those that are not prime.

This simple method we owe to Eratosthenes, a Greek mathematician, who called it his *sieve* for sifting out prime numbers. The advanced student will see that the method given above can be simplified still more.

Before applying this method which is tedious, Propositions I to VII of Art. 73 should be applied to every case.

Ex. Ascertain if 23 is a prime number.

By the Rule we have

$$1,\ 2,\ 3,\ \dot{4},\ 5,\ \ddot{6},\ 7,\ \dot{8},\ \dot{9},\ \ddot{10},\ 11,\ \ddot{12},\ 13,$$
$$\ddot{14},\ \ddot{15},\ \dot{16},\ 17,\ \ddot{18},\ 19,\ \ddot{20},\ \ddot{21},\ \ddot{22},\ 23.$$

Thus we see that 23 is a prime number.

The student should notice that the following are all the prime numbers below 100 :—

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

75. With the help of Arts. 73 and 74 and the Multiplication Table, we can resolve numbers into their elementary factors.

Ex. Resolve 234 into its elementary factors.

By Art. 73, 2 and 9 are factors of 234,

and $234 = 2 \times 117 = 2 \times 9 \times 13 = 2 \times 3 \times 3 \times 13$,

and these factors being all prime numbers, are elementary factors.

Examples VI.

1. Find the G. C. M. of

(1) 24 and 30 ; 32 and 48 ; 24 and 60 ; 32 and 80 ; 28 and 70 ; 38 and 95 ; 95 and 133 ; 115 and 161.

(2) 84 and 144 ; 133 and 228 ; 175 and 300 ; 144 and 208 ; 171 and 247 ; 153 and 289 ; 171 and 380 ; 323 and 361.

(3) 105 and 600 ; 56 and 128 ; 64 and 276 ; 121 and 209 ; 289 and 425 ; 289 and 867 ; 121 and 1331 ; 361 and 1805.

(4) 456 and 646 ; 442 and 646 ; 532 and 588 ; 516 and 817.

(5) 1164 and 1455 ; 948 and 1659 ; 876 and 2190 ; 852 and 2343.

(6) 1414 and 2222 ; 1605 and 2247 ; 1359 and 3624 ; 2172 and 2896.

(7) 2796 and 2868; 2358 and 3006; 1245 and 2265; 2589 and 9493.

(8) 916 and 2164; 2164 and 6332; 9498 and 21426.

(9) 123456789 and 987654321; 999999 and 9903300.

2. Find the G. C. M. of

(1) 24, 30, and 48.

(2) 108, 144, and 192.

(3) 150, 225, and 365.

(4) 144, 180, 216, and 324.

(5) 171, 228, 342. and 380.

(6) 198, 264, and 924.

3. Find the L. C. M. of

(1) 12 and 27; 14 and 42; 14 and 63; 18 and 42; 18 and 78; 23 and 95; 35 and 119; 49 and 133.

(2) 78 and 117; 78 and 143; 75 and 225; 119 and 289; 361 and 570; 276 and 368.

(3) 852 and 2343; 948 and 1659; 1414 and 2222.

(4) 285714 and 999999; 10353 and 14875.

(5) 123456789 and 987654321.

4. Find the L. C. M. of

(1) 1, 2, 3, 4, 5, 6, 7, 8, and 9.

(2) 1, 3, 5, 7, 9, 11, 13, and 15.

(3) 2, 4, 6, 8, 10, 12, 14, and 16.

(4) 1, 4, 9, 16, 25, 36, 49, 64, and 81.

(5) 5, 10, 15, 20, 25, 30, 35, and 40.

(6) 3, 4, 8, 6, 9, 18, and 30.

(7) 8, 20, 176, 165, 45, and 1233.

(8) 8, 1328, 166, 121, 22, and 40.

(9) 9, 16, 23, 30, 37, 44, and 51.

(10) 10, 18, 26, 34, 42, and 51.

(11) 9, 24, 75, 144, 180, and 508.

(12) 8, 18, 22, 176, 540, and 550.

5. Resolve into elementary factors.

(1) 92, 108, 111, 119, 171, and 189.

(2) 204, 216, 260, 289, and 304.

(3) 324, 361, 402, 403, and 404.

(4) 406, 407, 415, 512, and 618.

(5) 759, 781, 790, 828, and 945.

Miscellaneous Questions and Examples.

76. We have noticed in Art. 33 that when an expression contains brackets, it is to be understood that in simplifying it, we are to perform first of all the operations indicated within the brackets, and then the operations indicated outside. It now remains to notice a Rule of *convention* which relates to the order in which the operations in either case are to be performed. It is this :—

Rule. Within the brackets, perform first of all the operations of Division with the numbers immediately on the two sides of the sign $\div$; then perform all the operations of Multiplication with the resulting numbers on both sides of the sign $\times$; then add together all the resulting numbers that are preceded by the sign $-$, *i.e.*, are meant to be subtracted ; and then subtract the sum so obtained from the sum of the other resulting numbers.

Having thus cleared the brackets, observe the same rule outside the brackets.

Ex. Simplify

$$654\div 3\times 2-18\div(2\times 3)\times 4+(51\div 3-15)\times 8-(12\times 4-80\div 2)$$

By the Rule, the expression

$$\begin{aligned}
&=654\div 3\times 2-18\div 6\times 4+(17-15)\times 8-(48-40)\\
&=654\div 3\times 2-18\div 6\times 4+2\times 8-8\\
&=218\times 2-3\times 4+2\times 8-8\\
&=436-12+16-8\\
&=452-20\\
&=432.
\end{aligned}$$

We may here insert the following Propositions :-

Prop. I. The sum of the sum and difference of any two numbers = twice the greater number.

For, greater number + smaller number
+ greater number − smaller number
= twice the greater number.

Prop. II. A number is multiplied by 9, 99, 999 &c. by annexing to it as many ciphers as there are nines in the multiplier and subtracting it from the result.

For $9=10-1$, $99=100-1$, $999=1000-1$, &c.

Therefore any number $\times 9$ = the number $\times 10$ − the number ;
.... $\times 99$ = ... $\times 100$ − the number ;
.... $\times 999$ = ... $\times 1000$ − the number ;
&c. = &c.

Prop. III. A number is multiplied by 5, 5^2 or 25, 5^3 or 125 &c by affixing to it one, two, three &c. ciphers, and then dividing the result by 2, 2^2 or 4, 2^3 or 8, &c.

The reason for this is clear.

$$\begin{aligned} \text{For } 5 &= 10 \div 2 \\ 5^2 &= (10 \div 2)^2 = 100 \div 4 \\ 5^3 &= (10 \div 2)^3 = 1000 \div 8 \\ \text{\&c.} &= \text{\&c.} = \text{\&c.} \end{aligned}$$

$$\begin{aligned} \text{Therefore a number} \times 5 &= \text{the number} \times 10 \div 2 \\ \ldots\ldots\ldots\ldots \times 5^2 &= \ldots\ldots\ldots\ldots \times 100 \div 4 \\ \ldots\ldots\ldots\ldots \times 5^3 &= \ldots\ldots\ldots\ldots \times 1000 \div 8 \\ \text{\&c.} &= \text{\&c.} \end{aligned}$$

And a number is multiplied by 10, 100, 1000 &c. by affixing one, two, three &c. ciphers to it (Art. 44 Prop. II.)

77. In working out Examples given in language *other than the pure language of Arithmetic* we must first of all clearly understand the question, and then *translate* it into the language of Arithmetic, and state clearly, concisely, and *methodically* all the steps of the process. The statement of the steps of the process must consist of connected and complete intelligible *sentences*, differing from ordinary sentences only in this respect, that signs, symbols, and abbreviations are for the most part used here instead of words. These remarks will be illustrated by the following Examples.

Ex. 1. A person A has in a box 300 rupees in silver and 200 pieces of currency notes each worth 50 rupees. Of this, the sum of 240 rupees belongs to another person B. C and D each owe 100 rupees to A, and A owes rupees 150 and 250 to E and F respectively. If A has no other property and owes nothing more to any body, how much is he really worth in rupees?

Here, evidently

$$\begin{aligned} \text{the money } A \text{ is worth} &= \text{the amount in his possession} \\ &\quad + \text{the amount due to him} \\ &\quad - \text{the amount not his own} \\ &\quad - \text{also the amounts he owes to others} \\ &= 300 \text{ rupees} + 200 \times 50 \text{ rupees} \\ &\quad + 100 \text{ rupees} + 100 \text{ rupees} \\ &\quad - 240 \text{ rupees} \\ &\quad - (150 \text{ rupees} + 250 \text{ rupees}) \\ &= (300 + 10000 + 200 - 240 - 400) \text{ rupees} \\ &= (10500 - 640) \text{ rupees} \\ &= 9860 \text{ rupees.} \end{aligned}$$

Ex. 2. The sum of the ages of three boys A, B, and C, is 30 years ; the sum of the ages of A and B is 18 ; and that of the ages of A and C is 20. What is the age of each ?

Since age of A + age of B + age of C = 30,
and ... ... A + ... B = 18,
∴ by Subtraction age of C = 12.
Again ∵ age of A + age of C = 20,
and age of C = 12,
∴ age of A = 8.
And ∵ age of A + age of B = 18,
and age of A = 8,
∴ age of B = 10.

Hence the ages of A, B, and C are 8, 10, and 12 years respectively.

Ex. 3. In a field there are 20 rows of trees, each row containing 25 trees. What ts the total number of trees ?

Here, the total no. reqd. = the no. of trees in a row repeated as often as there are rows
= 25 repeated 20 times
= 25×20
= 500.

Ex. 4. A bookseller having sold several copies of a book at 3 rupees a copy, finds that he has realized 63 rupees by the sale. How many copies did he sell ?

Here the no. of copies reqd. = the no. of times that 3 must be repeated to produce 63
= the no. of times that 3 is contained in 63
= $63 \div 3$
= 21.

Ex. 5. A wheel 3 cubits in circumference is made to roll over a length of 18 cubits. How many revolutions does it make ? and what length will it roll over in making 8 revolutions ?

Here we must first of all know what length the wheel rolls over in one revolution.

Now one revolution is completed when the wheel, after having any particular point in its circumference in contact with the surface rolled over, comes to have the same point again in contact with that surface ; and this happens after successive points on the whole

circumference have come in contact, one after another, with points on the path rolled over.

Hence the length rolled over in one revolution = circumference of the wheel
= 3 cubits.

Therefore the no. of revolutions reqd. = the no. of times that 3 is contained in 18
$= 18 \div 3$
$= 6.$

And the length rolled over in 8 revolutions
$= 8 \times 3$ cubits
$= 24$ cubits.

Ex. 6. A man brought to the market a certain number of mangoes for sale. He sold 5 mangoes to A: to B, 3 more than to A; and to C, 2 less than to B; and found that he had sold half of the whole number. How many mangoes did he bring for sale?

Here,
the no. of mangoes sold to $A = 5$,
.... $B = 5 + 3 = 8$,
and .... $C = 8 - 2 = 6$;
$\therefore$ .... in all $= 5 + 8 + 6$
$= 19$,
which = half the no. brought;
$\therefore$ .... brought for sale $= 2 \times 19$
$= 38.$

Ex. 7. The first book of a poem contains 495 lines, and the second book, 900 lines. Find the largest number of lines that a page can contain, so that every page may contain the same number of lines, and each of the two books may consist of an integral number of pages.

Since each book is to consist of an integral number of pages, and every page is to contain the same number of lines, the number of lines in a page must be a common measure of 495 and 900, and $\therefore$ the largest number of lines that a page can contain
= the G. C. M. of 495 and 900
= 45, by the ordinary process for finding the G. C. M.

Ex. 8. Find the least number of rupees which can be divided equally among 2, 3, 4, 5 or 6 men.

Since the number required is to be divisible by 2, 3, 4, 5, and 6, it must be a common multiple of the numbers;

and $\because$ it is to be the least number that is so divisible,
$\therefore$ it must be their L. C. M. which by the ordinary Rule
$= 4 \times 5 \times 3 = 60.$

Examples VII.

1. What is Arithmetic? Define the terms Unity and Number, and distinguish between Abstract and Concrete Numbers, and between Integers and Fractions.

2. Define Notation and Numeration, and describe briefly the Common System of Notation.

3. Write down in figures, twenty millions thirty thousand and forty, and read the result according to the Indian Numeration Table.

4. What is meant by the Local Value of a digit? How is this value affected by affixing ciphers to the right of a number?

5. Three boys *A*, *B*, and *C*, have each a certain number of marbles. The total number of marbles is 18, and of these *A* and *B* together have 9, and *A* and *C* together 12. How many marbles has each?

6. A gentleman whose age is 33 has two sons. The age of the second son is 6 years, and the difference between the age of the father and the sum of the ages of the sons is 18 years. Find the age of the first son.

7. A woman brought a certain number of mangoes to the market for sale. She sold 10 mangoes to Ram; to Syam 5 more than to Ram; and to Hari as many as to Ram and Syam together; and she found that had she sold 5 more to each of the three, Ram, Syam, and Hari, there would have remained unsold just as many as she would have sold in that case. How many mangoes did she bring?

8. The number of boys in the first two classes of a school is 47. In the third class there are 15 boys less than in the first two classes taken together, and in the second class, there are 3 boys more than in the third. How many boys are there in each class?

II.

1. Define the terms Summand and Sum.

Add together:—Seventeen thousand three hundred and four; nineteen hundred and twenty; and eleven hundred and twelve.

2. In the Subtraction of integers, when any figure in the subtrahend is greater than the corresponding figure in the minuend, how do you proceed, and why in that way?

3. Shew that the sum of any two numbers added to their difference is equal to twice the greater number.

4. How do you account for the fact that the number ten forms the basis of our numerical computation?

5. The total number of pages in three books taken together is 129. The number of pages in the first two taken together is 62; and the third has 31 pages more than the second. How many pages are there in each?

6. A person had 25 rupees in his pocket. After the purchase of some books and stationery, he finds that that he has only five rupees left, and that if he had not purchased any books he would have had 20 rupees left. How much did he spend in the purchase of stationery?

7. Of three horses, the first is worth 175 rupees, the second, 25 rupees more than the first, and the third, 50 rupees more than the first and the second together. What is the price of the third horse?

8. Given that the beginning of the year 1799 of the Saka era corresponds with the year 1877 of Christian era; find the year of the Christian era when the Saka era commenced.

III.

1. Define the terms Product, Factor, and Power.

Shew that Multiplication is only a concise method of Addition.

2. Shew that the local value of every figure after the units' figure in any number expressed in the Common System of Notation, has some power of 10 as one of its factors.

3. Can you multiply one concrete number by another? If not, give your reason.

4. Shew that in Multiplication, the order of the factors is immaterial, so far as the numerical value of the product is concerned.

5. In a gurden there are 15 rows of trees, in each row there are 18 trees, and in each tree, 60 fruits. How many fruits are there in all?

6. If the price of a single copy of a book is 5 rupees, what is the price of 64 copies, and what is the price of 6 times 64 copies?

7. If you can have 9 mangoes for 1 rupee, how many of the same quality can you have for 16 rupees?

8. There are 4 pice in an anna, 16 annas in a rupee, and 16 rupees in a gold mohur. How many pice are there in one rupee? and how many in one gold mohur?

IV.

1. Define the terms Dividend and Divisor. What are the two different meanings which the Quotient can have?

2. Can you divide one concrete number by another? If you can, what is the meaning of the quotient? Can you divide an abstract number by a concrete number?

3. Shew that

dividend—remainder = divisor × quotient.

4. In Division by factors, how do you get the true remainder?

5. How many revolutions will the wheel of a carriage, 13 feet in circumference, make, in going over one-fourth part of a mile, there being 1760 yards in a mile, and 3 feet in a yard?

6. If 55 copies of a book can be had for 275 rupees, what is the price of a copy?

7. If 9 mangoes can be had for 1 rupee, what is the price of 117 mangoes?

8. If 192 mangoes can be had for 16 rupees, how many can be had for a rupee, and how many for 12 rupees?

V.

1. What do you mean by the Measure of a number, and what by the Greatest Common Measure of two or more numbers?

2. Shew that when two numbers are divided by their G. C. M., the quotients are prime to each other.

3. Shew that the number of measures of any number must be limited, but the number of its multiples is unlimited.

4. Shew that when the L. C. M. of two numbers is divided by each of those numbers, the quotients are prime to each other.

5. A gentleman has 24 rupees in his pocket, and he wishes to distribute the sum amongst the poor, in such a manner that each man shall receive the same number of rupees. In how many ways can he make the distribution?

6. *A* offers to distribute 36 pice amongst a number of beggars in such a manner that each shall receive the same number of pice; and *B* offers to distribute 24 pice amongst the same number in the same way. What is the largest number of beggars amongst whom the distribution can be made, and how many pice will each receive?

7. What is the least number of rupees that can be sorted in groups of 3, 4, 5, or 6 each?

8. Find the least number that is exactly divisible by the first nine odd numbers.

VI.

1. What is an Expression? and what is an Equation?

2. In simplifying an expression, what is the order in which you must proceed?

3. Simplify

$$68 \div (3 \times 4 + 5) + 16 \times (20 \div 2 \times 3 - 30) - (4 - 8 \div 4).$$

4. Find the value of

$$2 \times 3 \times 4 - 5 \times (21 - 4 \times 5) + 5 - 6 \div (32 - 3 \times 10).$$

5. Find the difference between

$$6 \times (7 + 8) - 9 + 10 \div (11 - 2 \times 3) \times 12,$$

and $7 \times (8 + 9) - 10 + 11 \div (11 - 2 \times 5) \times 13.$

6. Simplify

$$10 \times 12 - (13 \times 3 - 19 \times 2) \div (3 \times 7 - 4 \times 5) + 60.$$

7. Divide $22 \div 2 - (2 \times 3 + 5) + 656 - (4 \times 8 - 16 \div 2)$

by $20 \div 2 - (2 \times 5 - 5) + 56 - (64 \div 2 + 3).$

8. Find the value of

$$(3 - 2)^4 + (4 \times 2 - 18 \div 3)^4 - (66 \div 11 - 10 \div 5)^2.$$

VII.

1. Shew that the local value of every significant digit after the units' figure in any number is a multiple of some power of ten.

2. Shew that a number is multiplied by any power of ten by affixing to its right a number of ciphers equal to the index of the power.

3. A woman brought a certain number of oranges to the market for sale. She sold 20 oranges to A; to B, 15 more than to A; and to C, 22 less than the number sold to A and B together; and she found that had she sold 10 more to each of the three persons A, B, and C, there would have remained only 3 oranges unsold. How many oranges were there at first?

4. A boy read 30 pages of a book in one week; in the next week he read 5 pages more than in the first; in the third week, 6 pages more than in the second; and in the fourth week as many pages as in the first two weeks together; and he found that had he read 5 pages less every week, he would have gone through exactly one-third of the book. How many pages were there in the book?

5. A horse and a carriage are together worth 1860 rupees, and the carriage is worth five times as much as the horse. What is the price of each?

6. Two horses are together worth 850 rupees, and one of them is worth 250 rupees more than the other. What is the price of each horse?

7. A boy spends 15 rupees in the purchase of books and fruits, the price of the books being four times as much as that of the fruits. How much are the books worth?

8. A gentleman divides 1500 rupees amongst his 3 sons and 4 daughters, giving to each son twice as much as to each daughter. How much does each son get?

VIII.

1. How can you ascertain by inspection whether a number is divisible by 4?

2. Shew that a number is divisible by 9 if the sum of its digits is divisible by 9.

3. A man dies leaving 5 sons and 2 daughters. His property is worth 36192 rupees. What is the value of the property inherited by each son, supposing a son's share to be twice as much as that of a daughter?

4. Given that the quotient is 3025 and the dividend 36300, find the divisor.

5. What number multiplied by 627 will produce 11913? and what number added to the former will produce the latter?

6. A father at the age of 60 is twice as old as his eldest son, and four times as old as the youngest. Find the difference between the ages of these sons.

7. In a certain town, it is found that the death rate is 3 per cent. and the birth rate 5 per cent. of the population at the beginning of the year. Supposing the population at the beginning of the year to be 20,000 what will be the population at the end of the same?

8. The product of two numbers is 864, and their L. C. M. is 72. Find their G. C. M.

CHAPTER II.

THE FUNDAMENTAL OPERATIONS WITH ABSTRACT FRACTIONS.

INTRODUCTION.

78. Besides *Integers* or *multiples* of unity which we considered in the preceding Chapter, we have frequently to deal with *Fractions* or *parts* of unity. Thus, if we have to divide 6 units into 4 equal parts, we get as the result the quotient 1, and the remainder 2, which means that 6 units divided into 4 equal parts give 1 unit for each part, and their remain besides 2 units which cannot be divided into 4 equal parts in integers. To complete the division, we must *break* these two units each into *two equal parts or halves*, thus getting 4 halves, which divided into 4 equal parts give a *half* for each part : so that the complete quotient is *one and a half*; *i.e.*, 6 units divided into 4 equal parts give a unit and a half for each part.

The word **Fraction** (from the Latin *frangere*, to break) literally means a *broken part*.

79. By *multiplying* the *primary* unit *one* by 1, 2, 3, 4, &c., we obtain an unlimited series of numbers *viz.*, 1, 2, 3, 4, &c., which contains all possible integers, and no other numbers. If we *divide* the *primary* unit *one* into 2, 3. 4, &c., equal parts, we obtain an unlimited series of *secondary* units or parts of the primary unit, *viz.*, *one-half*, *one-third*, *one-fourth*, &c., by multiplying each of which by 1, 2, 3, 4, &c., we obtain an unlimited number of unlimited series, *viz.*,

One-half, two-halves, three-halves, four-halves, &c. ;
One-third, two-thirds, three-thirds, four-thirds, &c. ;
One-fourth, two-fourths, three-fourths, four-fourths, &c. ;
&c., &c., &c. ;

which contain all possible numbers, whether integral or fractional. Thus, the second term of the first series, *two-halves*, is really the integer 1 ; so its fourth term *four-halves* is really the integer 2 ; and so on ; and that these series give all possible fractional numbers or parts of unity, and collections of parts of unity not composing entire units, is obvious.

Again, as *Integers* are expressed in the Common System of Notation in an *ascending* scale of *tens*, *hundreds &c.*, so for *Fractions*, we may, by dividing the primary unit *one* into 10, 100, &c., equal parts, and thus getting a series of *secondary* or fractional units, *viz.*, *one-tenth*, *one-hundredth*, *&c.*, obtain by the repetition of

these last a *descending* scale of *tenths*, *hundredths*, *&c*. Here as we divide the primary unit into 10, 100, &c, equal parts to obtain our secondary units, it is not evident that in this mode, all possible fractions can be expressed. Whether every possible fraction can be really expressed in this mode, will be considered in Sections VII and XII of this Chapter.

Def. Fractions of the former class, that is, fractions consisting of parts of unity obtained by dividing unity into *any* number of equal parts, are called **Vulgar Fractions** or *ordinary* fractions, to distinguish them from fractions of the latter class, that is, fractions consisting of parts of unity obtained by dividing unity into *ten*, *hundred*, *&c.*, equal parts, which are called **Decimal Fractions** or **Decimals**, from their having *ten* (which is *decem* in Latin) for their basis.

80. Each of the above systems of fractions has its peculiar advantages and disadvantages, and we shall consider each in a separate Division of this Chapter.

DIVISION I. VULGAR FRACTIONS.

SECTION I. NOTATION AND NUMERATION OF FRACTIONS.

81. We have seen that a fraction is an integral number of parts of unity, which parts result from the division of unity by some integer. Thus the fraction *three-fourths* is one in which 3 parts of unity are taken, such parts resulting from the division of unity by 4.

When the number of parts composing a fraction is a multiple of the number of parts into which unity is divided, the fraction, though a fraction *in form*, is *really* an integer. For the fraction really consists of *all* the parts into which unity is divided, *i. e.*, of the *entire* unit itself, taken a certain number of times.

Thus, the fraction *six-thirds* means *six* times the *third* part of unity, *i. e.*, *twice three* times the *third* part of unity, *i. e.*, *two* units.

When the number of parts composing a fraction is greater than the number of parts into which unity is divided, any portion of the former which is equal to, or is a multiple of, the latter, may be separated from the rest, and expressed in the form of an integer. Thus, the fraction *seven-thirds* may be separated into two portions, *six-thirds* and *one-third*, whereof the former is equal to the integer *two*, and thus the whole fraction is equal to *two* and *one-third.*

Def. In any fraction the *number* of parts into which unity is *divided* is called the **Denominator** of the fraction, and the *number* of parts *taken* is called the **Numerator**; and both these are called the **Terms** of the fraction.

The propriety of these names is evident. The former is called the *denominator*, because it shews the *denomination* of the secondary units or parts of which the fraction is composed; and the latter is called the *numerator*, because it shews the *number* of such parts or secondary units intended to be taken.

A fraction is *named* by naming the numerator or the *number* of parts of unity taken, and after it the *ordinal* of the denominator or the *number* by which unity is divided to give one of such parts.

Thus where unity is divided into *seven* equal parts, and *three* of such parts are taken, the fraction is named *three-sevenths*

82. A fraction is *represented* by writing the numerator and the denominator, which are integers, according to the Common System of Notation, the former above the latter, with a line between.

Thus, the fraction *three-fifths* is written $\frac{3}{5}$;

.........*seven-sixteenths*.............$\frac{7}{16}$;

&c. &c. &c.

Hence the secondary or fractional units will be written $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c.

Def. The fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. are called the **Reciprocals** of 2, 3, 4, 5, &c.

83. This mode of Notation is obviously *complete*. For as in the Common System of Notation we can express every integral numerator and denominator, we can express every fraction, whatever its numerator and denominator may be, in this mode.

It is also a *convenient* mode. For, as in naming a fraction, such as *three-fourths*, we *name* first the numerator, and then the denominator, so in our Notation, we *write* in the same order, first the numerator, and then the denominator.

84. It now remains to be seen whether this Notation of fractions is *consistent* with our *previous conventions*.

For, the symbol $\frac{\text{numerator}}{\text{denominator}}$ by which we have expressed fractions, not being a new symbol, but being one which *has already been used* in Art. 14 to indicate the division of the upper number by the lower, we must shew that a fraction also denotes the division of the numerator by the denominator, in order to establish the *consistency* of our Notation, and to be able to use it without confusion, such as would result from the expression of *really different things* by the *same symbol* in the same science.

Now to shew that a fraction indicates the division of the numerator by the denominator, or in other words, is equal to the quotient arising from the division of the numerator by the denominator, take any fraction *four-fifths*.

This fraction indicates that unity is divided into 5 equal parts of which 4 parts are taken; *i. e.*, it denotes 4 *times the fifth part of* 1 *unit*.

But when 4 units are divided into 5 equal parts, then also, each part is equal to 4 *times the fifth part of* 1 *unit*.

Thus the fraction *four-fifths* means the same thing as 4 units divided into 5 equal parts. And so for any other fraction.

Hence a fraction denotes *the magnitude of each part* of the numerator when it is divided into the number of equal parts

indicated by the denominator. And this is one of the meanings attached to the term quotient. (Art. 50.)

Next let us see how far a fraction has the other meaning of the term, *i. e.*, indicates the *number of times* that the numerator contains the denominator.

Taking the same fraction *four-fifths*, we see that 5, being 5 times as large as 1, is contained in 1 *to the extent only of its fifth part*, and is contained in 4 *to the extent of 4 times its fifth part*, or in other words *to the extent of its four-fifths*. Now, as when one number is contained in another to the extent of its *double*, *triple*, *&c.*, we say that it is contained in that other *two times*, *three times*, *&c.*, so when one number is contained in another to the extent of its *half*, *third*, *&c.*, part, we may by a *stretch of language* say, that it is contained in that other *one-half*, *one-third*, *&c.*, *part of a time*. And in this sense, as 5 is contained in 4 to the extent of its *four-fifths*, we can say that 5 is contained in 4 *four-fifths of a time*.

Thus the fraction *four-fifths* represents the *number of times* or rather, the *number of parts of a time*, that 5 is contained in 4. And so for any other fraction.

Hence a fraction also denotes *the number of times* that the denominator is contained in the numerator.

This establishes the consistency of our Notation.

The above considerations enable us to express *numerically* the *complete* quotient arising from the Division of integers, in cases in which the dividend is not exactly divisible by the divisor.

Thus, take as an example $19 \div 3$.

The result of the division in this case, expressed in integers, is the quotient 6 and the remainder 1, which means that one part of 19, namely, 18, contains 3, 6 times, and the other part 1 not containing 3 any integral number of times is represented as a part of the dividend that *remains* to be divided by 3 This part, 1, however contains 3 *one-third part of a time*; and therefore by a *stretch of language* we can say that 3 is contained in the entire dividend 19, 6 times together with the *third* part of a time, or shortly, $6+\frac{1}{3}$ times; so that $6+\frac{1}{3}$ (written also $6\frac{1}{3}$) is the *complete* quotient arising from the division.

It may be observed that the same *symbolical* expression for the quotient may be deduced without reference to the Notation of fractions. Thus :—

$$\tfrac{19}{3}=\tfrac{18+1}{3}=\tfrac{18}{3}+\tfrac{1}{3}=6+\tfrac{1}{3}.$$

85. Defs. A fraction of which no part is expressed in an integral form, and whose numerator and denominator are integers, is called a **Simple Fraction**.

Thus, $\frac{2}{5}$, $\frac{7}{8}$, $\frac{3}{10}$ are simple fractions.

A fraction whose numerator is less than the denominator, is called a **Proper Fraction.**

Thus, $\frac{1}{6}$, $\frac{3}{7}$, $\frac{2}{9}$ are proper fractions.

A fraction whose numerator is not less than the denominator, is called an **Improper Fraction.**

Thus, $\frac{7}{5}$, $\frac{8}{8}$, $\frac{13}{5}$ are improper fractions.

A number expressed partly in an integral and partly in a fractional form is called a **Mixed Number.**

Thus, $1\frac{1}{3}$, $3\frac{4}{5}$, $7\frac{2}{3}$ are mixed numbers.

86. Prop. I. Any integer may be represented in the form of a fraction by making that integer the numerator, and unity the denominator or by making the product of that integer and any other integer the numerator, and that other integer the denominator.

Thus, 5 may be represented as $\frac{5}{1}$; or $\frac{5\times2}{2}$, *i.e.*, $\frac{10}{2}$; or $\frac{5\times3}{3}$, *i. e.*, $\frac{15}{3}$; &c.

For $\frac{5}{1}$ means either that 5 is divided into 1 part, *i. e.*, kept entire; or that unity is divided into 1 part or kept entire, and 5 such parts are taken; *i. e.*, in either sense it means 5 units: $\therefore \frac{5}{1} = 5$.

So $\frac{10}{2}$ means either that 10 is divided into two equal parts and one of such parts is taken; or, that unity is divided into 2 equal parts or halves, and 10 such parts are taken; *i. e.*, in either sense it means 5 units: $\therefore \frac{10}{2} = 5$. Similarly $\frac{15}{3} = 5$. And so on.

Prop. II. Every fraction whose numerator is a multiple of the denominator, is really an integer equal to the quotient arising from the division of the numerator by the denominator.

Thus, $\frac{10}{5} = 2$.

87. Besides the kinds of fractions defined in Art. 85, there are two other kinds which are defined below.

Defs. A Compound Fraction is a fraction of a fraction.

Thus, $\frac{2}{3}$ of $\frac{3}{4}$ is a compound fraction.

A Complex Fraction is a fraction having a mixed number or a fraction for its numerator or denominator or both.

Thus, $\frac{2\frac{1}{2}}{3}$, $\frac{4}{5\frac{6}{7}}$, $\frac{\frac{3}{8}}{6}$, $\frac{8}{\frac{3}{4}}$, $\frac{2\frac{1}{2}}{3\frac{3}{8}}$, $\frac{\frac{5}{8}}{\frac{3}{16}}$, are complex fractions.

The meanings of these two forms of fractions are not evident from the considerations in Arts 78 to 81. We must therefore *interpret* these forms, *i. e.*, see what they mean.

1st. *Compound Fractions.* Take as an example $\frac{3}{4}$ of $\frac{4}{5}$.

Then ∵ to obtain $\frac{3}{4}$, or $\frac{3}{4}$ of 1, we divide 1 into 4 equal parts, and take 3 of such parts, ∴ to obtain $\frac{3}{4}$ of $\frac{4}{5}$, we must *divide* $\frac{4}{5}$, *or the secondary units that compose* $\frac{4}{5}$ *into* 4 *equal parts, and take* 3 *of such parts.* And this is the meaning of $\frac{3}{4}$ of $\frac{4}{5}$.

How this division of the secondary units into parts and the multiplication of those parts are to be effected, in other words, how the value of a compound fraction is to be expressed in the form of a simple fraction, will be shewn in Art. 92.

2nd. *Complex Fractions.* Take as an example $\frac{2\frac{3}{4}}{5\frac{2}{5}}$.

Then ∵ $\frac{\text{one number}}{\text{another number}}$ means the former divided by the latter,

∴ $\frac{2\frac{3}{4}}{5\frac{2}{5}}$ will mean $2\frac{3}{4}$ *divided by* $5\frac{3}{5}$, *i. e., the secondary units of which the former is composed divided by those of which the latter is composed.*

How this division is to be effected, in other words, how the value of a complex fraction is to be expressed in the form of a simple fraction, will be shewn in Art. 93.

Examples VIII.

1. Express as fractions—

(1) The integer 5 having 2, 3, and 4 for the denominator.
(2) ... ... 9 ... 3, 4, ... 5 ....
(3) ... ... 7 ... 6, 7, ... 8 ....
(4) ... ... 8 ... 3, 5, ... 7 ....
(5) ... ... 16 ... 11, 12, ... 13 ....
(6) ... ... 25 ... 4, 8, ... 16 ....

2. Convert the following fractions into their equivalent integers :—

(1) $\frac{16}{8}$. (2) $\frac{9}{3}$. (3) $\frac{27}{9}$. (4) $\frac{28}{7}$. (5) $\frac{38}{19}$. (6) $\frac{121}{11}$.
(7) $\frac{256}{8}$. (8) $\frac{171}{9}$. (9) $\frac{324}{9}$. (10) $\frac{1728}{12}$. (11) $\frac{360}{60}$. (12) $\frac{210}{70}$.

SECTION II. TRANSFORMATION OF FRACTIONS.

88. We can *transform* one fraction into another, *i. e*, we can change the *form* of a fraction without altering its value. In effecting this, the Propositions in the following Article will be of use.

89. Prop. I. The value of a fraction is not altered if we multiply or divide both the numerator and the denominator by the same number.

Thus take any fraction $\frac{4}{6}$. Here unity is divided into 6 equal parts and 4 of such parts are taken.

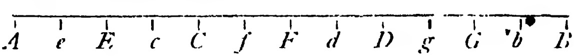
Multiplying both its terms by 2, we have $\frac{4\times2}{6\times2}=\frac{8}{12}$, which means that unity is divided into 12 equal parts whereof 8 are taken. Now although each part in the latter case being a *twelfth* part of unity is only *half* as large as each part in the former case, namely a *sixth*, yet as in the latter case we take 8 parts, which are *twice* as many as the parts taken in the former case, in fact we take just as much of unity now as we took before.

Hence $\frac{4}{6}=\frac{8}{12}$.

Again, dividing both its terms by 2, we have $\frac{4\div2}{6\div2}=\frac{2}{3}$, which means that unity is divided into 3 equal parts whereof 2 are taken. Now although here each part being *one-third* is *twice* as large as *one-sixth*, yet as we now take only 2 parts, which are *half* as many as the parts taken before, in fact we take just as much of unity as we took before.

Hence $\frac{4}{6}=\frac{2}{3}$.

This may be illustrated thus :—



Let the line AB represent unity.

Let AB be divided into 6 equal parts AE, EC, CF, FD, DG, and GB; then 4 of these parts taken together, *i. e.*, $AE+EC+CF+FD$ make up AD.

Thus $\frac{4}{6}$ of $AB=AD$.

Again, let AB be divided into 12 equal parts by bisecting, or dividing into 2 equal parts, each of the above 6 parts, so that we have Ae, eE, Ec, cC, Cf, fF, Fd, dD, Dg, gG, Gb, and bB for the 12 parts; then 8 of these parts taken together, *i. e.*, $Ae+eE+Ec+cC+Cf+fF+Fd+dD$ make up AD.

Thus $\frac{8}{12}$ of $AB=AD=\frac{4}{6}$ of AB.

Lastly, let AB be divided into 3 equal parts by taking

$AE+EC$, $CF+FD$, and $DG+GB$, to be the parts; then 2 of these parts together, *i. e.*, $(AE+EC)+(CF+FD)$ make up AD.

Thus $\frac{2}{3}$ of $AB=AD=\frac{4}{6}$ of AB.

Hence, *if the numerator and the denominator contain any common factor, it may be struck out without altering the value of the fraction.*

For, to strike out a factor of any number is to divide the number by that factor.

Thus $\frac{4}{6}=\frac{2\times2}{3\times2}=\frac{2\times2\div2}{3\times2\div2}=\frac{2}{3}$.

Prop. II. A fraction is multiplied or divided by an integer by multiplying its numerator or denominator by that integer.

Thus, let it be required to multiply $\frac{4}{8}$ by 3.

Since $\frac{4}{8}\times3$ means that the secondary units or parts of which the fraction $\frac{4}{8}$ is composed are to be multiplied by 3, the product will be obtained by multiplying the numerator 4, which denotes the number of parts composing the fraction, by 3, and keeping the denominator unchanged; so that

$$\frac{4}{8}\times3=\frac{4\times3}{8}=\frac{12}{8}.$$

Again, let it be required to divide $\frac{4}{8}$ by 3.

Since $\frac{4}{8}\div3$ means that the secondary units or parts of which the fraction $\frac{4}{8}$ is composed are to be divided into 3 equal parts, the quotient will be obtained by making each part *one-third* of what it is, and this is done by dividing the primary unit into *thrice* as many parts as before, *i. e.*, into 3×8 parts, and keeping unchanged the number of parts taken; so that

$$\frac{4}{8}\div3=\frac{4}{3\times8}=\frac{4}{24}.$$

Prop. III. A fraction is multiplied or divided by an integer by dividing its denominator or numerator by that integer.

Thus $\frac{2}{6}\times2=\frac{2\times2}{6}$ (by Prop. II) $=\frac{2\times2\div2}{6\div2}$ (by Prop. I) $=\frac{2}{3}$.

So $\frac{2}{6}\div2=\frac{2}{6\times2}$ (by Prop. II) $=\frac{2\div2}{6\times2\div2}$ (by Prop. I) $=\frac{1}{6}$.

Prop. IV. A proper fraction is less than unity; and an improper fraction is greater than or equal to, unity, according as the numerator is greater than, or equal to, the denominator.

For, in a proper fraction, the numerator being less than the denominator, we take fewer parts than those into which unity is divided, *i. e.*, something less than unity.

Again, in an improper fraction, when the numerator exceeds the denominator, we take more parts than those into which unity is divided, *i. e.*, something more than unity.

Lastly, when the numerator is equal to the denominator, we take as many parts as those into which unity is divided, *i. e.* exactly the whole of unity.

90. *To reduce an improper fraction to a mixed number.*

Rule. Divide the numerator by the denominator; put the quotient for the integral part, and the remainder for the numerator of the fractional part, and the former denominator for its denominator.

The *reason for the Rule* will be clear from the following Example.

Ex. Reduce $\frac{19}{5}$ to a mixed number.

We have $\frac{19}{5} = 19 \div 5 = 3$ together with 4 remaining to be divided by $5 = 3 + \frac{4}{5} = 3\frac{4}{5}$.

91. *To reduce a mixed number to a simple fraction.*

Rule. To the numerator add the product of the denominator and the integral part for the new numerator, and put the denominator below.

The *reason for the Rule* will appear from the Example below.

Ex. Reduce $6\frac{2}{3}$ to an improper fraction.

We have $6\frac{2}{3} = 6 + \frac{2}{3} = \frac{6 \times 3}{3} + \frac{2}{3}$ (Art. 86, Prop. I)

$= \frac{18}{3} + \frac{2}{3} = 18$ *thirds* of unity $+ 2$ *thirds* of unity

$= (18 + 2)$ *thirds* of unity

$= \frac{18+2}{3}$

$= \frac{20}{3}$.

92. *To reduce a compound fraction to a simple fraction.*

Rule. After having reduced (if necessary) the component fractions to simple ones, write down the product of the several numerators for the new numerator, and the product of the several denominators for the new denominator, and cancel or strike out all the factors that are common to the numerator and the denominator.

The *reason for the Rule* will appear from the Example below.

Ex. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{8}$ to a simple fraction.

We have $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{8} = \frac{2}{3}$ of ($\frac{3}{4}$ of $\frac{4}{8}$).

Now $\quad \frac{3}{4}$ of $\frac{4}{8} = (\frac{4}{8} \div 4) \times 3$ (Art. 87)

$= \frac{4}{8 \times 4} \times 3 = \frac{4 \times 3}{8 \times 4}$ (Art. 89, Prop. II);

$\therefore \quad \frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{8} = \frac{2}{3}$ of $\frac{4 \times 3}{8 \times 4}$

$= \frac{4 \times 3 \times 2}{8 \times 4 \times 3}$ (in the same way)

$= \frac{1}{4}$ (after striking out common factors).

93. *To reduce a complex fraction to a simple fraction.*

Rule. After having reduced (if necessary) the numerator and the denominator to the form of simple fractions, put the product of the numerator of the upper fraction and the denominator of the lower for the new numerator, and the product of the denominator of the upper fraction and the numerator of the lower for the new denominator, and strike out all the factors that are common to the numerator and the denominator.

The *reason for the Rule* will appear from the Example below.

Ex. Reduce $\dfrac{2\frac{1}{3}}{1\frac{2}{5}}$ to a simple fraction.

We have $\dfrac{2\frac{1}{3}}{1\frac{2}{5}} = \dfrac{\frac{7}{3}}{\frac{7}{5}}$ (Art. 91)

$= \frac{7}{3} \div \frac{7}{5}$ (Art. 87).

Now $\quad \frac{7}{3} = \frac{7 \times 5}{3 \times 5} = \frac{7 \times 5}{15}$

$= (7 \times 5)$ secondary units whereof each is $\frac{1}{15}$th of 1.

So $\quad \frac{7}{5} = \frac{7 \times 3}{15}$

$= (7 \times 3)$..

$\therefore \quad \frac{7}{3} \div \frac{7}{5} =$ the number of times that (7×3) secondary units each being $\frac{1}{15}$ of 1 are contained in (7×5) seondary units of the same value

$= (7 \times 5) \div (7 \times 3)$

$=$ the fraction $\frac{7 \times 5}{7 \times 3}$ (Art. 84)

$= \frac{5}{3}$ (after striking out the common factor 7).

94. When the numerator and the denominator of a simple fraction contain any common factor, we have seen (Art. 89, Prop. I) that it may be struck out or cancelled, without altering the value of the fraction, and the fraction may thus be expressed by a numerator and a denominator which are *less* than what they were before. The *lowering* of the *terms* of a fraction is,

however, not possible when they are prime to each other. Accordingly we have the following Definition.

Def. A simple fraction is said to be in its **Lowest Terms** when its numerator and denominator are prime to each other.

95. *To reduce a simple fraction to its lowest terms.*

Rule. After striking out from the numerator and the denominator any common factors that may be found by inspection, divide the resulting numerator and denominator by their G. C. M., and write the corresponding quotients as the new numerator and the new denominator.

The *reason for the Rule* is evident. The division of both the terms by their G. C. M. does not alter the value of the fraction, and the new numerator and the new denominator are prime to each other by Art. 68.

Ex. 1. Reduce $\frac{936}{2368}$ to its lowest terms.

We have $\frac{936}{2368}=\frac{117}{296}$ (by striking out the factor 8).

Now the G. C. M. of 117 and 296 is 1, or in other words 117 and 296 are prime to each other.

Hence $\frac{936}{2368}$ reduced to its lowest terms

$$=\frac{117}{296}.$$

We may, without proceeding to find the G. C. M. of 117 and 296, say that they are prime to each other. For $117=9\times13=3\times3\times13$, and neither 3 nor 13 is a factor of 296.

Ex. 2. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{9}$ to a simple fraction in its lowest terms.

We have $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{9}=\frac{2\times3\times8}{3\times4\times9}=\frac{2\times3\times2\times4}{3\times4\times9}$

$$=\frac{2\times2}{9}=\frac{4}{9}.$$

96. *To reduce fractions to their equivalent ones having the least common denominator.*

Rule. Having reduced the fractions to simple fractions in their lowest terms, find the L. C. M. of the resulting denominators, and write it as the new denominator of each.

Divide it by each denominator, multiply the quotient by the corresponding numerator, and put down the product as the corresponding new numerator.

The *reason for the Rule* is clear. The fractions being first reduced to their lowest terms, have their denominators the

least possible; and the only way in which we can reduce these fractions to the required form without altering their values is by multiplying both the terms of each by the same quantity such that the resulting denominator will in each case be the same, and the least possible.

Now ∵ the required least common denominator results from the multiplication of each denominator by some integer, it must be the L. C. M. of the denominators; and the number by which the terms of each fraction must be multiplied, is the the quotient arising from the division of this L. C. M. by the corresponding denominator. By this process, the resulting denominator will be the same in each case, being the L. C. M. of the denominators.

Ex. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{6}$, and $\frac{3}{9}$ to their equivalent fractions with the least common denominator.

Reducing the fractions to their lowest terms we have

$$\frac{1}{2},\ \frac{3}{4},\ \frac{2}{3},\ \text{and } \frac{1}{3}.$$

The L. C. M. of the denominators is 12, and the several quotients are 6, 3, 4, 4; so that the required fractions are

$$\frac{1\times 6}{2\times 6},\quad \frac{3\times 3}{4\times 3},\quad \frac{2\times 4}{3\times 4},\quad \text{and}\quad \frac{1\times 4}{3\times 4},$$

$$\text{or}\quad \frac{6}{12},\ \frac{9}{12},\ \frac{8}{12}\ \text{and}\ \frac{4}{12}.$$

97. Prop. I. Of two fractions having the same denominator, the one with the greater numerator is the greater.

For, the denominators being the same, unity is divided into the same number of parts in both cases, and ∴ the magnitude of the parts is the same in both; and hence that fraction is the greater which is composed of the larger number of parts, *i. e.*, which has the greater numerator.

Prop. II. Of two fractions having the same numerator, the one with the smaller denominator is the greater.

For, the same number of parts being taken in both cases, that fraction is the greater in which the magnitude of each part is the greater, *i. e.*, in which unity is divided into the smaller number of parts, *i. e.*, in which the denominator is the smaller.

98. By the preceding Article, we can compare the values of fractions by reducing them to their equivalent ones with the least common denominator, and then comparing their numerators; or by comparing their denominators, if they have the same numerator.

Ex. 1. Compare the values of $\frac{1}{6}$, $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{5}{8}$.

The given fractions reduced to their equivalent ones with the least common denominator become $\frac{12}{24}$, $\frac{20}{24}$, $\frac{9}{24}$, and $\frac{16}{24}$, and these in the descending order are

$$\frac{20}{24},\ \frac{16}{24},\ \frac{12}{24},\ \frac{9}{24};$$

$\therefore$ the given fractions similarly arranged are

$$\frac{5}{6},\ \frac{2}{3},\ \frac{1}{2},\ \text{and } \frac{3}{8}.$$

Ex. 2. Compare the values of $\frac{1}{3}$, $\frac{2}{10}$, $\frac{3}{12}$, and $\frac{5}{30}$.

Reduced to their lowest terms, the fractions are

$$\frac{1}{3},\ \frac{1}{5},\ \frac{1}{4},\ \text{and } \frac{1}{6}.$$

By Art. 97, Prop. II, these in the descending order are

$$\frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \text{and } \frac{1}{6};$$

$\therefore$ the given fractions in the same order are

$$\frac{1}{3},\ \frac{3}{12},\ \frac{2}{10},\ \text{and } \frac{5}{30}.$$

Examples IX.

1. Express the following improper fractions as mixed or whole numbers :—

(1) $\frac{3}{2}$. (2) $\frac{5}{3}$. (3) $\frac{10}{5}$. (4) $\frac{18}{15}$.

(5) $\frac{18}{7}$. (6) $\frac{24}{5}$. (7) $\frac{113}{13}$. (8) $\frac{602}{26}$.

(9) $\frac{456}{12}$. (10) $\frac{123}{45}$. (11) $\frac{799}{56}$. (12) $\frac{1000}{99}$.

2. Reduce the following mixed numbers to the form of simple fractions :—

(1) $1\frac{1}{3}$. (2) $3\frac{3}{5}$. (3) $5\frac{2}{7}$. (4) $8\frac{9}{10}$.

(5) $11\frac{11}{12}$. (6) $13\frac{9}{19}$. (7) $29\frac{7}{17}$. (8) $19\frac{5}{8}$.

(9) $16\frac{6}{11}$. (10) $121\frac{19}{32}$. (11) $100\frac{10}{11}$. (12) $1000\frac{1}{100}$.

3. Reduce the following compound fractions to simple fractions in their lowest terms :—

(1) $\frac{1}{3}$ of $\frac{3}{4}$. (2) $\frac{1}{4}$ of $\frac{2}{3}$. (3) $\frac{5}{8}$ of $\frac{8}{10}$.

(4) $\frac{1}{9}$ of $\frac{9}{5}$. (5) $\frac{3}{2}$ of $\frac{8}{9}$. (6) $\frac{10}{11}$ of $\frac{121}{100}$.

(7) $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$. (8) $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$. (9) $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$.

(10) $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.

(11) $\frac{3}{4}$ of $\frac{6}{15}$ of $\frac{8}{9}$ of $\frac{5}{6}$.

(12) $3\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{1}{9}$ of [illegible] of $9\frac{2}{3}$ of $\frac{7}{12}$ of $\frac{92}{121}$.

(13) $\frac{7}{8}$ of $\frac{120}{136}$ of $\frac{2}{[illegible]}$ of $3\frac{[illegible]}{11}$ of $\frac{3}{5}$.

(14) $2\frac{84}{100}$ of $\frac{4}{13}$ of $\frac{5}{9}$ of $\frac{31}{82}$ of $\frac{19}{52}$.

(15) $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$.

(16) $1\frac{1}{2}$ of $1\frac{1}{3}$ of $1\frac{1}{4}$ of $1\frac{1}{5}$ of $1\frac{1}{6}$ of $1\frac{1}{7}$ of $1\frac{1}{8}$ of $1\frac{1}{9}$.

(17) $\frac{11}{12}$ of $\frac{21}{22}$ of $\frac{31}{32}$ of $\frac{41}{42}$ of $\frac{51}{52}$ of $\frac{61}{62}$.

(18) $\frac{12}{21}$ of $\frac{34}{43}$ of $\frac{56}{65}$ of $\frac{78}{87}$ of $\frac{90}{9}$.

4. Reduce the following complex fractions to simple fractions in their lowest terms :—

(1) $\dfrac{2\frac{2}{3}}{3\frac{3}{4}}$. (2) $\dfrac{1\frac{1}{3}}{2\frac{1}{4}}$. (3) $\dfrac{5\frac{1}{2}}{2\frac{1}{3}}$. (4) $\dfrac{7\frac{1}{2}}{4\frac{1}{6}}$.

(5) $\dfrac{28}{4\frac{1}{5}}$. (6) $\dfrac{21\frac{1}{4}}{17}$. (7) $\dfrac{33\frac{1}{3}}{16\frac{2}{3}}$. (8) $\dfrac{27\frac{1}{2}}{2\frac{3}{4}}$.

(9) $\dfrac{\frac{1}{3}\text{ of }\frac{1}{4}}{12}$. (10) $\dfrac{\frac{2}{3}\text{ of }\frac{3}{4}}{\frac{3}{4}\text{ of }\frac{4}{5}}$. (11) $\dfrac{\frac{1}{3}\text{ of }\frac{1}{3}}{\frac{1}{30}}$. (12) $\dfrac{8\frac{1}{10}}{\frac{3}{5}\text{ of }\frac{9}{10}}$.

(13) $\dfrac{\frac{1}{2}\text{ of }\frac{1}{3}\text{ of }\frac{1}{4}}{\frac{1}{2}\text{ of }\frac{1}{3}}$. (14) $\dfrac{\frac{2}{3}\text{ of }\frac{6}{7}\text{ of }\frac{8}{9}}{\frac{4}{5}\text{ of }\frac{5}{9}\text{ of }\frac{1}{2}}$.

(15) $\dfrac{2\frac{1}{2}\text{ of }3\frac{3}{5}}{1\frac{1}{2}\text{ of }2\frac{2}{3}}$. (16) $\dfrac{28\frac{1}{3}\text{ of }1\frac{1}{17}\text{ of }\frac{1}{30}}{18\frac{1}{3}\text{ of }1\frac{1}{5}\text{ of }\frac{1}{22}}$.

5. Reduce the following fractions to their lowest terms :-

(1) $\frac{9}{15}$. (2) $\frac{19}{22}$. (3) $\frac{16}{52}$. (4) $\frac{39}{91}$.
(5) $\frac{121}{990}$. (6) $\frac{361}{456}$. (7) $\frac{324}{558}$. (8) $\frac{825}{945}$.
(9) $\frac{169}{273}$. (10) $\frac{585}{780}$. (11) $\frac{195}{260}$. (12) $\frac{460}{690}$.
(13) $\frac{123}{456}$. (14) $\frac{789}{1011}$. (15) $\frac{135}{999}$. (16) $\frac{399}{963}$.
(17) $\frac{24}{642}$. (18) $\frac{248}{810}$. (19) $\frac{256}{720}$. (20) $\frac{700}{910}$.

6. Reduce the fractions in each of the following sets to equivalent fractions having the least common denominator :—

(1) $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. (2) $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$.
(3) $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{3}{10}$. (4) $\frac{5}{6}$, $\frac{2}{9}$, and $\frac{7}{12}$.
(5) $\frac{4}{15}$, $\frac{5}{18}$, and $\frac{6}{21}$. (6) $\frac{5}{12}$, $\frac{6}{18}$, and $\frac{7}{21}$.
(7) $\frac{3}{8}$, $\frac{4}{9}$, $\frac{5}{10}$, and $\frac{6}{12}$. (8) $\frac{2}{6}$, $\frac{3}{7}$, $\frac{4}{8}$, and $\frac{5}{9}$.
(9) $\frac{21}{22}$, $\frac{31}{33}$, $\frac{41}{44}$, and $\frac{1}{15}$. (10) $\frac{8}{10}$, $\frac{10}{12}$, $\frac{12}{14}$, and $\frac{14}{16}$.
(11) $\frac{54}{81}$, $\frac{30}{90}$, $\frac{49}{91}$, and $\frac{13}{14}$. (12) $\frac{11}{111}$, $\frac{22}{222}$, $\frac{33}{333}$, and $\frac{2}{3}$.
(13) $\frac{2}{2}$, $\frac{2}{3}$, $\frac{2}{4}$, $\frac{2}{5}$, $\frac{2}{6}$, $\frac{2}{7}$, $\frac{2}{8}$, and $\frac{2}{9}$.
(14) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{8}{9}$.
(15) $\frac{2}{3}$, $\frac{2}{4}$, $\frac{2}{6}$, $\frac{2}{8}$, and $\frac{2}{10}$.
(16) $\frac{1}{36}$, $\frac{2}{69}$, $\frac{3}{912}$, and $\frac{4}{1215}$.
(17) $\frac{1}{12}$, $\frac{2}{24}$, $\frac{4}{48}$, and $\frac{8}{816}$. (18) $\frac{9}{10}$, $\frac{9}{18}$, $\frac{9}{100}$ $\frac{9}{180}$, and $\frac{9}{1000}$.

7. Compare the values of

(1) $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{5}$.
(2) $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
(3) $\frac{3}{5}$, $\frac{5}{8}$, and $\frac{7}{10}$.
(4) $\frac{7}{8}$, $\frac{9}{16}$, and $\frac{22}{24}$.
(5) $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{6}{13}$, and $\frac{27}{18}$.
(6) $\frac{3}{7}$ of $2\frac{1}{3}$, $\frac{2}{5}$ of $\frac{3}{8}$, and $\frac{29}{36}$.
(7) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, and $\frac{1}{9}$.
(8) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{8}{9}$.
(9) $\frac{5}{9}$, $\frac{4}{8}$, $\frac{3}{7}$, $\frac{2}{6}$, and $\frac{2}{5}$.
(10) $\frac{4}{5}$, $\frac{5}{8}$, $\frac{6}{7}$, $\frac{3}{6}$, $\frac{2}{3}$, and $\frac{1}{4}$.
(11) $\frac{12}{21}$, $\frac{22}{31}$, $\frac{32}{41}$, and $\frac{42}{51}$.
(12) $\frac{9}{20}$, $\frac{13}{25}$, $\frac{12}{273}$, $\frac{7}{12}$, and $\frac{7}{36}$.
(13) $\frac{12}{16}$, $\frac{23}{25}$, $\frac{22}{56}$, $\frac{28}{24}$, and $\frac{24}{128}$.
(14) $\frac{5}{13}$, $\frac{10}{24}$, $\frac{15}{21}$, and $\frac{25}{105}$.
(15) $\frac{2}{10}$, $\frac{22}{100}$, $\frac{90}{1000}$, and $\frac{224}{900}$.

SECTION III. ADDITION OF FRACTIONS.

99. Rule. Reduce compound and complex fractions to simple ones, improper fractions to mixed numbers, and all the proper fractions to their lowest terms.

Add together the integral parts of the summands, if any, as in Simple Addition.

Reduce the proper fractions in the several summands to their equivalent ones having the least common denominator, add together the new numerators of these fractions, and put down the result as the numerator of the sum, and the least common denominator as its denominator.

The resulting fraction, reduced if possible, together with the sum of the integral parts already found, will be the sum required.

Reason for the Rule. To add numbers together is to add together their integral parts and also their fractional parts; and to add fractions together is to add together the *numbers* of parts of unity that they contain when reduced to a common denominator. Thus, take the following Example :—

Ex. Add together $2\frac{1}{2}$, $\frac{1}{3}$ of $\frac{3}{5}$, $5\frac{2}{10}$, and $\frac{3}{8}$.

We have $2\frac{1}{2}+\frac{1}{3}$ of $\frac{3}{5}+5\frac{2}{10}+\frac{3}{8}=2\frac{1}{2}+\frac{1}{5}+5\frac{1}{5}+\frac{3}{8}$

$$=2+5+\frac{1}{2}+\frac{1}{5}+\frac{1}{5}+\frac{3}{8}$$

$$=7+\frac{20}{40}+\frac{8}{40}+\frac{8}{40}+\frac{15}{40}.$$

Now $\frac{20}{40}+\frac{8}{40}+\frac{8}{40}+\frac{15}{40}$

= 20 times the $\frac{1}{40}$th part of unity
+ 8..
+ 8..
+ 15..
= 51..
$=\frac{51}{40}$;

$\therefore$ sum reqd. $=7+\frac{51}{40}=7+1+\frac{11}{40}=8+\frac{11}{40}$.

The process is shortly represented thus :—

$$2\frac{1}{2}+\frac{1}{3}\text{ of }\frac{3}{5}+5\frac{3}{10}+\frac{3}{8}=2\frac{1}{2}+\frac{1}{5}+5\frac{1}{5}+\frac{3}{8}=2+5+\frac{1}{2}+\frac{1}{5}+\frac{1}{5}+\frac{3}{8}$$
$$=7+\frac{20+8+8+15}{40}=7+\frac{51}{40}$$
$$=8\frac{11}{40}.$$

Examples X.

1. Add together

(1) $\frac{1}{2}$ and $\frac{1}{4}$. (2) $\frac{2}{3}$ and (3) $\frac{1}{8}$ and $\frac{1}{5}$.

(4) $\frac{2}{3}$ and $\frac{3}{10}$. (5) $\frac{11}{14}$ and $\frac{9}{21}$. (6) $\frac{13}{14}$ and $\frac{19}{21}$.

(7) $1\frac{1}{2}$ and $3\frac{3}{4}$. (8) $2\frac{3}{4}$ and $5\frac{6}{7}$. (9) $8\frac{9}{10}$ and $10\frac{8}{15}$.

(10) $11\frac{13}{18}$ and $\frac{8}{39}$. (11) $\frac{13}{18}$ and $\frac{7}{21}$. (12) [illegible] and $\frac{5}{27}$.

(13) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.

(14) $\frac{11}{12}$, $\frac{11}{16}$, $\frac{13}{18}$, and $\frac{7}{24}$.

(15) $\frac{1}{5}$ of $\frac{3}{8}$, $\frac{2}{7}$ of $\frac{35}{36}$, $\frac{7}{13}$, $\frac{8}{25}$, and $38\frac{9}{10}$.

(16) $3\frac{3}{4}$ of $5\frac{1}{3}$, $\frac{3}{8}$, $\frac{2}{8}$ of $\frac{5}{60}$, and $1\frac{11}{21}$.

(17) $\frac{1}{2}$, $\frac{11}{12}$, $\frac{21}{22}$, $\frac{31}{32}$, $\frac{41}{42}$, and $\frac{51}{52}$.

(18) $\frac{1}{3}$, $\frac{2}{15}$, $\frac{3}{27}$, $\frac{4}{39}$, and $\frac{5}{78}$.

2. Find the value of

(1) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$. (2) $\frac{1}{6}+\frac{1}{7}+\frac{2}{8}+\frac{2}{9}+\frac{2}{10}$.

(3) $\frac{1}{12}+\frac{2}{15}+\frac{3}{18}+\frac{4}{21}$. (4) $\frac{5}{24}+\frac{6}{27}+\frac{7}{30}+\frac{8}{33}$.

(5) $\frac{2}{4}+\frac{2}{8}+\frac{3}{12}+\frac{4}{16}$. (6) $\frac{2}{5}+\frac{2}{10}+\frac{3}{15}+\frac{4}{20}$.

(7) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}$.

(8) $1\frac{1}{2}+2\frac{2}{3}+3\frac{3}{4}+4\frac{4}{5}+5\frac{5}{6}+6\frac{6}{7}+7\frac{7}{8}+8\frac{8}{9}+9\frac{9}{10}$.

(9) $\frac{1}{13}+\frac{2}{27}+\frac{3}{27}+\frac{4}{72}+\frac{5}{36}+\frac{6}{63}$.

(10) $\frac{4}{5}$ of $\frac{2}{7}$ of $3\frac{19}{24}+\frac{7}{15}+\frac{19}{30}$ of $\frac{3}{70}+2$.

(11) $2\frac{4}{5}$ of $\frac{8}{11}$ of $2\frac{2}{5}+6\frac{3}{10}+\frac{13}{14}+22\frac{1}{2}$.

(12) $\frac{3}{11}+\frac{5}{9}$ of $\frac{4}{22}$ of $\frac{3}{4}+7\frac{7}{8}+33\frac{1}{3}+66\frac{2}{3}$.

(13) $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}$ of $1\frac{1}{3}$ of $\frac{2}{5}+\frac{3}{8}+\frac{7}{10}+28\frac{3}{11}$.

(14) $\dfrac{3\frac{1}{3}}{8\frac{1}{3}}+\dfrac{5\frac{1}{7}}{1\frac{5}{6}}+\frac{9}{100}$ of $3\frac{1}{3}+28$ [illegible].

(15) $\frac{51}{72}+\frac{72}{81}+\frac{81}{102}+\frac{102}{306}$ of $\frac{1}{2}+8$ [illegible]

SECTION IV. SUBTRACTION OF FRACTIONS.

100. Rule. Having reduced the fractions as in Addition, subtract the integral part of the subtrahend from that of the minuend.

Reduce the fractional parts of the minuend and the subtrahend to their equivalent fractions having the least common denominator, and subtract the new numerator of the subtrahend, if possible, from that of the minuend, and put down the difference as the numerator of the remainder, and the least common denominator as its denominator. If the new numerator of the minuend is less than that of the subtrahend, take 1 from the integral remainder already found, add it to the fractional part of the minuend, reduced as aforesaid, and having reduced the mixed number so obtained to an improper fraction, from its numerator subtract the new numerator of the subtrahend, and proceed as before.

Reason for the Rule. To subtract one number from another is to subtract separately the integral and the fractional parts of the former from the corresponding parts of the latter ; and to subtract one fraction from another is to subtract the number of parts of unity contained in the former from the number of parts in the latter, when they are reduced to a common denominator. When the number of parts of unity contained in the minuend is less than that in the subtrahend, we borrow 1 or an entire unit from the integral remainder, and add it, or in other words, the number of parts in an entire unit, to the number of parts in the minuend, and subtract from the result the number of parts in the subtrahend.

Ex. 1. Subtract $2\frac{1}{4}$ from $6\frac{1}{3}$.

We have
$$6\tfrac{1}{3}-2\tfrac{1}{4}=(6-2)+\tfrac{1}{3}-\tfrac{1}{4}$$
$$=4+\tfrac{4}{12}-\tfrac{3}{12}$$
$$=4\tfrac{1}{12}.$$

Ex. 2. Subtract $3\frac{5}{6}$ from $6\frac{1}{4}$.

We have
$$6\tfrac{1}{4}-3\tfrac{5}{6}=(6-3)+\tfrac{1}{4}-\tfrac{5}{6}$$
$$=3+\tfrac{3}{12}-\tfrac{10}{12}$$
$$=2+(1+\tfrac{3}{12})-\tfrac{10}{12}$$
$$=2+\tfrac{15}{12}-\tfrac{10}{12}$$
$$=2\tfrac{5}{12}.$$

Ex. 3. Subtract $\frac{3}{4}$ from 3.

We have
$$3-\tfrac{3}{4}=2+1-\tfrac{3}{4}=2+\tfrac{4}{4}-\tfrac{3}{4}$$
$$=2\tfrac{1}{4}.$$

Ex. 4. Find the value of $\frac{3}{4}$ of $\frac{2}{5}-\frac{1}{3}+\frac{7}{8}-\frac{3}{5}$ of $\frac{5}{12}$.

We have $\frac{3}{4}$ of $\frac{2}{5}-\frac{1}{3}+\frac{7}{8}-\frac{3}{5}$ of $\frac{5}{12}=\frac{3}{10}-\frac{1}{3}+\frac{7}{8}-\frac{1}{4}$

$$=\frac{3}{10}+\frac{7}{8}-\left(\frac{1}{3}+\frac{1}{4}\right),$$

for, $\frac{3}{10}$ and $\frac{7}{8}$ are meant to be added together, and $\frac{1}{3}$ and $\frac{1}{4}$ are both meant to be subtracted from the sum of $\frac{3}{10}$ and $\frac{7}{8}$;

$$\text{and } \frac{3}{10}+\frac{7}{8}-\left(\frac{1}{3}+\frac{1}{4}\right)=\frac{12}{40}+\frac{35}{40}-\left(\frac{4}{12}+\frac{3}{12}\right)$$
$$=\frac{47}{40}-\frac{7}{12}$$
$$\frac{141}{120}-\frac{70}{120}$$
$$=\frac{71}{120}.$$

Ex. 5. What proper fraction must be added to $2\frac{3}{4}$ to make the result an integer?

Evidently, the required fraction must be such, that being added to $\frac{3}{4}$, it will give 1 for the sum;

$\therefore$ the fraction required $=1-\frac{3}{4}=\frac{4}{4}-\frac{3}{4}=\frac{1}{4}$.

Examples XI.

1. Find the difference between

(1) $\frac{1}{2}$ and $\frac{3}{8}$. (2) $\frac{3}{5}$ and $\frac{3}{8}$. (3) $\frac{2}{4}$ and $\frac{2}{8}$.

(4) $\frac{2}{3}$ and $\frac{3}{4}$. (5) $\frac{3}{4}$ and $\frac{4}{5}$. (6) $\frac{4}{5}$ and $\frac{5}{6}$.

(7) $\frac{3}{4}$ and $\frac{3}{8}$. (8) $\frac{2}{3}$ and $\frac{7}{10}$. (9) $\frac{3}{16}$ and $\frac{2}{12}$.

(10) $2\frac{3}{4}$ and $7\frac{2}{8}$. (11) $5\frac{3}{4}$ and $2\frac{2}{3}$. (12) $12\frac{5}{12}$ and $17\frac{3}{10}$.

(13) $22\frac{2}{9}$ and $11\frac{3}{5}$ of $\frac{3}{19}$. (14) 25 and $\frac{3}{5}$ of $\frac{5}{9}$.

(15) $\frac{2}{3}$ of $\frac{3}{11}$ and $17\frac{7}{33}$. (16) $\frac{2}{7}$ of $\frac{56}{58}$ and $\frac{3}{29}$ of $\frac{9}{12}$.

(17) $11\frac{23}{222}$ and $3\frac{8}{37}$. (18) $1\frac{2}{20}$ and $\frac{2}{3}$ of $\frac{3}{10}$ of $\frac{5}{18}$.

(19) 2 and $\frac{2}{3}$. (20) 22 and $33\frac{2}{5}$.

2. Find the value of

(1) $\frac{1}{2}+\frac{2}{3}-\frac{1}{4}-\frac{2}{5}$. (2) $\frac{2}{3}-\frac{2}{5}+\frac{6}{7}-\frac{3}{8}$.

(3) $\frac{3}{5}+\frac{7}{10}-\frac{2}{15}-\frac{2}{25}$. (4) $\frac{2}{3}$ of $\frac{3}{4}-\frac{2}{13}+\frac{3}{8}$.

(5) $\frac{2}{21}+\frac{2}{22}-\frac{2}{11}+\frac{2}{12}$. (6) $\frac{199}{100}-\frac{9}{10}+\frac{4}{30}-\frac{40}{330}$.

3. What fraction must be added to $\frac{3}{4}$ to make the sum, equal to the difference between $2\frac{1}{2}$ and $1\frac{1}{4}$.

4. What proper fraction must be added to $5\frac{7}{18}$ to make the result an integer?

5. What fraction must be subtracted from $\frac{8}{15}$ to give $\frac{1}{3}$?

6. What fraction must be added to $\frac{7}{21}$ to give $\frac{2}{3}$?

SECTION V. MULTIPLICATION OF FRACTIONS.

101. Before giving the Rule for the Multiplication of Fractions, let us see how far the meaning we have attached to the operation of Multiplication in our Definition in Art. 13, applies to the case in which the multiplier is not an integer.

When a number is taken to the extent of its *double*, *triple*, *&c.*, we say that it is taken 2, 3, &c. times, *i. e.*, is multiplied by 2, 3, &c. Similarly, when a number, whether integral or fractional, is taken to the extent of its *half*, *one-third*, *two-thirds*, *&c.*, we may, by a *stretch of language*, say, that it is taken *half*, *one-third*, *two-thirds*, *&c.*, times, or rather parts of a time, *i. e.*, by our Definition in Art. 13, that it is multiplied by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, &c.

Thus, to multiply any number $2\frac{1}{3}$ by $\frac{2}{7}$ is to take $\frac{2}{7}$ of $2\frac{1}{3}$, *i. e.*, $\frac{2}{7}$ of $\frac{7}{3}$; *i. e.*, it is to divide $\frac{7}{3}$ into 7 equal parts, and then to take 2 of such parts;

$$\therefore\ 2\tfrac{1}{3} \times \tfrac{2}{7} = (\tfrac{7}{3} \div 7) \times 2 = \tfrac{7}{3 \times 7} \times 2 = \tfrac{7 \times 2}{3 \times 7} \text{ (Art. 89, Prop. II)}$$

$$= \tfrac{2}{3} \text{ (after striking out 7).}$$

Similarly, if there is another factor $\frac{3}{4}$, we have

$$\tfrac{7}{3} \times \tfrac{2}{7} \times \tfrac{3}{4} = \tfrac{7 \times 2}{3 \times 7} \times \tfrac{3}{4} = \tfrac{7 \times 2 \times 3}{3 \times 7 \times 4} = \tfrac{1}{2},$$

(after cancelling the common factors 7, 3, and 2).

Hence we deduce the Rule for the Multiplication of Fractions given in the next Article.

102. Rule. Reduce the factors, if necessary, to the form of simple fractions, multiply all the numerators together for the numerator of the product, and all the denominators together for its denominator, and simplify the result by cancelling all the factors common to the numerator and the denominator.

Ex. Multiply $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{4}$ and $\frac{5}{6}$ together.

By the Rule we have

$$\tfrac{2}{3} \times \tfrac{4}{5} \times \tfrac{3}{4} \times \tfrac{5}{6} = \tfrac{2 \times 4 \times 3 \times 5}{3 \times 5 \times 4 \times 6} = \tfrac{2}{6} = \tfrac{1}{3}.$$

103. In the Multiplication of Fractions, the order of the factors is immaterial.

Thus $\frac{2}{3} \times \frac{6}{7} = \frac{2 \times 6}{3 \times 7} = \frac{6 \times 2}{7 \times 3}$ (Art. 41)

$$= \tfrac{6}{7} \times \tfrac{2}{3}.$$

104. It may be here observed that a compound fraction denotes the product of the component fractions.

Examples XII.

1. Multiply—

(1) $\frac{1}{2}$ by $\frac{2}{3}$. (2) $\frac{2}{3}$ by $\frac{3}{4}$. (3) $1\frac{1}{4}$ by $\frac{4}{5}$.

(4) $\frac{5}{6}$ by $\frac{3}{10}$. (5) $\frac{6}{7}$ by $\frac{5}{12}$. (6) $\frac{7}{8}$ by $\frac{20}{21}$.

(7) $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{4}{5}$ of $\frac{5}{7}$. (8) $3\frac{2}{3}$ of $\frac{9}{22}$ by $\frac{15}{16}$.

(9) $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{4}{5}$ of $\frac{6}{7}$. (10) $\frac{5}{16}$ of $\frac{20}{11}$ by $\frac{22}{27}$.

(11) $\frac{9}{20}$ of $\frac{6}{7}$ by $\frac{20}{27}$ of $\frac{28}{24}$. (12) $\frac{13}{57}$ of $\frac{27}{32}$ by $\frac{8}{85}$ of $1\frac{7}{12}$.

2. Find the continued product of

(1) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{4}$, and $\frac{2}{3}$. (2) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{8}{9}$.

(3) $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{9}$, $\frac{5}{11}$, and $\frac{9}{14}$. (4) $\frac{1}{2}$, $\frac{8}{17}$, $\frac{23}{24}$, $\frac{34}{45}$, and $\frac{45}{46}$.

(5) $\frac{15}{16}$, $\frac{35}{36}$, $1\frac{8}{65}$, $1\frac{2}{75}$, and $\frac{5}{7}$. (6) $\frac{7}{23}$, $\frac{10}{21}$, $\frac{102}{100}$, $\frac{5}{57}$, and $\frac{9}{10}$.

3. Find the value of

(1) $\frac{1}{2}$ of $\frac{8}{9} \times \frac{9}{10} + 3\frac{2}{3} - 2\frac{2}{5}$. (2) $\frac{22}{21} + \frac{12}{13} \times \frac{117}{120} \times \frac{5}{9} - \frac{1}{105}$.

(3) $3\frac{2}{3} - \frac{3}{5} \times \frac{5}{8} + \frac{2}{3} \times \frac{9}{22}$.

SECTION VI. DIVISION OF FRACTIONS.

105. The meaning which we have attached to Division in Art. 14, is applicable to the Division of an integer or a fraction by a fraction, as we have already seen in Art. 93, and as will further appear from the following Examples.

Ex. 1. Divide 3 by $\frac{4}{5}$.

Here we have to find how often the secondary units in $\frac{4}{5}$ are contained in 3 primary units, *i. e*, how often 4 secondary units, whereof each is $\frac{1}{5}$th of 1, are contained in 3×5 secondary units of the same value;

$\therefore$ the quotient reqd. $= 3 \times 5 \div 4 = \frac{3 \times 5}{4}$

$$= \frac{\text{dividend} \times \text{denr. of divisor}}{\text{numr. of divisor}}.$$

Ex. 2. Divide $\frac{2}{3}$ by $\frac{3}{5}$.

Here we have quotient reqd.

$= \frac{2}{3} \div \frac{3}{5}$

$= \frac{2 \times 5}{3 \times 5} \div \frac{3 \times 3}{5 \times 3}$ (Art. 89, Prop. I)

$= \frac{2 \times 5}{15} \div \frac{3 \times 3}{15}$

$= 2 \times 5$ secondary units whereof each is $\frac{1}{15}$ of 1

$\div\, 3 \times 3$ ....

$= \frac{2 \times 5}{3 \times 3}$

$$= \frac{\text{numr. of dividend} \times \text{denr. of divisor}}{\text{denr. of dividend} \times \text{numr. of divisor}}.$$

Hence we deduce the Rule for the Division of Fractions given in the following Article.

106. Rule. Reduce the dividend and the divisor, if necessary, to the form of simple fractions, invert the divisor, and then proceed as in Multiplication.

Ex. Divide $2\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{3}{8}$ of $\frac{2}{3}$.

By the Rule we have $(2\frac{1}{2} \text{ of } \frac{3}{4}) \div (\frac{3}{8} \text{ of } \frac{2}{3}) = (\frac{5}{2} \text{ of } \frac{3}{4}) \div (\frac{3}{8} \text{ of } \frac{2}{3})$

$$= \frac{5 \times 3}{2 \times 4} \div \frac{3 \times 2}{8 \times 3}$$

$$= \frac{5 \times 3}{2 \times 4} \times \frac{8 \times 3}{3 \times 2}$$

$$= \frac{5 \times 3 \times 8 \times 3}{2 \times 4 \times 3 \times 2} = \frac{15}{2}$$

$$= 7\frac{1}{2}.$$

107. It now remains to be seen how far the second meaning of Division noticed in Art. 50, is applicable to the division of a number by a fraction. Division in this sense is the method of finding the magnitude of each part of the dividend when it is divided into the number of parts indicated by the divisor, or in other words, of finding a part of the dividend, such that when multiplied by the divisor, it will reproduce the dividend. In this sense, to divide $\frac{2}{3}$ by $\frac{3}{5}$ is to find a fraction, such that when multiplied by $\frac{3}{5}$ in the sense in which Multiplication is understood in Art. 101, it will produce $\frac{2}{3}$.

Now the quotient of $\frac{2}{3} \div \frac{3}{5}$ according to the Rule in Art. 106 is $\frac{2 \times 5}{3 \times 3}$;

and $\frac{2 \times 5}{3 \times 3} \times \frac{3}{5} = \frac{2}{3}$.

Hence, the quotient obtained by the preceding Rule also expresses numerically the magnitude of a part of the dividend, such that when multiplied by the divisor it will reproduce the dividend.

108. From the above we see that in the Division of Fractions also,

quotient × divisor = dividend.

Examples XIII.

1. Divide—

(1) 1 by $\frac{1}{2}$. (2) 2 by $\frac{2}{3}$. (3) $1\frac{1}{3}$ by $2\frac{2}{3}$.

(4) $\frac{5}{16}$ by $\frac{5}{8}$. (5) $\frac{9}{10}$ by $\frac{9}{[illegible]}$. (6) $3\frac{1}{30}$ by $4\frac{7}{15}$.

(7) $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{3}{4}$ by $\frac{5}{12}$ of $\frac{9}{16}$ of $\frac{7}{32}$.

(8) $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ by $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$.

(9) $1\frac{1}{4}$ of $2\frac{1}{3}$ of $3\frac{1}{12}$ by $2\frac{1}{2}$ of $3\frac{2}{3}$ of $4\frac{1}{3}$.

(10) $7\frac{1}{3}$ of $\frac{3}{11}$ of $\frac{17}{81}$ by $\frac{7}{13}$ of $\frac{8}{14}$ of $\frac{9}{15}$.

(11) $101\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{9}{13}$ by $\frac{19}{32}$ of $\frac{64}{65}$.

(12) $1\frac{1}{50}$ of $2\frac{2}{9}$ of $3\frac{3}{8}$ by $3\frac{7}{11}$ of $4\frac{8}{15}$ of $\frac{1}{5}$.

2. What number multiplied by $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ will give $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$?

3. What number must be multiplied by the continued product of the reciprocals of the first five integers to give the reciprocal of the sixth?

4. Divide the difference between ten and one-tenth by the sum of one and one-tenth.

MISCELLANEOUS QUESTIONS AND EXAMPLES.

109. In simplifying expressions containing fractions, the Rule in Art. 76 should be borne in mind.

Ex. 1. Simplify $\dfrac{2\frac{1}{2} \text{ of } \frac{3}{10} - \frac{1}{3} \text{ of } \frac{1}{4}}{2\frac{1}{2}} - \left(\frac{1}{10} - \frac{1}{100}\right) + \dfrac{2\frac{3}{4}}{2\frac{1}{5}} \div 3\frac{1}{3}$.

The given expression

$$= \frac{\frac{5}{2} \times \frac{3}{10} - \frac{1}{3} \times \frac{1}{4}}{\frac{5}{2}} - \frac{10-1}{100} + \frac{\frac{11}{4}}{\frac{11}{5}} \div \frac{10}{3}$$

$$= \frac{\frac{3}{4} - \frac{1}{12}}{\frac{5}{2}} - \frac{9}{100} + \frac{11 \times 5}{4 \times 11} \times \frac{3}{10} = \frac{\frac{9}{12} - \frac{1}{12}}{\frac{5}{2}} - \frac{9}{100} + \frac{3}{8}$$

$$= \frac{\frac{8}{12}}{\frac{5}{2}} - \frac{9}{100} + \frac{3}{8} = \frac{8 \times 2}{12 \times 5} - \frac{9}{100} + \frac{3}{8}$$

$$= \frac{4}{15} - \frac{9}{100} + \frac{3}{8} = \frac{160}{600} - \frac{54}{600} + \frac{225}{600}$$

$$= \frac{331}{600}.$$

Ex. 2. *A* owns $\frac{2}{5}$ of an estate and *B* $\frac{3}{10}$ of the same. For a certain price, *A* offers to sell you $\frac{1}{4}$ of his share, and for the same price, *B* offers to sell $\frac{2}{3}$ of his. Which is the more profitable offer to you; what is your gain by accepting it; and what share of his property must *B* offer to make the two offers equally profitable?

Here there are three questions. Take them one after another.

(1) To find which is the more profitable offer.

For the same price,

A offers $\frac{1}{4}$ of his share, *i. e.*, $\frac{1}{4}$ of $\frac{2}{5}$ of the estate,

and *B* ... $\frac{2}{3}$ ... ... $\frac{2}{3}$ of $\frac{3}{10}$ ...

Hence, the question is reduced to the comparison of the two fractions $\frac{1}{4}$ of $\frac{2}{5}$ and $\frac{3}{8}$ of $\frac{3}{10}$

$$i.\ e.,\ \frac{1}{10} \text{ and } \frac{9}{80} \qquad i.\ e.,\ \frac{8}{80} \text{ and } \frac{9}{80}$$

The latter being the greater, B's offer is the more profitable of the two.

(2) To find the gain.

This is the difference between the two offers, *i. e.*, between $\frac{8}{80}$ and $\frac{9}{80}$ and equals $\frac{9}{80}-\frac{8}{80}=\frac{1}{80}$.

So that by accepting B's offer, the purchaser gets $\frac{1}{80}$ of the estate more than what he gets by accepting the other offer.

(3) To find the share of his property that B must agree to sell, to make his offer equal to A's.

A's offer is $\frac{1}{10}$ of the whole estate,

and B owns $\frac{3}{10}$ ... ...

Hence the question is reduced to finding a fraction such that $\frac{3}{10}$ being multiplied by it, will produce $\frac{1}{10}$.

This fraction $=\frac{1}{10}\div\frac{3}{10}=\frac{1}{3}$.

So that B must agree to sell $\frac{1}{3}$ of his property to make his offer equal to A's.

Ex. 3. What is the total number of rupees whereof $\frac{1}{2}$ being spent for one purpose, and $\frac{1}{5}$ for another, there remain 6 rupees left?

Taking $\frac{1}{2}+\frac{1}{5}$ from the whole or unity,

we have $1-(\frac{1}{2}+\frac{1}{5})=1-\frac{7}{10}=\frac{3}{10}$.

Now $\frac{3}{10}\times$ the total no. reqd. $=6$;

$\therefore$ the total no. reqd. $=6\div\frac{3}{10}=\frac{6\times 10}{3}=20$.

Ex. 4. A book contains 4 chapters, whereof the 1st and the 2nd together contain $\frac{13}{20}$ of the whole number of pages in the book; the 2nd and the 3rd together, $\frac{3}{5}$ of the whole number; the 2nd contains twice as many pages as the 1st and the 3rd together; and there are 20 pages in the 4th chapter. What is the total number of pages in the book, and how many pages are there in each of the first 3 chapters?

Here, we have

no. of pages in Ch. I + no. in Ch. II $=\frac{13}{20}$ of total no.;

... ... Ch. II + ... Ch. III $=\frac{3}{5}$ ... ...

$\therefore$ ... ... Ch. I + ... Ch. III + 2 × no. in Ch. II

$=(\frac{13}{20}+\frac{3}{5})$ of total no. $=\frac{25}{20}$ of total no.

But by the question,

no. in Ch. II $=2\times$(no. in Ch. I + no. in Ch. III);

$\therefore$ no. in Ch. I + no. in Ch. III + $2\times2\times$(no. in Ch. I + no. in Ch. III),

or $5\times$(no. in Ch. I + no. in Ch. III) $=\frac{25}{20}$ of total no.;

and $\therefore$ no. in Ch. I + no. in Ch. III $=\frac{1}{5}$ of $\frac{25}{20}$ ...

$=\frac{1}{4}$ ...

Hence no. in Ch. II $=2\times\frac{1}{4}$

$=\frac{1}{2}$

and $\therefore$ no. in Ch. I + $\frac{1}{2}$ of total no. $=\frac{13}{20}$

or no. in Ch. I $=\frac{13}{20}-\frac{1}{2}$

$=\frac{3}{20}$

Similarly, no. in Ch. III $=\frac{3}{5}-\frac{1}{2}$

$=\frac{1}{10}$

Hence no. of pages in the first 3 Ch. $=\frac{1}{2}+\frac{3}{20}+\frac{1}{10}$

$=\frac{15}{20}$

and ... ... Ch. IV $=1-\frac{15}{20}$

$=\frac{5}{20}$

But by the question,

no. of pages in Ch. IV $=20$;

$\therefore \frac{5}{20}$ of total no. $=20$;

and $\therefore$ the total no. of pages $=20\div\frac{5}{20}$

$=80.$

And hence no. of pages in Ch. I $=\frac{3}{20}$ of $80=12$,

..................Ch. II $=\frac{1}{2}$ of $80=40$,

and..................Ch. III $=\frac{1}{10}$ of $80=8$.

Ex. 5. A man dies leaving his father who gets $\frac{1}{6}$ of his estate, 3 widows who divide $\frac{1}{8}$ of his estate equally amongst themselves, and 2 sons and 3 daughters who take the remainder in such a manner that each son gets twice as much as each daughter. Divide the estate into the least number of equal parts such that each claimant may get an integral number of those parts.

The father gets $\frac{1}{6}$ of the estate;

each widow, $\frac{1}{3}$ of $\frac{1}{8}$ or $\frac{1}{24}$.................;

and there remain $1-(\frac{1}{6}+\frac{1}{8})$ or $\frac{17}{24}$.................

Now each son takes 2 parts while each daughter takes 1; $\therefore$ for the children there must be $2+2+1+1+1$ or 7 parts, whereof each son takes 2, and each daughter 1.

Hence each son takes $\frac{2}{7}$ of $\frac{17}{24}$ or $\frac{17}{84}$ of the estate,

and each daughter $\frac{1}{7}$ of $\frac{17}{24}$ or $\frac{17}{168}$..................

Thus the shares of the claimants are

$$\frac{1}{6},\ \frac{1}{24},\ \frac{17}{84},\text{ and }\frac{17}{168};$$

and these fractions reduced to their equivalent ones having the least common denominator, are

$$\frac{28}{168},\ \frac{7}{168},\ \frac{34}{168},\text{ and }\frac{17}{168}.$$

Hence if we divide the estate into 168 equal parts, the father will get 28, each widow, 7, each son, 34, and each daughter, 17 parts; so that 168 is the number required.

Examples XIV.

1. What is a Fraction and why is it so called? What are the two systems of fractions in common use?

2. State the method of Notation of fractions. Point out clearly the relation between the value of a fraction and the result of the division of its numerator by its denominator.

3. Shew that the value of a fraction is not altered if its numerator and its denominator are both multiplied or both divided by the same number.

Reduce [illegible] to its lowest terms.

4. What is a Compound Fraction? Shew how to reduce a Compound Fraction to the form of a Simple Fraction.

Simplify $\frac{3}{100}$ of [illegible] of [illegible] of [illegible].

5. What is a Complex Fraction? Shew how to simplify it.

Simplify [illegible] of $3\frac{3}{10}$.

6. A workman is offered 6 rupees per week (*i. e.*, 7 days) by one employer, and 25 rupees per month (*i. e.*, 30 days) by another. Which is the better remuneration?

II.

1. A person inherits [illegible] of [illegible] of an estate; he next obtains by gift [illegible] of that estate; and he lastly purchases [illegible] of the same. How much of the estate does he now own, and what fraction of the estate remains to be acquired by him, to make him the sole proprietor of the whole?

2. A post has $\frac{1}{3}$ of its length in the mud, $\frac{2}{5}$ in the water and 12 feet above the water. What is the whole length of the post?

3. A gentleman has two sons and a daughter. His age equals the sum of the ages of his children; the age of the eldest child is $\frac{1}{2}$ of his age; that of the second, $\frac{1}{3}$ of his age; and that of the third, 10 years. Find the ages of the father and the first two children.

4. Two numbers are respectively $\frac{1}{2}$ and $\frac{1}{6}$ of a third; and the difference between this last and the sum of the other two is 19. Find the numbers.

5. Find the difference between the reciprocal of the sum of the first four natural numbers and the sum of the reciprocals of the first four even numbers.

6. Find the least fraction which being added to the sum of $\frac{3}{8}$ and $\frac{3}{4}$ will make the result an integer.

III.

1. Find the number of which the double and the triple together exceed the half by $\frac{3}{8}$.

2. If I pay away $\frac{2}{3}$ of my money, then $\frac{3}{4}$ of what remains, and then $\frac{5}{8}$ of what still remains, what fraction of the original amount have I still left?

3. What number must be added to $\frac{\frac{1}{2}}{\frac{7}{18}}+\frac{\frac{2}{7}}{\frac{4}{21}}$ to make the result equal to $\frac{\frac{1}{2}}{\frac{3}{8}}+\frac{\frac{2}{3}}{\frac{3}{8}}$?

4. Each of 3 bags contains a certain number of rupees, such that the amount in the first and the third taken together is twice the amount in the second which is 600 rupees; and the amount in the first and the second together is $\frac{5}{6}$ of the total amount: find the total number of rupees, and the number in the first bag.

5. What number must be taken from $\frac{\frac{2}{3}}{\frac{1}{4}}$ to make the result equal to $\frac{\frac{1}{2}}{\frac{3}{4}}$?

6. Three boys A, B, and C, have each a certain number of marbles. The number belonging to B is $\frac{1}{3}$ of the number belonging to all the three; the number belonging to A and B together is $\frac{5}{6}$ of the total number; and the number belonging to C is 16. Find the total number of marbles, and the number belonging to A.

IV.

1. What do you understand by the Multiplication of a number by a proper fraction?

Give the reason for the Rule for the Multiplication of Fractions.

2. Multiply the sum of [illegible], [illegible], and [illegible], by the difference between [illegible] and [illegible].

3. What number multiplied by [illegible] of [illegible]—[illegible] of $\frac{1}{10}$ will produce [illegible] of [illegible]?

4. The sum of the ages of two boys is 24 years, and the difference of their ages is [illegible] of the age of the younger. What is the age of each?

5. The sum of two fractions is 2[illegible] times their difference, and the greater is [illegible]. Find the less.

6. Find the continued product of [illegible], 4[illegible], 5[illegible], and 6[illegible].

V.

1. What is the meaning of the quotient arising from the Division of one fraction by another?

State the Rule for the Division of Fractions, and give the reason for that Rule.

2. A number divided by 2 becomes half of what it is, and a number divided by $\frac{1}{2}$ becomes double of what it is. Explain clearly the reason for this. What do you understand by Division in the latter case?

3. What number divided by 3[illegible] will produce 5[illegible]?

4. What number multiplied by the sum of [illegible] and $\frac{1}{10}$ will equal the quotient arising from the division of the difference of [illegible] and [illegible] by the sum of [illegible] and [illegible]?

5. Divide the product of $\frac{3[illegible]}{2[illegible]}$ and [illegible] by the quotient arising from the division of $\frac{4[illegible]}{3[illegible]}$ by 2[illegible].

6. By what number must you divide 9[illegible] to make the quotient equal to 23?

VI.

1. Simplify :—

(1) $\frac{2 \div [illegible]}{3 \times [illegible]} \times \frac{5[illegible]}{7[illegible]} \div 7$. (2) $\frac{[illegible] + [illegible]}{[illegible] + [illegible]}$ of [illegible].

2. *A* owns [illegible] of an estate, and sells [illegible] of his share to *B*. How much more of his share must *A* sell to *B* to make their shares equal, and what share of the entire estate will each own in that case?

3. In a certain field there are 66 trees arranged in 3 rows. The number of trees in the 1st row is 3 times that in the 3rd, and 2 times that in the 2nd. How many trees are there in each row?

4. Find the sum of the sum and the difference of $2\frac{1}{3}$ and $3\frac{1}{4}$, without performing the operations of Addition and Subtraction of fractions.

5. What part of $\frac{3}{4}$ is $\frac{2}{3}$, and what part of $\frac{3}{10}$ is $\frac{2}{3}$?

6. A man dies leaving 2 widows, 3 sons, and 4 daughters. His widows are entitled to $\frac{1}{8}$ of his property, to be divided equally between them, and his children to the remainder, to be divided amongst them in such a manner that the share of a son shall be double of that of a daughter. Divide the property into the least number of parts such that each claimant may get an integral number of those parts.

DIVISION II. DECIMALS.

SECTION VII. NOTATION AND NUMERATION OF DECIMALS.

110. We have seen in Art. 79, that decimal fractions are those that consist of secondary units or parts which are tenths, hundredths, &c., of the primary unit. Hence, a decimal is a fraction having 10, 100, *i. e.* 10^2, or some other power of 10, for its denominator, and the number of tenths, hundredths, or other parts that it consists of, for its numerator; and it may be expressed in the same manner as a vulgar fraction.

Thus

three-tenths, *seven hundredths*, *twenty-six thousandths*,

may be written

$$\tfrac{3}{10}, \qquad \tfrac{7}{100}, \qquad \tfrac{26}{1000}.$$

111. The numerator of a decimal fraction may be analyzed into its constituent digits which would represent the number of tenths, the number of hundredths, &c., that compose the fraction.

Thus, take as an example the decimal fraction $\frac{236}{1000}$.

Then $\frac{236}{1000} = \frac{200+30+6}{1000} = \frac{200}{1000} + \frac{30}{1000} + \frac{6}{1000}$

$$= \tfrac{2}{10} + \tfrac{3}{100} + \tfrac{6}{1000}.$$

Take another decimal fraction $\frac{705}{10000}$.

Then $\frac{705}{10000} = \frac{700+0\times10+5}{10000} = \frac{700}{10000} + \frac{0\times10}{10000} + \frac{5}{10000}$

$$= \tfrac{7}{100} + \tfrac{0}{1000} + \tfrac{5}{10000}.$$

From the above we see that a decimal fraction may be regarded as a number composed of the figures of its numerator, the first figure on the left representing so many tenths or hundredths, or the like, as the case may be, and the others having their local values decreasing tenfold at each step towards the right.

112. The preceding Article *suggests* another method of Notation for decimals, as will be seen below.

In the Common System of Notation, the local value of every figure *increases* tenfold at each step towards the *left*, *i. e.*, *decreases* tenfold at each step towards the *right*. If then we consider this scale of tenfold decrease of local value *extended* to the right of the units' place, and put figures there, separated from the other figures by a mark such as a dot, we shall have to the right of the dot a series of figures whose local values are so many *tenths*, *hundredths*, *&c.*, and these figures will in every case represent some decimal fraction or other.

Thus, $267\cdot203 = 267 + \frac{2}{10} + \frac{0}{100} + \frac{3}{1000}$
$= 267\frac{203}{1000}$,

i. e., ·203 represents the decimal fraction $\frac{203}{1000}$.

So, $\cdot023 = \frac{0}{10} + \frac{2}{100} + \frac{3}{1000}$
$= \frac{23}{1000}$,

i. e., ·023 represents the decimal fraction $\frac{23}{1000}$.

And so in other cases.

Hence we may deduce the following Rule for the Notation of decimals :—

Rule. Having written in figures the integral part of the number, if any, place a dot on its right, and on the right of the dot write the numerator of the decimal fraction, preceded by ciphers, if necessary, to make the number of figures to the right of the dot equal to the number of ciphers in the denominator.

Ex. Express in figures, two hundred and fifty-six, and fifty-three thousandths.

Here, there being 3 ciphers in the denominator, the numerator 53 must have 1 cipher prefixed to it, before it is put after the dot ; and the number will be written thus :—

256·053.

And hence a decimal expressed in the above mode can be reduced to the form of a vulgar fraction by the following Rule :—

Rule. Write the decimal, omitting the dot and the ciphers just after it, as the numerator, and 1 followed by as many ciphers as there are figures to the right of the dot, for the denominator.

Ex. Express ·028 as a vulgar fraction.

We have $\cdot028 = \frac{28}{1000} = \frac{7}{250}$.

Def. The dot separating the integral part of a number from the decimal is called the **Decimal Point,** and the places of figures to the right of the dot are called the **Decimal Places.**

113. The above is the usual method of Notation for decimals. We have thus a uniform *ascending* and *descending* scale of Notation extending without limit to the *left* and the *right* of the units' place, the former or the ascending part of the scale consisting of tens, hundreds, &c., and being sufficient for the expression of all possible integers ; and the latter or the descending part of the scale consisting of tenths, hundredths, &c., and being sufficient for the expression of all possible *decimal* fractions.

It is this capability of being expressed in a uniform system of Notation with integers, that constitutes the peculiar advantage of decimals over vulgar fractions, and makes them peculiarly adapted for numerical calculation, as the student will hereafter see.

114. The above system is evidently convenient and complete for the expression of *decimal* fractions. It now remains to be seen whether *every possible* fraction can be expressed in this system.

Take the fraction $\frac{1}{2}$.

Then $\frac{1}{2} = \frac{1}{2} \times \frac{10}{10} = \frac{5}{10} = {}^{\cdot}5$.

Next take $\frac{1}{3}$.

$$\text{Then } \frac{1}{3} = \frac{1}{3} \times \frac{10}{10} = \frac{\frac{1}{3} \times 10}{10} = \frac{3 + \frac{1}{3}}{10}$$

$$= {}^{\cdot}3 + \frac{1}{10} \times \frac{1}{3};$$

$$\text{or } = \frac{1}{3} \times \frac{100}{100} = \frac{\frac{1}{3} \times 100}{100} = \frac{33 + \frac{1}{3}}{100}$$

$$= {}^{\cdot}33 + \frac{1}{100} \times \frac{1}{3},$$

$$\text{or similarly} = {}^{\cdot}333 + \frac{1}{1000} \times \frac{1}{3}.$$

&c. &c.

This shews that the operation will never terminate, or in other words, that $\frac{1}{3}$ can never be *exactly* expressed as a decimal, though by continuing the operation, and taking more and more places of of decimals, the difference between $\frac{1}{3}$ and the decimal becomes successively $\frac{1}{30}$, $\frac{1}{300}$, $\frac{1}{3000}$, &c., and will grow less and less ; *i.e.*, we can have decimals *approximating* to the value of $\frac{1}{3}$, without ever being exactly equal to it.

From the first of the above two examples, we see that *some* vulgar fractions can be exactly expressed as decimals ; and from the second we see that some again cannot be so expressed, though we can have decimals approximating to their value.

115. A Numeration Table for decimals may be given similar to that for integers.

&c.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	&c.

Accordingly, taking as an example, any number, 139·0232, it may be read as, one hundred and thirty-nine, and two hundredths,

three thousandths and two ten thousandths. But 2 hundredths + 3 thousandths + 2 ten thousandths

$$=\frac{2}{100}+\frac{3}{1000}+\frac{2}{10000}$$

$$=\frac{232}{10000},$$

or two hundred and thirty-two ten thousandths. Thus the decimal part may be read in two ways, whereof the latter is the shorter, and being similar to the mode of naming ordinary fractions by naming separately the numerator and the denominator, is the one usually adopted. Besides these, there is another common mode of naming decimals, in which the figures in the successive decimal places are read out one after another. Thus the above number will be read thus :—one hundred and thirty-nine, decimal, nought, two, three, two.

116. Prop. I. A decimal is multiplied or divided by 10 by removing the decimal point one place towards the right or the left.

Thus, taking any decimal $2{\cdot}307$,

we have $2{\cdot}307\times10=(2+\frac{307}{1000})\times10=\frac{2307}{1000}\times10$

$$=\frac{2307}{100}=23\tfrac{7}{100}=23{\cdot}07\ ;$$

and $2{\cdot}307\div10=\frac{2307}{1000}\times\frac{1}{10}=\frac{2307}{10000}$

$$={\cdot}2307.$$

Prop. II.—The value of a decimal is not altered by affixing ciphers to its right.

Thus, taking any decimal ${\cdot}237$ we have

$${\cdot}237=\frac{237}{1000}=\frac{237\times10}{1000\times10}=\frac{2370}{10000}={\cdot}2370.$$

Prop. III. The value of a decimal not having an part is decreased tenfold by prefixing a cipher to its left right of the decimal point.

Thus, ${\cdot}0237=\frac{237}{10000}=\frac{237}{10\times1000}=\frac{1}{10}\times\frac{237}{1000}=\frac{1}{10}\times{\cdot}237.$

These effects are very different from the effects of affixing and prefixing ciphers to integers.

Examples XV.

1. Express in figures the following :

(1) Three-tenths ; seven-tenths ; five hundredths ; sixty-six hundredths ; five and five hundredths ; six hundred and sixty and sixty-nine hundredths ; one hundred and one hundredth.

(2) One thousandth ; ninety-nine and nine thousandths ; one hundred and twenty-three and forty-five thousandths.

(3) One million, and one millionth; five millions and fifty, and two thousand and fifty-three millionths.

2. Express in words the following :—

(1) ·02 ; 1·03 ; 21·12 ; 1·0001 ; 20·0002.

(2) 123·456 ; 7891·01112 ; 13·1517.

(3) ·000001 ; ·00000050 ; ·00500.

3. Convert the following decimals into vulgar fractions :—

(1) ·1 ; ·12 ; 12·34 ; 567·8 ; ·01.

(2) ·0001 ; ·002 ; ·03 ; 100·002.

(3) 35·970 ; ·00200 ; 12·321.

4. Express as vulgar fractions in their lowest terms the following :—

(1) ·25 ; 2·5 ; ·0025 ; ·002500.

(2) 56·64 ; 72·0028 ; 1·002.

(3) ·128 ; 17·28 ; ·0032 ; 61·64.

5. Express as decimals :—

(1) $\frac{1}{10}$; $\frac{5}{10}$; $2\frac{3}{10}$; $5\frac{6}{20}$; $9\frac{8}{40}$.

(2) $\frac{11}{10000}$; $\frac{82102}{1000}$; $\frac{6814}{100}$; $57\frac{6}{80}$.

(3) $\frac{12345}{100}$; $\frac{123456789}{1000000}$; $\frac{100100}{1000000}$.

6. Multiply

(1) ·3 by 10, 100, and 1000.

(2) ·003 by 10, 100, and 1000.

(3) 20·00020 by 100 and 10000.

(4) ·156 by 10000 and 100000.

(5) 20·200 by 10 and 100.

7. Divide

(1) ·300 by 100 and 100000.

(2) 3·156 by 100 and 1000.

(3) 3567·1 by 10000 and 10.

(4) 98765·005 by 100 and 10000.

(5) 3·141500 by 100 and 10000.

SECTION VIII. ADDITION OF DECIMALS.

117. **Rule.** Place the numbers so that units may be under units, tens under tens, &c., and tenths under tenths, hundredths under hundredths, &c.

Commencing with the column of figures on the extreme right, add the numbers as in the Addition of integers, and in the sum, place the decimal point just below the line of decimal points above.

Ex. Add together 237·008, 5·53298, ·023 and 1·61001.

By the Rule we have

$$\begin{array}{r} 237\cdot008 \\ 5\cdot53298 \\ \cdot023 \\ 1\cdot61001 \\ \hline 244\cdot17399 \end{array}$$

Reason for the Rule. Since in our Notation, there is a *progressive increase* of local values *tenfold* at each step from right to left, the number of *tens* resulting from the addition of the intrinsic values of the figures in every column must be carried and added to the column to its left; and the figure in the sum below each column will have the local value of that column. In other words, the addition is to performed as in the case of integers, and the decimal point is to be placed below the line of decimal points above.

The *reason for the Rule* may be also shewn thus :—

$$237\cdot008 = 237\tfrac{8}{1000} = \tfrac{237008}{1000}.$$

Similarly,

$$5\cdot53298 = \tfrac{553298}{100000}\,;\ \cdot023 = \tfrac{23}{1000}\,;\ \text{and } 1\cdot61001 = \tfrac{161001}{100000}.$$

$$\text{Therefore the sum} = \tfrac{237008}{1000} + \tfrac{553298}{100000} + \tfrac{23}{1000} + \tfrac{161001}{100000}$$

$$= \frac{23700800+553298+2300+161001}{100000}$$

$$= \tfrac{24417399}{100000} = 244\cdot17399.$$

Examples XVI.

1. Add together

(1) ·12345, 1·2345, 12·345, 123·45 and 1234·5.

(2) 1000·1, 200·02, 30·003, 4·0004 and ·50005.

(3) ·000123, ·0045, ·067, ·89, ·10 and 11.

(4) 123·4589, 1234·56789 and 12345·6789.

(5) 27·0039, ·00009, 1000, 556, and ·00556.

(6) 1·003, 40·0005, 600·00007, and 80000·000009.

2. Find the sum of

(1) Three-tenths; seven hundredths; seventeen thousandths; and two hundreds, and fifty-three millionths.

(2) Fifty-five hundredths; sixty-six thousandths; and seventy-seven millionths.

(3) One hundred and twenty thousandths; one million and one millionth; and fifty thousands, and fifty thousandths.

8. Find the value of

(1) 200·003 + 123·789 + 88·009 + 35·005.

(2) ·0073 + ·173 + 128·359 + 627·047.

(3) 57·68 + 68·79 + 79·810 + 810 + ·911.

(4) ·0047 + ·00059 + ·0000611 + ·00000713.

(5) 1200·3 + 567·27 + 827·447 + 1·2.

(6) 55·55 + 66·66 + 77·77 + 88·88 + 99·99.

SECTION IX. SUBTRACTION OF DECIMALS.

118. Rule. Place the subtrahend below the minuend as in the Addition of decimals.

Affix ciphers to the right of either decimal, if necessary, to make the number of decimal places the same in both.

Then perform the subtraction as in the case of integers, and place the decimal point in the difference below the decimal point above.

Ex. Subtract 2·932 from 26·03.

By the Rule we have

$$\begin{array}{r} 26·030 \\ \underline{2·932} \\ 23·098 \end{array}$$

Reason for the Rule. The affixing of ciphers to the right does not alter the value of a decimal (Art. 116). The rest of the reason is the same as that given in the case of the Addition of decimals.

The reason for the Rule may be also shewn thus :—

$$26·03 - 2·932 = \frac{2603}{100} - \frac{2932}{1000}$$
$$= \frac{26030 - 2932}{1000} = \frac{23098}{1000}$$
$$= 23·098.$$

Examples XVII.

1. Subtract

(1) 1·23 from 45·6. (2) 23·45 from 67·89.

(3) 13·3 from 15·27. (4) 29·02 from 30·30.

(5) 41·0056 from 59·9. (6) 356·01 from 1000·0004.

2. Find the difference between

(1) One and one tenth.

(2) Three and three hundredths.

(3) One and one millionth.

(4) One million and one millionth.

(5) Seven and seventenths.

(6) Nine tenths and nine hundredths.

3. Find the value of

(1) ·0235 − ·0087956520. (2) ·3 − ·00999.

(3) 2·37 − 1·0047. (4) 53·008 + ·6279 − 7·08.

(5) 7·777 + 99·99 − 11·111. (6) ·1056 + 5600 − 5·600.

SECTION X. MULTIPLICATION OF DECIMALS.

119. Rule. Multiply the numbers as if they were integers, and in the product, mark off a number of decimal places equal to the sum of the number of decimal places in the multiplicand and the number of decimal places in the multiplier, prefixing ciphers to the left, if necessary.

Ex. 1. Multiply 67·51 by 2·06

By the Rule we have

$$\begin{array}{r} 6751 \\ 206 \\ \hline 40506 \\ 135020 \\ \hline 1390706 \end{array}$$

As the multiplicand has 2, and the multiplier 2, decimal places, in the product there will be 2 + 2 or 4 decimal places; ∴ the product is 139·0706.

Ex. 2. Multiply ·0027 by 15.

Since ·0027 regarded as an integer is the same as 27, we have

$$\begin{array}{r} 27 \\ 15 \\ \hline 135 \\ 27 \\ \hline 405 \end{array}$$

The number of decimal places in the product = 4 + 0 = 4; and we ∴ prefix 1 cipher to 405 to make the number of decimal places 4; and the product required is ·0405.

The reason for the Rule will be seen below.

Taking Ex. 1, we have 67·51 × 2·06

$$= \frac{6751}{100} \times \frac{206}{100} = \frac{6751 \times 206}{10000} = \frac{1390706}{10000} = 139·0706.$$

Next taking Ex. 2, we have ·0027 × 15

$$= \frac{27}{10000} \times 15 = \frac{27 \times 15}{10000} = \frac{405}{10000} = ·0405.$$

These Examples shew that to multiply decimals is the same thing as to multiply them as integers, and then mark off in the product a number of decimal places according to our Rule.

Examples XVIII.

1. Multiply—

(1) 9·8 by 11·10. (2) 7·65 by 43·21.

(3) ·0023 by 2300. (4) 56 by ·0056.

(5) 357·701 by 3·003. (6) 4729·01 by ·0076.

2. Find the product of

(1) One million and one millionth.

(2) Seven tenths and eight hundredths.

(3) One hundred and one thousandth.

(4) Sixty-seven and sixty-seven hundredths.

(5) One hundred and fifty thousandths.

(6) Three hundredths and six.

3. Find the value of

(1) 3·2 × 32 × ·02 × ·002. (2) 57·29 × 5·729 × ·00006.

(3) 127·358 × 359·009. (4) ·00056 × ·0067 × ·00001.

(5) ·001 × 200 × 3·30. (6) ·127 × 450·054 × ·09.

SECTION XI. DIVISION OF DECIMALS.

120. Rule. Perform the division as if the dividend and the divisor were integers.

If the number of decimal places in the dividend equals that in the divisor, the quotient obtained will be the one required; if it exceeds that in the divisor, mark off in the quotient a number of decimal places equal to the difference between the two, prefixing ciphers to the left if necessary; and if it is less than the number of decimal places in the divisor, affix a number of ciphers to the right of the quotient equal to the difference between the two.

If, when regarded as integers, the divisor exceeds the dividend, or if the division does not terminate, then affix ciphers to the right of the dividend, and carry on the operation as long as necessary, taking care, in pointing the quotient, to regard these ciphers as so many additional places of decimals in the dividend.

Ex. 1. Divide 1·2 by ·025.

By the Rule we have (∵ ·025 regarded as an integer is the same as 25)

$$
\begin{array}{r l}
25) & 12{,}00\ (48 \\
 & \underline{100} \\
 & \ \ 200 \\
 & \ \ \underline{200}
\end{array}
$$

We put a comma to separate the additional ciphers affixed. Here the total number of decimal places in the dividend being 3, *i. e.*, the same as that in the divisor, the quotient required is 48.

Ex. 2. Divide ·272 by 2·9 to 4 places of decimals in the quotient.

By the Rule we have

$$
\begin{array}{r l}
29) & 272{,}00\ (937 \\
 & \underline{261} \\
 & \ \ 110 \\
 & \ \ \ \underline{87} \\
 & \ \ 230 \\
 & \ \ \underline{203} \\
 & \ \ \ \ 27
\end{array}
$$

We need not carry on the division further, as we have now the number of decimal places in the quotient $= 5 - 1 = 4$, the required number, and the quotient required is ·0937...

The reason for the Rule will be seen below.

Taking Ex. 1, we have $1{\cdot}2 \div {\cdot}025 = \frac{12}{10} \div \frac{25}{1000}$

$$= \frac{12}{10} \times \frac{1000}{25} = \frac{12}{25} \times \frac{1000}{10} = \frac{12 \times 100}{25} \times \frac{1}{100} \times \frac{1000}{10} = 12 \times 4 = 48.$$

Next, taking Ex. 2, we have ${\cdot}272 \div 2{\cdot}9 = \frac{272}{1000} \div \frac{29}{10}$

$$= \frac{272}{1000} \times \frac{10}{29} = \frac{272}{29} \times \frac{10}{1000} = \frac{27200}{29} \times \frac{1}{100} \times \frac{10}{1000} = \frac{27200}{29} \times \frac{1}{10000}.$$

Now $\frac{27200}{29} = 937......$

$\therefore\ \frac{27200}{29} \times \frac{1}{10000} = {\cdot}0937......$

These Examples shew that in the Division of decimals, we divide the numbers as if they were integers, affixing ciphers to the dividend if necessary, care being taken to place the decimal point in the quotient according to our Rule.

Examples XIX.

1. Divide

(1) ·5568 by 2·32.	(2) 2·292 by ·0135.
(3) 78·8977 by 26·33.	(4) 66·4488 by 9·9.
(5) 78·78 by ·0026.	(6) ·00624 by 2·08.
(7) 1122·3333 by ·99.	(8) 121416·3 by ·009.
(9) 2122·2 by ·0018.	(10) 6·33 by ·0025.
(11) 33·363 by ·00275.	(12) 94·5 by ·225.

2. Divide to four places of decimals

(1) 247·943 by 13·3.	(2) 1516·17 by ·0023.
(3) 78·007 by ·0135.	(4) ·0023 by 6·69.
(5) 64·005 by 17·6.	(6) ·0008 by 3·3.
(7) 22·25 by 23·8.	(8) ·0025 by ·009.
(9) 135·002 by ·121.	(10) ·007 by ·00073.

3. Find the value of

(1) 2·5 ÷ ·0025.	(2) ·25 ÷ ·025.
(3) ·11 ÷ ·0011.	(4) ·0143 ÷ 1·3.
(5) 1710 ÷ ·0171.	(6) ·0247 ÷ ·0013.

4. What number multiplied by ·25 will produce 63?

5. What number multiplied by ·225 will produce 126?

6. What number multiplied by the quotient arising from the division of 12 by ·075 will produce 13?

SECTION XII. CONVERSION OF VULGAR FRACTIONS INTO DECIMALS. RECURRING DECIMALS.

121. We have seen in Arts. 110-112 how to convert a decimal into a vulgar fraction. We have also seen in Art. 114 that some vulgar fractions can be converted into decimals consisting of a *finite* number of figures, and others cannot. We will now give a general Rule for the conversion of vulgar fractions into decimals, and ascertain where such conversion is *exactly* possible and where not.

122. *To convert a vulgar fraction into a decimal.*

Rule. Having reduced the fraction to its lowest terms, divide the numerator by the denominator as in the Division of decimals, affixing ciphers to the numerator, and carrying on the division as far as necessary.

Since a fraction denotes the quotient of the numerator by the denominator the *reason for the Rule* is evident.

Ex. 1. Convert $\frac{2}{125}$ into a decimal.

By the Rule we have

$$\begin{array}{r} 125)\ 2{,}000\ (16 \\ \underline{125} \\ 750 \\ \underline{750} \end{array}$$

$\therefore \frac{2}{125} = \cdot 016.$

The following mode will at once indicate the process and the reason for it.

$$\frac{2}{125} = \frac{2}{5\times 5\times 5} \times \frac{10\times 10\times 10}{10^3} = \frac{2}{5\times 5\times 5} \times \frac{5\times 2\times 5\times 2\times 5\times 2}{10^3}$$
$$= \frac{2\times 2\times 2\times 2}{10^3} = \cdot 016.$$

Here we resolve the denominator into factors, and multiply the numerator and denominator by such a power of 10, as will enable us to cancel all these factors, *i. e.*, as is divisible exactly by the denominator.

Ex. 2. Convert $\frac{3}{80}$ into a decimal.

Following the second mode, we have

$$\frac{3}{80} = \frac{3}{10\times 2\times 2\times 2} \times \frac{10\times 10\times 10}{10^3} = \frac{3\times 5\times 5\times 5}{10^4} = \cdot 0375.$$

Ex. 3. Convert $\frac{5}{18}$ into a decimal.

By the Rule we have

$$\begin{array}{r} 18)\ 5{,}0000\ (2777 \\ \underline{36} \\ 140 \\ \underline{126} \\ 140 \\ \underline{126} \\ 14 \end{array}$$

$\therefore \frac{5}{18} = \cdot 2777.$

123. Prop. I.—A proper fraction in its lowest terms can be converted into a *terminating* decimal, only when the denominator is composed *solely* of the factors 2 and 5.

For, the process for converting a fraction into a decimal, as we have seen in Art. 122, consists only in affixing ciphers to the numerator, *i. e.*, multiplying it by some power of 10, and after dividing the result by the denominator, marking off the proper number of decimal places in the quotient. And as, the fraction

being in its lowest terms, the numerator and the denominator have no common factor, ∴ the division will terminate and we shall have a terminating decimal, only when the denominator will divide the power of 10 exactly *i. e.*, when the denominator consists solely of the factors 2 and 5, which are the only factors of which 10 and its powers are composed.

Prop. II. When the division of the numerator of a proper fraction, with ciphers affixed, by the denominator, does not terminate, the figures in the quotient must begin to recur before we have got a number of them equal to the number of units in the denominator.

For, in carrying on the division, as we always bring down the same figure 0 from the dividend to constitute the successive partial dividends, these partial dividends, and therefore the figures in the quotient, will begin to recur as soon as we get a remainder equal to one of the former remainders. Now ∵ every remainder must be less than the divisior, *i.e.*, the denominator, ∴ the different possible remainders are 1, 2, &c., up to the integer next below the denominator : and ∴ the *number* of *different* possible remainders = *the denominator minus unity.* And ∵ the numerator, which forms with affixed ciphers the first partial dividend, is also less than the denominator, ∴ before we have got a number of different partial dividends and a corresponding number of figures in the quotient, equal to the number of units in the denominator, one of the former partial dividends must recur, and thence the figures in the quotient will begin to recur.

Thus take as examples $\frac{1}{3}$, $\frac{3}{7}$, and $\frac{53}{370}$.

We have

I.
```
3)1·00(·33
  9
  10
   9
   1
```

II.
```
7)3·0000000(·4285714
  28
   20
   14
    60
    56
     40
     35
      50
      49
       10
        7
       30
       28
        2
```

III.
```
370)53·00000(·14324
    370
    1600
    1480
     1200
     1110
       900
       740
       1600
       1480
        120
```

In I, we see that the partial dividends recur from the first, and the figures of the quotient are 33......

In II, the partial dividends recur, after we get all the possible different partial dividends, *viz.*, 10, 20, 30, 40, 50, 60, in the order 30, 20, 60, 40, 50, 10, and the figures in the quotient recur after the sixth, *viz.*, 1, in the order 4, 2, &c.

In III, the partial dividends recur after the fourth, and the figures in the quotient recur after the fourth in the order 432 &c., the first figure 1 not recurring at all.

124. Defs. Non-terminating decimals in which the figures thus recur are called **Recurring** or **Circulating Decimals**.

A circulating decimal is called **Pure** or **Mixed** according as its figures recur from the first or not.

The set of recurring figures is called the **Period** or **Repetend**.

Thus, ·333..., ·428571428571428571..., are pure circulating decimals, and ·1432432432..., ·2777..., are mixed circulating decimals.

125. Circulating decimals are written by writing the figures only to the end of the first period, and putting dots over the first and last figures of the period.

Thus, ·333...is written $\cdot\dot{3}$,

·428571428571... ... $\cdot\dot{4}2857\dot{1}$,

·1432432... ... $\cdot 1\dot{4}3\dot{2}$

·277... ... $\cdot 2\dot{7}$

We now proceed to the converse process of converting recurring decimals into vulgar fractions.

126. *To convert a pure circulating decimal into a vulgar fraction.*

Rule.* Write the period as the numerator; and for the denominator write as many *nines* as there are figures in the period; and then reduce the fraction to its lowest terms, if necessary.

Ex. 1. Convert $\cdot\dot{3}$ into a vulgar fraction.

By the Rule we have $\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}$.

Ex. 2. Convert $\cdot\dot{4}2857\dot{1}$ into a vulgar fraction.

We have $\cdot\dot{4}2857\dot{1} = \frac{428571}{999999} = \frac{47619}{111111} = \frac{4329}{10101} = \frac{1443}{3367} = \frac{3}{7}$.

Reason for the Rule. Take Ex. 2.

Let the circulating decimal $\cdot428571428571\ldots$ be represented by the symbol x.

$$\text{Then} \qquad x = \cdot428571428571\ldots$$
$$\text{and} \qquad 1000000 \times x = 1000000 \times \cdot428571428571\ldots$$
$$= 428571\cdot428571428\ldots \text{(Art 116, Prop. I)};$$
$$\therefore \quad 1000000 \times x - x = 428571\cdot428571428571\ldots$$
$$- \cdot428571428571\ldots,$$
$$i.\ e., \ 999999 \times x = 428571.$$
$$\text{Hence} \qquad x = \tfrac{428571}{999999}.$$

Here, the *artifice* employed is to multiply x and the circulating decimal by such a power of 10 as will make the first period an integer, and then to get rid of the non-terminating part by subtraction.

Next, take Ex. 1, and let $x = \cdot333\ldots\ldots$

Then $\because$ the period has only one figure 3, it will be made an integer by multiplication by 10; and we have

$$10 \times x = 3\cdot333\ldots\ldots;$$
$$\therefore \quad 10 \times x - x = 3\cdot333\ldots\ldots$$
$$- \cdot333\ldots\ldots,$$
$$\text{or } 9 \times x = 3,$$
$$\text{and } \therefore x = \tfrac{3}{9}.$$

127. *To convert a mixed circulating decimal into a vulgar fraction.*

Rule. From the decimal down to the end of the first period, subtract the non-recurring part, regarding both as integers, and write the remainder as the numerator; and for the denominator write as many *nines* as there are figures in the period, followed by as many *ciphers* as there are figures in the non-recurring part.

Ex. Convert $\cdot1\dot{4}3\dot{2}$ into a vulgar fraction.

By the Rule we have $\cdot1\dot{4}3\dot{2} = \frac{1432-1}{9990} = \frac{1431}{9990} = \frac{159}{1110} = \frac{53}{370}$.

Reason for the Rule.

Let $x = \cdot1432432\ldots$

Then, employing the same artifice as in Art. 126, successively to make the decimal to the end of the first period an integer, and the decimal to the end of the non-recurring part an integer, we have

$$10000 \times x = 1432\cdot432\ldots$$
$$\text{and} \quad 10 \times x = \quad 1\cdot432432\ldots$$

$\therefore$ by subtraction

$$9990 \times x = 1431$$
$$\text{and } \therefore \qquad x = \tfrac{1431}{9990}.$$

128. To perform *accurately* the fundamental operations with recurring decimals, we must reduce them to vulgar fractions, and then perform the operations with these last; and then we can reduce the resulting vulgar fraction to a decimal.

We can however perform these operations *approximately* without reducing circulating decimals to vulgar fractions, as will be seen in the next Section.

Examples XX.

1. Convert into decimals the following vulgar fractions :—

(1) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ and $\frac{1}{32}$. (2) $\frac{3}{8}, \frac{3}{16}, \frac{9}{32}, \frac{9}{25}$ and $\frac{27}{80}$.

(3) $\frac{1}{5}, \frac{1}{25}, \frac{1}{125}$ and $\frac{1}{625}$. (4) $\frac{3}{4}, \frac{4}{5}, \frac{5}{8}, \frac{9}{16}$ and $\frac{9}{625}$.

(5) $\frac{131}{275}, \frac{626}{640}, \frac{329}{400}$ and $\frac{76}{190}$. (6) $\frac{70}{128}, \frac{56}{175}, \frac{26}{104}$ and $\frac{27}{72}$.

2. Reduce the following vulgar fractions to decimals correctly to 5 places of decimals :—

(1) $\frac{9}{37}, \frac{8}{26}, \frac{7}{13}, \frac{8}{15}$ and $\frac{10}{11}$. (2) $\frac{1}{3}, \frac{1}{7}, \frac{1}{6}, \frac{1}{11}$ and $\frac{1}{13}$.

(3) $\frac{2}{13}, \frac{3}{17}, \frac{4}{19}, \frac{5}{21}$ and $\frac{6}{23}$. (4) $\frac{11}{12}, \frac{12}{13}, \frac{13}{14}$ and $\frac{14}{15}$.

(5) $\frac{21}{22}, \frac{22}{23}, \frac{23}{24}$ and $\frac{24}{25}$. (6) $7\frac{2}{7}, 8\frac{4}{9}, \frac{7}{131}$ and $\frac{8}{169}$.

3. Reduce the following vulgar fractions to recurring decimals :—

(1) $\frac{3}{7}, \frac{5}{9}, \frac{9}{11}, \frac{11}{13}, \frac{13}{14}$ and $\frac{14}{15}$. (2) $\frac{5}{18}, \frac{20}{21}, \frac{10}{13}$ and $\frac{8}{14}$.

(3) $\frac{37}{63}, \frac{89}{91}, \frac{43}{63}$ and $\frac{54}{121}$. (4) $\frac{62}{81}, \frac{16}{70}, \frac{16}{700}$ and $\frac{18}{81}$.

(5) $\frac{11}{33}, \frac{14}{35}, \frac{21}{36}$ and $\frac{167}{187}$. (6) $\frac{7}{9}, \frac{49}{64}, \frac{144}{1728}$ and $\frac{8}{880}$.

4. Convert the following decimals into vulgar fractions :—

(1) $\cdot\dot{7}$, $\cdot\dot{8}\dot{9}$, $\cdot8\dot{1}$, $\cdot\dot{9}\dot{3}$ and $\cdot\dot{1}2\dot{3}$.

(2) $5\cdot7\dot{8}$, $4\cdot6\dot{2}$, $2\cdot3\dot{7}$ and $9\cdot1\dot{7}$.

(3) $1\cdot3\dot{6}$, $5\cdot7\dot{3}$, $6\cdot\dot{2}\dot{8}$ and $\cdot7\dot{5}$.

(4) $\cdot7\dot{2}$, $\cdot8\dot{3}$, $\cdot8\dot{1}$, $8\cdot\dot{5}$ and $\cdot6\dot{7}$.

(5) $\cdot0\dot{2}$, $\cdot003\dot{2}$, $\cdot67\dot{9}$ and $1\cdot\dot{3}6\dot{9}$.

(6) $\cdot\dot{8}0\dot{3}$, $\cdot00\dot{3}$, $\cdot0\dot{7}$ and $\cdot\dot{8}6\dot{5}$.

SECTION XIII. APPROXIMATE DECIMAL OPERATIONS.

129. The symbol $<$ read *less than* indicates that the number before it, is less than that after it, and the symbol $>$ read *greater than*, indicates that the number before it, is greater than that after it.

130. We have seen in *theory*, what millions, billions, trillions, &c., are; but in *practice*, we seldom have to deal with numbers above hundreds of millions. Thus, the population of India is about 200 millions: and that of the world is estimated at 1110 millions. The annual revenue of all India is about 60 crores or 600 millions of rupees. The mean distance of the Sun from the Earth is less than 96 millions of miles.

The higher numbers, billions, &c., are very large, indeed so very large, that we cannot readily form any adequate conception of their magnitude. Thus, if it is asked, "How much will a trillion of grains of rice weigh?", one who has never considered the question may, considering the smallness of a grain of rice, answer that the weight will be a few maunds, or at the most, a few hundred maunds. But on a little consideration it will be found that, the weight is something much greater. For, 1 maund $=40$ seers, and 1 seer $=80$ tolas; $\therefore$ 1 maund $=80\times40$ tolas $=3200$ tolas, and on counting the number of grains contained in a quantity of rice weighing 1 tola, it will be found, that it never exceeds 1000, so that 1 maund can never contain more than 3200×1000 or 3200000 grains of rice; and $\therefore$ the number of maunds in one trillion of grains of rice will be at least equal to $\frac{1000000000000000000}{3200000}=\frac{10000000000000}{32}=312500000000$, which is an immense quantity.

131. As numbers above millions are very large numbers, and seldom occur in practice, so numbers below millionths are very small, and may be *neglected* in practice without any *appreciable* error in the results of most of our calculations. Thus, taking the ordinary unit of money, 1 rupee, $\frac{1}{100}$th part of 1 rupee $=\frac{64}{100}$ pice which is <1 pice; $\therefore$ $\frac{1}{1000000}$ of 1 rupee $<\frac{1}{10000}$ of 1 pice, *i. e.*, ·000001 of 1 rupee $<$ ·0001 of 1 pice, which is almost inappreciable. Hence, in dealing with decimals, where a rupee is the unit, if we reject the decimal places after the 6th, we shall be rejecting what is almost inappreciable. And the same thing may be shewn of other units. Generally, *if we carry on our operations correctly to 6 or 7 places of decimals, we shall have an approximation sufficient for all practical purposes.*

We now proceed to give Rules for these approximate or contracted operations.

132. Notation.—Rule. Retain the decimal to the required number of places, increasing by 1 the last figure retained, if the first figure rejected is greater than 4.

Ex. Write ·30579734 retaining only 5 places of decimals so as to be approximately correct.

By the Rule, increasing 9 by 1, we have 10 in the place of 9 or 80 in the place of 79 ; and the required decimal is ·30580.

Reason. ·30580 − given decimal ·30579734 = ·00000266, and ·30579734 − ·30579 = ·00000734. But ·00000266 < ·00000734 ; ∴ ·30580 is *nearer* to the given decimal than ·30579.

133. Addition.—Rule. In each summand retain 2 or 3 more figures than the required number, observing the Rule in Art. 132 ; perform the addition, and then in the sum retain the required number of places.

The *reason for the Rule* will appear from a comparison of the contracted and full operations given below.

Ex. Add together 2·53789637, 15·00785678, 20·000087654 and ·1000345678, correctly to 5 places of decimals.

Contracted form.

2·5378964
15·0078568
20·0000877
·1000346
37·6458755

Full form.

2·537896	37
15·007856	78
20·000087	654
·100034	5678
37·6458755	3718

∴ the sum required is 37·64587.

134. Subtraction.—Rule. Retain 2 or 3 figures more than the required number of places in the minuend and subtrahend, observing the Rule in Art. 132, and then perform the subtraction, and in the difference retain the required number of places.

The reason for the Rule will appear below.

Ex. Subtract $\cdot 0\dot{6}$ from $\cdot 1\dot{4}$ so as to be correct to 5 places of decimals.

Contracted form

$\cdot 1\dot{4} = \cdot 1444444$

$\cdot 0\dot{6} = \cdot 0666667$

$\cdot 0777777$

∴ the diff. = ·07777.

Full form. (Art. 128.)

$\cdot 1\dot{4} = \frac{14-1}{90} = \frac{13}{90}$

$\cdot 0\dot{6} = \frac{6}{90}$

$\therefore \cdot 1\dot{4} - \cdot 0\dot{6} = \frac{13}{90} - \frac{6}{90} = \frac{7}{90}$

$= \cdot 0\dot{7} = \cdot 07777777\ldots$

135. Multiplication.—Rule. Under the multiplicand write the figures of the multiplier in the reverse order, placing the units' figure below that decimal place of the multiplicand to which the operation is to be carried.

Multiply by each figure of the multiplier all the figures of the

multiplicand that are above and to the left of itself, neglecting figures to its right except for the purpose of seeing what should be carried and whether the first figure in any partial product is to be increased by 1 according to Art. 132.

Place the first figures of the several partial products in the same vertical line, add those products together, and in the sum mark off the required number of decimal places.

The sum will be the product required, true or nearly true to the required number of decimal places.

The *reason for the Rule* will appear from a comparison of the contracted and full forms of operation given below. It is based on the following considerations :—

Units × units	give	units.
Units × tenths		tenths.
Units × hundredths		hundredths.
&c		&c.
Tens × units		tens.
Tens × tenths		units.
Tens × hundredths		tenths.
&c.		&c.
Tenths × units		tenths.
Tenths × tenths		hundredths.
Tenths × hundredths		thousandths.
&c.	..	&c.

Ex. 1. Multiply 25·7056 by 18·6203 correctly to 4 places of decimals.

Contracted form.	*Full form.*
25·7056	25·7056
302681	18·6203
2570560	771168
2056448	5141120
154234	154233
5141	2056448
77	257056
478·6460	478·6459|8368

Explanation of the process. Above 1 there being no figure, we suppose a 0 supplied, which does not alter the value of the multiplicand, and then we put down 1 × 6, 1 × 5, &c., successively. In the next line we put the product by 8. In the third line, 6 × 6 = 36, from which we carry 3 and add it to 6 × 5, thus getting 33, and we then increase the 3 in the units' place of 33 by 1, since 6 the figure rejected is > 4; and thus we have 34 of which we put 4 below 8, and then proceed on. In the next line 2 × 5 = 10,

so we carry 1, and add it to 2×0 or 0 and put 1 below 4, and then proceed on. Similarly we get the last line. If we look to the partial products in the contracted operation from below upwards, we see that they are the same as the cut off portions of the partial products in the full form from above downwards.

Ex. 2. Multiply $\cdot\dot{3}$ by $\cdot 1\dot{6}$ correctly to 4 places of decimals.

Contracted form.

$$\cdot\dot{3} = 0\cdot 333333\ldots$$
$$\cdot 1\dot{6} = 0\cdot 166666\ldots$$

Hence by the Rule we have

$$\begin{array}{r} 0\cdot 333333\ldots \\ 66610 \\ \hline 0333 \\ 200 \\ 20 \\ 2 \\ \hline \cdot 0555 \end{array}$$

Full form. (Art. 128).

$$\cdot\dot{3} = \tfrac{1}{3}\,;\ \cdot 1\dot{6} = \tfrac{16-1}{90} = \tfrac{1}{6}\,;$$
$$\therefore\ \cdot\dot{3} \times \cdot 1\dot{6} = \tfrac{1}{3} \times \tfrac{1}{6} = \tfrac{1}{18}$$
$$= \tfrac{5}{90} = \cdot 0\dot{5}$$
$$= \cdot 0555\ldots.$$

136. Division.—Rule. In the divisor, retain a number of figures equal to the required number of decimal places together with the number of integral places which the quotient must contain, and make this the new divisor; and in the dividend retain a number of figures that will contain this new divisor less than 10 times but not less than once.

Having obtained the first figure of the quotient, to find the next figure, cut off the last figure of the new divisor, and regard the first remainder as the partial dividend. Having obtained the second figure of the quotient, proceed to find its third figure in the same way; and so on.

In multiplying the divisor by each figure of the quotient take into account figures of the divisor that are cut off to see what should be carried.

The *reason for the Rule* will appear from a comparison of the contracted and full operations given below.

Ex. 1. Divide 8·6[illegible]452 by 7·35243 correctly to 4 places of decimals.

Contracted form.

```
7·3524,3) 8·6134,52 (1·1715
           73524
           12610
            7352
            5258
            5147
             111
              74
              37
              37
```

Full form.

```
7·35243) 8·6134|52 (1·1715
         73524|3
         12610|22
          7352|43
          5257|790
          5146|701
           111|0890
            73|5243
            37|56470
            36|76215
```

Ex. 2. Divide $\cdot1\dot{6}$ by $\cdot\dot{3}$.

Contracted form.

```
·333) ·1666 (.5
       1666
```

Full form. (Art. 128).

$\cdot\dot{3} = \frac{1}{3}$

$\cdot1\dot{6} = \frac{15}{90} = \frac{1}{6}$;

$\therefore$ quotient $= \frac{1}{6} \times \frac{3}{1} = \frac{1}{2} = \cdot5$.

Ex. 3. Divide 9·0072 by $\cdot\dot{0}\dot{7}$ correctly to 2 places of decimals.

Contracted form.

```
·070707) 9·0072 (127·38
         70707
         19365
         14141
          5224
          4949
           275
           212
            63
            57
             6
```

Full form.

The quotient $= \frac{90072}{10000} \times \frac{99}{7}$

$= \frac{891\cdot7128}{7}$

$= 127\cdot387\ldots.$

137. In approximate operations with terminating decimals, the Rules in Arts. 132 to 136 should always be followed. In the case of recurring decimals, it is advisable to carry on the operations after reducing the decimals to vulgar fractions, as in this way we sometimes get very simple results, as we see in Ex. 2 of Art. 136.

138. We shall now compare vulgar fractions with decimals as regards their respective advantages and disadvantages.

Advantages of the use of vulgar fractions.

I. Vulgar fractions occur to us more readily and naturally than the corresponding decimals. Thus $\frac{1}{2}$ occurs to us more readily than $\frac{5}{10}$ or $\cdot 5$; $\frac{1}{4}$, more readily than $\cdot 25$; $\frac{1}{8}$, much more readily than $\cdot 125$; and there is no comparison between $\frac{1}{3}$ and $\cdot 333\ldots$, $\frac{1}{6}$ ond $\cdot 1666\ldots$, &c., in point of simplicity.

II. By vulgar fractions, we can exactly express all fractional parts with a finite number of figures; but this is not possible with decimals. Thus, the fractional part *one-third* can be expressed as a vulgar fraction by $\frac{1}{3}$, but as a decimal it will be expressed by $\cdot 333\ldots\ldots$ *ad infinitum.*

Advantages of the use of decimals.

I. The notation of decimals being only an extension of the Common System of Notation, is far more simple and convenient than the notation of vulgar fractions.

Though we cannot express some vulgar fractions *accurately* except by an infinite number of decimal places, yet by taking a sufficient number of these places, we can express them *by an approximation sufficient for all practical purposes.*

II. The fundamental operations are performed in the case of decimals far more easily than in the case of vulgar fractions. Further, as the student will hereafter see, decimals are incomparably better adapted for *Logarithmic* computation than vulgar fractions.

Examples XXI.

1. Write the following decimals retaining only 5 places of decimals, so as to be approximately correct :—

(1) $\cdot 06254678$; $\cdot 123456789$; $\cdot 987654321$.

(2) $\cdot 135791113$; $\cdot 24681012$; $\cdot 5115253$.

(3) $\cdot 8642867$; $\cdot 13934771$; $\cdot 72336872$.

(4) $\cdot 27\dot{3}$; $\cdot 02\dot{7}\dot{3}$; $\cdot 09\dot{2}\dot{3}$.

2. Find the value (correct to 5 places of decimals) of

(1) $12\cdot 3456789 + 23\cdot 4567891 + 34\cdot 5678912$.

(2) $\cdot 0036912 + \cdot 03691215 + \cdot 396121518$.

(3) $3\cdot\dot{3} + 2\cdot\dot{5}\dot{7} + 1\cdot 52\dot{4}\dot{7} + \cdot 0\dot{2}$.

(4) $\cdot 34\dot{5} + \cdot 4\dot{5} + 6\cdot\dot{8}\dot{1} + 56\cdot 05\dot{6}$.

(5) $12.34\dot{5} - .02\dot{7} + 72 - .\dot{1}2\dot{3}$.

(6) $9.\dot{9}\dot{5} + 6.3\dot{8} - 1.1\dot{6} - 11.\dot{6}$.

(7) $12.3456789 - 9.87654321$.

(8) $13.5791113 \times 24.681012$.

(9) $5.19152925 \times 39.354945$.

(10) $.\dot{3} \times 4$; $2.2\dot{7} \times .1\dot{3}$; $.\dot{1}\dot{5} \times .\dot{1}$.

(11) $.2\dot{4}\dot{7} \times 5$; $2.2\dot{2} \times 1\dot{8}$; $2.\dot{4} \times 5$.

(12) $.\dot{2} \times 4$; $1.\dot{4} \times 2.\dot{5}$; $6.\dot{7} \times 9.\dot{8}$.

(13) $1234.56789 \div 987.654321$.

(14) $357.9113 \div 246.81912$.

(15) $.\dot{3} \div 6$; $.\dot{6} \div 3$; $.1\dot{6} \div 9$; $15 \div .\dot{3}$.

MISCELLANEOUS QUESTIONS AND EXAMPLES.

139. In working out Examples in decimals, the remarks in Arts. 76 and 77 should be borne in mind.

Ex. 1. Simplify $\dfrac{.1 + .02 \times .8}{5 \div \frac{1}{2} - .\dot{3}} + \dfrac{3\frac{3}{4}}{.3}$.

The given expression

$$= \frac{.1 + .016}{10 - \frac{1}{3}} + \frac{\frac{15}{4}}{\frac{3}{10}} = \frac{\frac{116}{1000}}{\frac{29}{3}} + \frac{25}{2}$$

$$= \frac{116}{1000} \times \frac{3}{29} + \frac{25}{2} = \frac{12}{1000} + \frac{125}{10}$$

$$= .012 + 12.5 = 12.512.$$

Ex. 2. What decimal added to $\frac{1}{2} + \frac{1}{3} + .4 + .\dot{5}$ will make the sum equal to 6?

By the question,

the decimal required $= 6 - (\frac{1}{2} + \frac{1}{3} + .4 + .5)$ expressed as a decimal.

Now $6 - (\frac{1}{2} + \frac{1}{3} + .4 + .5) = 6 - (\frac{5}{6} + \frac{9}{10})$

$$= 6 - \frac{52}{30}$$

$$= 6 - \frac{26}{15}$$

$$= \frac{64}{15}$$

$$= 4.2\dot{6};$$

$\therefore$ $4.2\dot{6}$ is the decimal required.

Ex. 8. Find the value of $1+\frac{1}{1}+\frac{1}{1\times 2}+\frac{1}{1\times 2\times 3}+$ &c. to 5 places of decimals.

We have

$$
\begin{aligned}
1 &= 1\\
\tfrac{1}{1} &= 1\\
\tfrac{1}{1\times 2} &= \cdot 5\\
\tfrac{1}{1\times 2\times 3} &= \cdot 1666666\ldots\ldots\\
\tfrac{1}{1\times 2\times 3\times 4} &= \cdot 0416666\ldots\ldots\\
\tfrac{1}{1\times 2\times 3\times 4\times 5} &= \cdot 0083333\ldots\ldots\\
\tfrac{1}{1\times 2\times 3\times 4\times 5\times 6} &= \cdot 0013888\ldots\ldots\\
\tfrac{1}{1\times 2\times 3\times 4\times 5\times 6\times 7} &= \cdot 0001984\ldots\ldots\\
\tfrac{1}{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8} &= \cdot 0000248\ldots\ldots\\
\tfrac{1}{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9} &= \cdot 0000027\ldots\ldots\\
\tfrac{1}{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9\times 10} &= \cdot 0000002\ldots\ldots\\
\&c. &= \&c.
\end{aligned}
$$

$\therefore$ the sum $= 2\cdot 7182814$;

and $\therefore$ the value required $= 2\cdot 71828\ldots\ldots$

Here, to ensure accuracy we retain 7 places of decimals (*i. e.*, 2 more than the number required) in the values of the several fractions, and we stop after $\frac{1}{1\times 2\times 3\ldots\ldots\times 10}$ as the first seven figures in the values of all the succeeding fractions will be zeros.

Examples XXII.

I.

1. What is a Decimal Fraction, and why is it so called?

Shew that the ordinary Notation of decimals is an extension of the Common System of Notation.

2. Shew how to convert a decimal into a vulgar fraction.

Convert $\cdot 0325$ into a vulgar fraction in its lowest terms.

3. Shew that a decimal is multiplied or divided by any power of 10 by removing the decimal point to the right or to the left a number of places equal to the index of the power.

Find the value of $\frac{195\cdot 0035\times 10^4}{10^3} - \cdot 05 \div 10^3 + \cdot 25\times 10^4$.

4. Convert the value of $\frac{3}{25}+\frac{3}{625}-\frac{1}{16}$ into a decimal.

5. A number being multiplied by 1000 has 2 places of

decimals in the product. How many places of decimals had it originally?

6. A number being divided by 10^4 has 6 places of decimals in the quotient. Find the number of decimal places in the dividend.

II.

1. State the Rule for the Addition of decimals.

Add together 3·25, ·0928, ·0013, and 5·031.

2. Give the reason for the Rule for the Subtraction of decimals.

Find the difference between

$$\cdot 1234 + 1\cdot 234 + 12\cdot 34 + 123\cdot 4$$

and

$$1234 + 1234 + 1234 + 1234.$$

3. Find the value of

$$1000 - \cdot 0001 + 1\cdot 000 - \cdot 100000.$$

4. Point out the effects of affixing and prefixing ciphers to a decimal.

Find the difference between ·375 and ·0375, and between ·003750 and ·00375.

5. A person owns $\frac{3}{8}$ of an estate, and he subsequently purchases ·24 of the same. What decimal of the whole estate does he now possess, and what decimal of it must he dispose of, that he may have exactly half of the estate left for himself?

6. Find the sum of the sum and the difference of ·2357 and 2·357 without actually performing the operations of Addition and Subtraction.

III.

1. Give the reason for the Rule for the Multiplication of decimals.

Multiply ·035 by 5·23, and find the difference between the result obtained and the product of 523 and 35.

2. The sum of the ages of two boys is 15 years, and the age of one of them is ·5 of the other. What is the age of each?

3. The greater of two numbers is 1·75 times the less, and their difference is equal to ·54 ÷ ·09. Find the numbers.

4. A gentleman spends a certain sum in the first week of a month; 1·5 times that sum in the second week; and in the third week, 2·16 times the amount spent in the first two weeks

together; and he finds that he has altogether spent 790 rupees. How much did he spend in each week?

5. Find the value of $\frac{2 \times \cdot 25 - \frac{2}{3} \text{ of } \cdot 16}{1 \cdot 9}$.

6. Divide 1500 rupees amongst three persons, *A*, *B*, and *C*, in such a manner that *B* may get ·35 of what *A* gets, and *C*, $\frac{2}{7}$ of what *B* receives.

IV.

1. State the Rule for the Division of decimals. Divide 6·25 by 25, 2·5, and ·0025.

2. What number must be multiplied by ·023 to produce 1610?

3. What number must be divided by ·029 to produce 1·6?

4. Find the greatest and the least of the following three numbers:—

$$\cdot 5 \times \cdot 02; \quad 25 \times \cdot 10; \text{ and } 1 \cdot 05 \div 21.$$

5. Find the value of

$$\tfrac{1}{2} + \tfrac{3}{4} \text{ of } \cdot 264 \times \cdot 05 + \cdot 625 \div \tfrac{1}{2} - \cdot 0001.$$

6. What decimal of 2·75 is the number 2·2?

V.

1. What is a Recurring Decimal, and why is it so called?

Shew how to convert a recurring decimal into a vulgar fraction.

2. Shew that when a vulgar fraction cannot be converted into a terminating decimal, the figures of the decimal must recur.

Convert $\frac{2}{17}$ into a decimal.

3. Is the system of notation for decimal fractions sufficient for the expression of all fractions?

Shew that the only vulgar fractions that can be converted into terminating decimals are those in which in their lowest terms the denominators are composed exclusively of the factors 2 and 5.

Can the fraction $\frac{15}{768}$ be converted into a terminating decimal?

4. Multiply $\cdot \dot{2}7\dot{9}$ by ·45 and divide the result by ·31.

5. Shew that the decimal 3·1416 lies between $\frac{22}{7}$ and $\frac{355}{113}$.

6. Convert $\cdot 37\dot{8}$ and $\cdot 5\dot{2}\dot{7}$ into vulgar fractions, and $\frac{7}{8}$ and $\frac{9}{16}$ into decimals.

VI.

1. Find the value (correct to 4 places of decimals) of

$$\frac{1}{2}-\frac{1}{3\times 2^3}+\frac{1}{5\times 2^5}-\frac{1}{7\times 2^7}+\&c.$$
$$+\frac{1}{3}-\frac{1}{3\times 3^3}+\frac{1}{5\times 3^5}-\frac{1}{7\times 3^7}+\&c.$$

2. Explain clearly how the operations of Addition and Subtraction may be performed with recurring decimals, without converting them into vulgar fractions, so as to give results that are approximately correct.

Find the value of

$$\cdot\dot{3}+\cdot 3\dot{8}-\cdot 02\dot{7}+7\cdot\dot{8}-2\cdot 5.$$

3. Find the value of

$$1+\frac{1}{2\times 10}+\frac{2}{3\times 10^2}+\frac{3}{4\times 10^3}+\&c.$$

correctly to 5 places of decimals.

4. The first of three numbers is $\cdot\dot{3}$ of the second, and the second, $\cdot\dot{4}$ of the third; and their sum is 43. Find the numbers.

5. Find the value of

$$4\times\left\{\frac{1}{5}-\frac{1}{3\times 5^3}+\frac{1}{5\times 5^5}-\frac{1}{7\times 5^7}+\&c.\right\}$$
$$-\left\{\frac{1}{239}-\frac{1}{3\times 239^3}+\frac{1}{5\times 239^5}-\frac{1}{7\times 239^7}+\&c.\right\}$$

correctly to 4 places of decimals.

6. Express $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}$ as a decimal correctly to 5 places of decimals.

CHAPTER III.

THE FUNDAMENTAL OPERATIONS WITH CONCRETE INTEGERS.

SECTION I. TABLES OF MONEY, WEIGHTS AND MEASURES.

140. We have hitherto considered our operations with regard to abstract numbers, or concrete numbers of one denomination only. But we have also to deal with concrete numbers of different denominations. Thus, we may be required to add together several sums of money, each consisting of rupees, annas and pies; or we may have to find the difference between two sums each consisting of rupees, annas and pies; and so on.

141. The different kinds of concrete numbers which occur in Arithmetic are those expressing Values of Money, Weights of Substances and Measures of Length, of Surface, of Solidity, of Capacity, of Angular Magnitudes, of Number and of Time. For the numerical representation of quantities of each of these different kinds, a certain quantity of the same kind is taken as the unit of the highest denomination, so that any given quantity would be represented by the number indicating the number of times that it contains the unit of the same kind; and the unit of each kind is divided and subdivided into units of lower denominations, the object being to enable us to express quantities in *integers* of different denominations. Thus, suppose we have to express numerically the length of a line which contains the linear unit 1 foot 5 times together with $\frac{1}{2}$ of a time; the whole length expressed in feet will be represented by the mixed mumber $5\frac{1}{2}$; and if we divide the length of 1 foot into 12 equal parts, and call each part an inch, the *fraction* $\frac{1}{2}$ of a *foot* will be represented by the *integer* 6 *inches*, and the whole length, by 5 feet 6 inches.

The unit, adopted in each case, and its successive divisions and subdivisions, are purely *conventional*, and are different in different countries; though some grounds of convenience must form the basis of these conventions.

We subjoin the ordinary English and Indian Tables connecting the units of different denominations for each of the above kinds of concrete quantity, with short remarks on each.

ENGLISH TABLES.

TABLE OF MONEY.

142. In this Table, the different units are connected thus :—

4 Farthings	make	1 Penny	written 1*d*.
12 Pence	......	1 Shilling	...... 1*s*.
20 Shillings	......	1 Pound	...... £1.

The symbols £. *s.* *d.* and *q.* (the former symbol for a farthing) are the initials of the words *libra*, *solidus*, *denarius* and *quadrans*, Latin names of certain Roman coins or sums of money. The symbol *q* is not now in common use, the fractions ¼, ½ and ¾, annexed to pence, representing 1, 2 and 3 farthings respectively.

The following coins are now current in England :—

Bronze Coins.

A Farthing, the coin of least value.
A Half-penny = 2 Farthings.
A Penny = 4 Farthings.

Silver Coins.

A Threepenny-piece	= 3*d.*
A Fourpenny-piece	= 4*d.*
A Sixpence	= 6*d.*
A Shilling	= 12*d.*
A Florin	= 2*s.*
A Half-Crown	= 2*s.* 6*d.*
A Crown	= 5*s.*

Gold Coins.

A Half-Sovereign	= 10*s.*
A Sovereign	= 20*s.*

The following are some of the old English coins, but they are not now current :—

Silver Coins.

A Groat	= 4*d.*
A Tester	= 6*d.*

Gold Coins.

	£.	*s.*	*d.*
A Noble	= 0	6	8.
An Angel	= 0	10	0.
A Half-Guinea	= 0	10	6.
A Mark or Merk	= 0	13	4.
A Guinea	= 1	1	0.
A Carolus	= 1	3	0.
A Jacobus	= 1	5	0.
A Moidore	= 1	7	0.

The standard of gold coin in England is 22 parts of pure gold and 2 parts of alloy melted together. Each of these 24 parts is termed a *carat*, and the standard gold is said to be 22 carats fine. The weight of a Sovereign $= \frac{40}{1869}$ of 1 pound Troy. (See Art. 143.)

The standard of silver coin is 37 parts of pure silver and 3 parts of alloy. The weight of a shilling $= \frac{1}{66}$ of 1 pound Troy. The weight of a penny $= \frac{1}{81}$ of a pound Avoirdupois. (See Art. 145).

In England, the bronze coinage is not a *legal tender* for more than 12*d.*; nor is the silver coinage for more than 40*s.*; the gold coinage being a *legal tender* for all amounts.

Table of Troy Weight.

143. The name of this Table is derived by some from *Troyes* a city in France, and by others from *Troynovant* a name for London. It is used in weighing gold, silver and other costly articles, and also in philosophical investigations.

In this Table,

24 Grains (grs.)	make	1 Pennyweight	... 1 dwt.
20 Pennyweights	...	1 Ounce	... 1 oz.
12 Ounces	...	1 Pound	... 1 lb. or ℔.

Diamonds and other precious stones are weighed by *carats*, each carat weighing $3\frac{1}{5}$ grains.

Table of Apothecaries' Weight.

144. In this Table which is used in weighing medicines,

20 Grains (grs.)	make	1 Scruple	... 1 sc. or ℈i
3 Scruples	...	1 Dram	... 1 dr. or ʒi
8 Drams	...	1 Ounce	... 1 oz. or ℥i
12 Ounces	...	1 Pound	... 1 lb. or ℔.

In this Table, the pound, and consequently the ounce and the grain, are the same as in Troy Weight.

Table of Avoirdupois Weight.

145. The word Avoirdupois is derived from *Avoirs* (goods) and *Poids* (weight). Avoirdupois weight is used in weighing all heavy and coarse articles.

In this Table,

16 Drams	make	1 Ounce	oz.
16 Ounces	...	1 Pound	lb.
28 Pounds	...	1 Quarter	qr.
4 Quarters	...	1 Hundredweight	cwt.
20 Hundredweights		1 Ton	Ton.

1 Stone of meat or fish	=	8 lbs.
1 Stone (generally)	=	14 lbs.
1 Firkin of Butter	=	56 lbs.
1 Fodder of Lead	=	$19\frac{1}{2}$ cwt.
1 Pack of Wool	=	240 lbs.
1 lb. Avoirdupois	=	7000 grs. Troy.
1 lb. Troy	=	5760 grs. Troy.
1 lb. Avoirdupois	=	$\frac{7000}{5760}$ lbs. Troy $=\frac{175}{144}$ of 1 lb. Troy.

TABLE OF LINEAL MEASURE.

146. In this measure, used in measuring distances, lengths, breadths, and the like,

3 Barleycorns	make	1 Inch	1 in.
12 Inches	...	1 Foot	1 ft.
3 Feet		1 Yard	... 1 yd.
$5\frac{1}{2}$ Yards		1 Rod, Pole or Perch	1 po.
40 Poles		Furlong	1 fur.
8 Furlongs		Mile	1 m.
3 Miles		League	1 lea.
$69\frac{1}{8}$ Miles		Degree ...	... 1 deg. or 1°.

4 Inches	make	1 Hand	(used in measuring horses)
22 Yards		1 Chain	(........................ land)
100 Links	...	1 Chain	

1 Palm = 3 in., 1 Span = 9 in., 1 Cubit = 18 in.
1 Pace = 5 ft., 1 Geographical Mile* = $\frac{1}{60}$th of a degree.
1 Line = $\frac{1}{12}$th of an inch.
1 Mile = 1760 yards.

TABLE OF CLOTH MEASURE.

147. In this measure,

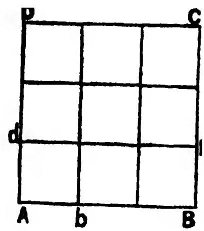
$2\frac{1}{4}$ Inches	make	1 Nail.	
4 Nails		1 Quarter ...	1 qr.
4 Quarters		1 Yard ...	1 yd.
5 Quarters		1 English Ell.	
6 Quarters		1 French Ell.	
3 Quarters		1 Flemish Ell.	

TABLE OF SUPERFICIAL OR SQUARE MEASURE.

148. In this measure, which is used in measuring areas, the units of the first four denominations, *viz.*, the Square Inch, the Square Foot, the Square Yard, and the Square Pole, are areas en-

closed by squares having for their sides an Inch, a Foot, a Yard, and a Pole of lineal measure; and as the lineal inch, foot, &c., have already got with one another the *conventional* relations given in Art. 146, the square inch, square foot, &c., must have with one another, certain relations which are not *conventional* but *necessary*. We must find out these relations before we give the table.

. To find the relation between a square foot and a square yard, take $ABCD$ to represent a square yard, so that $AB = AD = 1$ lineal yard; and divide AB, AD each into 3 equal parts like Ab, Ad; then each of these parts $= 1$ foot. Draw lines horizontally and vertically as in the figure; then each little square is 1 square foot.



Thus the no. of sq. feet in 1 sq. yard
= ....small squares in the large square
= ....horizontal rows like $ABEd$
× ....small squares in each row
$= 3 \times 3 = 9$.

. Similarly it may be shewn that the number of sq. inches in 1 sq. foot $= 12 \times 12$; and so on.

Our Table will therefore run thus :—

144 Square Inches	make	1 Square Foot	...	1 sq. ft.
9 Square Feet	...	1 Square Yard	...	1 sq. yd
$30\frac{1}{4}$ Square Yards	...	1 Square Pole	...	1 sq. po
40 Square Poles	...	1 Rood	...	1 ro.
4 Roods	...	1 Acre	...	1 ac.

1 Ac. $= 4$ roods $= 4 \times 40$ sq. poles
$= 4 \times 40 \times 30\frac{1}{4}$ sq. yds.
$= 4840$ sq. yds.
1 Sq. Chain $= (22 \times 22)$ sq. yds.
$= 484$ sq. yds.

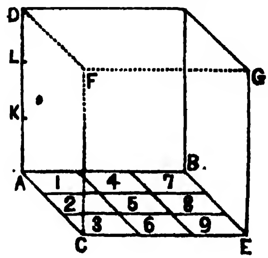
Hence 10 Sq. Chains $= 1$ ac.
1 Sq. Mile $= 1760 \times 1760$ sq. yds. $= 640$ acres.
A Rod of Brickwork $= 272\frac{1}{4}$ sq. ft.
A Square of Flooring $= 100$ sq. ft.
A Yard of Land $= 30$ ac.
A Hide of Land $= 100$ ac.

. *Two square feet* = twice the area of one square foot, but *two feet square* = area enclosed by a square having two feet for its side $= 2 \times 2$ or 4 square feet. This distinction should be borne in mind.

TABLE OF SOLID OR CUBIC MEASURE.

149. In this measure, which is used in measuring volumes, the successive units, *viz.*, the Cubic Inch, the Cubic Foot, and the Cubic Yard, are cubes having for their edges the lineal Inch, Foot, and Yard, respectively. Therefore, for the same reason as that stated in Art. 148, the cubic inch, cubic foot, and cubic yard must have with one another certain *necessary* relations, which may be ascertained thus :—

Let the annexed figure represent a cubic yard, then $AB = AC = AD = 1$ lineal yd., and the face $ABEC$ is a sq. yd. containing as in the figure, 9 sq. feet. Through the lines of division of $ABEC$, draw planes parallel to $ACFD$, and $CEGF$; then these planes by their intersection will divide the cube into 9 equal solid figures each standing on one of the 9 squares in $ACEB$. If now we draw through K and L (the points which divide AD into 3 equal parts) planes parallel to $ABEC$, we shall have each of these 9 solids divided into 3 small cubes, each of which is a cubic foot; and thus in a cubic yard there are altogether 3×9 or 27 cubic feet.



Similarly a cubic foot $= 12 \times 12 \times 12$ or 1728 cubic inches. Our Table will therefore stand thus :—

1728 Cubic Inches	make 1 Cubic Foot	...	1 cub. ft.
27 Cubic Feet	... 1 Cubic Yard	...	1 cub. yd.
A Load of Rough Timber	= 40 cub. ft.		
A Load of Squared Timber	= 50 cub. ft.		
A Ton of Shipping	= 42 cub. ft.		

TABLE OF WINE MEASURE.

150. In this measure by which wines and all liquids except malt liquors and water are measured,

4 Gills	make 1 Pint	...	1 pt.
2 Pints	1 Quart	...	1 qt.
4 Quarts	1 Gallon	...	1 gal.
10 Gallons	1 Anker	...	1 ank.
18 Gallons	1 Runlet	...	1 run.
42 Gallons	1 Tierce	...	1 tier.
2 Tierces	1 Puncheon	...	1 pun.
63 Gallons	1 Hogshead	...	1 hhd.
2 Hogsheads	1 Pipe	...	1 pipe.
2 Pipes	1 Tun	...	1 tun.

TABLE OF ALE AND BEER MEASURE.

151. In this measure used in measuring malt liquors and water,

2 Pints	make	1	Quart ...	1 qt.
4 Quarts	...	1	Gallon ...	1 gal.
9 Gallons	...	1	Firkin ...	1 fir.
2 Firkins	...	1	Kilderkin	1 kil.
2 Kilderkins	...	1	Barrel ...	1 bar.
$1\frac{1}{2}$ Barrels or 54 Gallons	...	1	Hogshead	1 hhd.
2 Hogsheads	...	1	Butt ...	1 butt.
2 Butts	...	1	Tun ...	1 tun.

TABLE OF CORN OR DRY MEASURE.

152. In this measure,

2 Quarts	make	1	Pottle	...	1 pot.
2 Pottles	...	1	Gallon	...	1 gal.
2 Gallons	...	1	Peck	...	1 pk.
4 Pecks	...	1	Bushel	...	1 bus.
2 Bushels	...	1	Strike	...	1 str.
2 Strikes	...	1	Coomb	...	1 coomb.
2 Coombs	...	1	Quarter	...	1 qr.
5 Quarters	...	1	Load	...	1 load.
2 Loads	...	1	Last	...	1 last.

TABLE OF COAL MEASURE.

153. In this measure used in measuring coal, lime, &c.,

4 Pecks	make	1 Bushel.
3 Bushels	....	1 Sack.
12 Sacks	...	1 Chaldron.

TABLE OF ANGULAR MEASURE.

154. The circumference of every circle is supposed to be divided into 360 equal parts, each of which is called a degree. The degree and its subdivisions are connected thus:—

60 Seconds	make	1 Minute	... 1′.
60 Minutes	...	1 Degree	... 1°.
1 Second is written	...	... 1″.	

TABLE OF NUMBER.

155.	12 Units	...... make	Dozen.
	12 Dozens	...	Gross.
	20 Units	...	Score.
	120 Units	...	Long Hundred.
	24 Sheets of Paper	...	Quire.
	20 Quires		Ream.
	10 Reams		Bale.

TABLE OF TIME.

156.	60 Seconds	...make	1 Minute	... 1′
	60 Minutes	...	1 Hour	... 1 hr.
	24 Hours	...	1 Day	... 1 day.
	7 Days	...	1 Week	... 1 wk.

1 Second is written 1″.

A year is divided into 12 parts called Calendar months, each containing a certain number of days as given in the following lines;—

Thirty days have September,
April, June, and November:
February has twenty-eight alone,
And all the rest have thirty-one:
But leap-year coming once in four,
February then has one day more.

The time from one vernal equinox to another, that is the solar year, consists of 365·242219 *mean* solar days. The ordinary year of 365 *mean* solar days differs therefore from the solar year by ·242219, or nearly $\frac{1}{4}$ of a day; and in 4 years this difference would nearly amount to a day. To prevent this *error*, Julius Cæsar introduced a method of correction by which every fourth year, called a *Leap* or *Bissextile* year, is made to consist of 366 days, an extra day, called the *Intercalary* day, being added to the month of February.

Thus, the average Julian year $=365·25$ days;
but the solar year $=365·242219$ days
$\therefore$ the difference $=$ ·007781 of a day;

and this difference in 400 years would amount to $400 \times .007781$ or 3·1124 days; that is, if in any year, the equinox fall on a certain day, 400 years after that year, it will fall 3 days earlier; and 1257 years after, it will fall $1257 \times ·007781$ or 9·780717 days, *i.e.*, nearly 10 days earlier, according to the Julian Calendar. This in fact was what was actually observed in the year 1582 of the Christian Era; for whereas in the year 325, the year of the Council of Nice, the vernal equinox fell on the 21st of March, in the year

1582, or 1257 years after, it fell on the 11th of March. To correct this error, Pope Gregory XIII caused 10 days to be omitted in that year, making the 15th of October follow the 4th; and to prevent the recurrence of this error in future, he ordered that in every 400 years, three of the Julian leap years should be regarded as ordinary years, *viz.*, those which complete centuries, the numbers expressing which centuries are not multiples of 4: thus, in the Gregorian or New Style, 1700, 1800, and 1900, are not leap years, but 1600, and 2000 are.

In England, the New Style was adopted on the 2nd of September 1752 when the error amounted to 11 days.

157. An Act of Parliament "To consolidate the Law relating to Weights and Measures" came into operation on the 1st of January 1879.

It enacts firstly: That the bronze bar *Standard Yard* then in the custody of the Warden of the Standards shall be the *Imperial Standard Yard* (the bar being at the temperature of 62° by Fahrenheit's thermometer); that it shall be the only standard measure of extension wherefrom all others are to be deduced; and that the $\frac{1}{36}$th part of this yard shall be an inch.

Now the length of the pendulum vibrating seconds in the latitude of London in a vacuum, and at the level of the sea, is found to be 39·1393 such inches. This affords the means of recovering the Standard Yard should it be lost.

It enacts secondly: That the platinum *weight of one Pound* then in the custody of the same officer, shall be the *Standard Measure* of Weight, and that the $\frac{1}{7000}$th part of it shall be a grain, and that 5760 such grains shall be contained in one Pound Troy.

Now the weight of a cubic inch of distilled water is 252.458 grains Troy, the barometer being at 30 inches and the thermomete at 62°. This affords the means of recovering the Imperial Standard Pound if it be lost.

It enacts thirdly: That the *Standard Measure of Capacity* shall be the *Gallon* containing 10 *Imperial Standard* Pounds Avoirdupois weight of distilled water, weighed in air against brass weights, the barometer being at 30 inches and the thermometer at 62°.

Now this weight fills 277.274 cub. in. and thus the Imperial Standard Gallon contains 277.274 cub. in.

158. An Act of Parliament was passed in 1864, legalizing the use of the Metric System of Weights and Measures. That Act has since been repealed, but the system is retained in use by the Weights Measures Act of 1878.

In this system, the several Tables run thus :—

I. MONEY. 1 FRANC (the unit) = about $9\frac{3}{4}d$.

10 Centimes (*c*)	make	1	Decime.
10 Decimes	...	1	FRANC.

II. WEIGHT. 1 GRAM (the unit) = 15.4323487 grs.

10 Milligrams	make	1	Centigram.
10 Centigrams	...	1	Decigram.
10 Decigrams	...	1	GRAM.
10 Grams	...	1	Dekagram.
10 Dekagrams	...	1	Hectogram.
10 Hectograms	...	1	Kilogram ($=2\frac{1}{5}$ lbs. Avoir. nearly.)
10 Kilograms	...	1	Myriagram.

III. LENGTH. 1 METRE (the unit) = 39.3708 inches.

10 Millimetres	make	1	Centimetre.
10 Centimetres	...	1	Deceimetere.
10 Decimetres	...	1	METRE.
10 Metres	...	1	Dekametre.
10 Dekametres	...	1	Hectometre.
10 Hectometres	...	1	Kilometre ($=\frac{5}{8}$ mile nearly).
10 Kilometres	...	1	Myriametre.

IV. SURFACE. 1 ARE (the unit) = 100 sq. metres.
= 119·6033 sq. yds.

10 Centiares	make	1	Deciare.
10 Deciares	...	1	ARE
10 Ares	...	1	Dekare.
10 Dekares	...	1	Heçtare.

V. SOLIDITY. 1 STERE (the unit) = 1 cub. metre.
= 35·317 cub. ft.

10 Decisteres	make	1	STERE.
10 Steres	...	1	Dekastere.

VI. CAPACITY. 1 LITRE (the unit) = $\frac{1}{1000}$th part 1 cub. metre
= 1·76077 pints.

10 Centilitres	make	1	Decilitre.
10 Decilitres	...	1	LITRE.
10 Litres	...	1	Dekalitre.
10 Dekalitres	...	1	Hectolitre.
10 Hectolitres	...	1	Kilolitre.

It will be observed that the several multiples and submultiples of the unit in each of the foregoing Tables, result from the multiplication and division of the unit by powers of 10.

INDIAN TABLES.

TABLE OF MONEY.

159. In this Table,

3 Pies	... ...	make	1 Pice.
12 Pies or 4 Pice	... ...		1 Anna.
16 Annas	... ...	....	1 Rupee.

15 Sicca Rupees = 16 current Rupees.

The following are the coins now in use.

Copper Coins.

A Pie.
A Half pice.
A Pice.
A Double pice.

Silver Coins.

A Two-anna piece.
A Four-anna piece.
A Half-rupee.
A Rupee.

The following are the only *Gold Coins* allowed to be coined under the Indian Coinage Act, 1870 :—

A Five-rupee piece.
A Ten-rupee piece.
A Gold Mohur or Fifteen-rupee piece.
A Double Gold Mohur or Thirty-rupee piece.

The gold mohur weighs 180 grs. Troy and consists of 11 parts of pure gold and 1 part of alloy. The other gold coins are of the same fineness, and their weights are proportional to their values.

The rupee weighs 180 grs. Troy and consists of 11 parts of pure sllver and 1 part of alloy.

The other silver coins are of the same fineness and of proportionate weight.

The double pice weighs 200 grs. Troy. The other copper coins are of the proportionate weight.

Under the Indian Coinage Act (Act XXIII of 1870) now in force, the gold coinage is not a legal tender; the rupee and the half rupee are a legal tender for any amount; and the copper and the other silver coins are a legal tender only for fractions of a rupee.

Approximately, 1 rupee = 2 shillings taking into account only the weight of the silver in each. But the exact value of a rupee in

shillings at any time depends upon the rate of exchange between England and India. At present a rupee is worth only a little over one shilling.

The following Table is used in keeping accounts in Bengali :—

4 Cowries	make	1	Ganda.
5 Gandas	...	1	Buri.
20 Gandas	...	1	Pan.
4 Pans	...	1	Chauk.
4 Chauks	...	1	Kahan.

1 Cowrie = 3 Krantis = 4 Kags = 7 Dips = 8 Bats = 9 Dantis.

None of these denominations is the name of any actual coin; but the kahan stands for 1 rupee, the chauk for 4 annas, and the pan for 1 anna.

The notation used in Bengali accounts is peculiar. The kahans are represented by the ordinary numerals, 1 *chauk* by the vertical stroke ৷•, 2 *chauks* and 3 *chauks* by ৷৷•, and ৸•, 1 *pan* by the oblique stroke ৴• and 2 *pans* and 3 *pans* by ৵• and ৶•.

Generally, in Bengali, if units of any denomination are represented by the ordinary numerals, their quarters are represented by ৷•, ৷৷• and ৸• and the quarters of these quarters by ৴• ৵• and ৶•.

Thus 2 Rs. 14 as. will be written as ২৸৵• in Bengali; so, 1 maund 20 seers (See art. 162) will be written as ১৷৷•.

Besides the theoretical cowries in the above Table, there are cowries or shells in actual use in the Bengal bazar for the payment of very small amounts, 1 pice being equal to 80 cowries generally.

In Behar and the North-Western Provinces, the following subdivisions of the pice are in use :—

2 Damris	make	1	Chhidam.
2 Chhidams ...	...	1	Adhela.
2 Adhelas ...	...	1	Paisa or Pice.

Table of Bengal gold and Silver Weight.

160. In this Table,

4 Punkos	make	1	Dhan.
4 Dhans ...	...	1	Ratti.
6 Rattis ...	...	1	Anna.
96 Rattis or 12 Mashas or 16 Annas ...	...	1	Tola

1 Tola = 180 grs. Troy.
32 Tolas = 1 lb. Troy.

TABLE OF BENGAL DOCTORS' WEIGHT.

161. In this Table,

4 Jabs	make	1	Ratti.
5 Ráttis	...	1	Dhán.
2 Dháns	...	1	Máshá.
12 Máshás		1	Tola.

1 Tola = 180 grs. Troy.

TABLE OF BRITISH INDIAN BAZAR WEIGHT.

162. In this Table used in weighing coarse articles,

5 Sikis or quarter rupees (in weight)	make	1	Kancha.
4 Kanchas or 5 rupees in weight	...	1	Chatak.
4 Chataks	...	1	Powa.
4 Powas	...	1	Seer.
5 Seers	...	1	Pusury.
8 Pusuries or 40 Seers	...	1	Maund.

1 Bazar Maund = 100 lbs. Troy = $82\frac{2}{7}$ lbs. Avoirdupois = $\frac{54}{49}$ of 1 Factory Maund = 1 Maund $4\frac{4}{49}$ Seers of Factory weight.

1 Seer = 80 rupees' weight = 80 × 180 or 14400 grs. Troy = $2\frac{1}{2}$ lbs. Troy = $\frac{72}{35}$ lbs. Avoir.

1 Factory Maund = $\frac{2}{3}$ cwt.

Bengal Regulation VII of 1833 (now repealed) first introduced the Tola of 180 grs. Troy as the standard unit of weight. The Table of weights given in that Regulation runs thus :—

8 Rattis	= 1 Masha	=	15 grs. Troy.
12 Mashas	= 1 Tola	=	180 grs. Troy.
80 Tolas	= 1 Seer	=	$2\frac{1}{2}$ lbs. Troy.
40 Seers	= 1 Maund	=	100 lbs. Troy.

The following is the Table of Bombay local weights :—

4 Dhans	make	1	Raktika.
8 Raktikas	...	1	Máshá.
4 Máshás	...	1	Tank.
72 Tanks	...	1	Seer.
40 Seers	...	1	Maund = 28 lbs. Avoir.
20 Maunds	...	1	Khandi.

The following is the Table of Madras local weights :—

10 Pagodas	make	Palam.
8 Palams		Seer = 10 oz. Avoir.
5 Seers		Bis.
8 Bis or 40 Seers		Maund = 25 lbs. Avoir.
20 Maunds		Khandi.

Table of Linear Measure.

163. In Bengal,

3	Jabs	make	1 Anguli.
4	Angulis	...	1 Muti.
3	Mutis	...	1 Bighat.
2	Bighats	...	1 Hát.
4	Háts	...	1 Dhanu.
1000	Dhanus	...	1 Kros or Kos.
2	Kroses	...	1 Gavyuti.
2	Gavyutis	...	1 Jojan.
4	Háts	=	1 Káthá.
20	Káthás or 80 Háts	=	1 Bigha or Rasi.

In the above Table, 1 Kros = 1 mile 1 furlong 3 poles 3½ yards English.

But generally, 1 Kros = 100 Rasis = 8000 Háts = 2 miles 2 furlongs 7 poles 1⅓ yds. English.

1 Hát = 18 inches generally.

In Bombay the half háth is called Vent; and the measuring rod or káthi for land is 9·4 ft.

In the North-Western Provinces,

1	Ilahi Guj	=	33 inches.
3	Ilahi Guj	=	1 Bans.
20	Bans	=	1 Jarib.

Table of Cloth Measure.

164. In Bengal,

3	Jabs	make	1 Anguli.
3	Angulis	...	1 Girah.
8	Girahs	...	1 Hát.
2	Háts	...	1 Guj = 1 yard.

In Bombay,

2	Angulis	=	1 Tasu.
24	Tasus	=	1 Guj = 27 in.

In Madras,

1	Kovid	=	18·6 in.

Table of Land Measure.

165. In Bengal,

1	Square Cubit	makes	1 Ganda.
20	Gandas or 5 Cubits long by 4 wide	make	1 Chatak.
16	Chataks or 80 Cubits long by 4 wide	...	1 Katha.
20	Kathas or 80 Cubits long by 80 wide	...	1 Bigha.

A Katha $= 80 \times 4$ or 320 sq. cubits.
$= 120 \times 6$ or 720 sq. ft.
A Bigha $= 80 \times 80$ or 6400 sq. cubits.
$= 120 \times 120$ or 14400 sq. ft.
$= \frac{14400}{9}$ or 1600 sq. yds.

From Art. 148,

1 acre $=$ 4840 sq. yds.
$= 3 \times 1600$ sq. yds. $+ 40$ sq. yds.
$= 3$ bighas $+ \frac{1}{40}$ bigha
$= 3\frac{1}{40}$ bighas.

The Benares or Ghazipur Bigha
$=$ 3600 sq. Benares guj $=$ 3136 sq. yds.
$=$ 2 Bengal bighas nearly.

In the North-Western Provinces,

20 Biswas $=$ 1 Bigha.
1 Bigha $=$ 3600 sq. Guj Ilahi
$=$ 3025 sq. yds.

In Bombay,

$39\frac{1}{4}$ Square Háts	make	1	Kathi.
20 Kathis			Pand.
20 Pands			Bigha $=$ 3897·41 sq. yds.
6 Bighas			Rukeh.
120 Bighas			Chahur.

In Madras,

1 Manai $=$ 2400 sq. ft.
24 Manais $=$ 1 Kani $=$ 6400 sq. yds.

Table of Bengal Lime Measure.

166. 2 ft. 3 in. long by 1 ft. 8 in. wide by 9 in. deep $=$ 1 Ferah weighing $1\frac{1}{4}$ mds.
80 Ferahs or 225 cub. ft. $=$ 100 maunds.

Table of Bengal Liquid Measure.

167. 4 Chataks	make	1	Powa.
4 Powas	...	1	Seer.
40 Seers	...	1	Maund.

Table of Corn Measure.

168. In Bengal,

5 Chataks	make	1	Kunki.
4 Kunkis	...	1	Rek.
4 Reks	...	1	Pally.
20 Pallies	...	1	Solly.
16 Sollies	...	1	Kahun $=$ 40 mds.

In Bombay,

36 Tanks	make	1	Tipari.
2 Tiparis	...	1	Ser.
4 Sers	...	1	Payli = $2\frac{1}{4}$ lbs. Avoir.
16 Paylis	...	1	Phera.
8 Pheras	...	1	Khandi.
25 Pheras	...	1	Muda.

$8\frac{1}{3}$ Paylis of lime	=	1	Phera.
$17\frac{1}{2}$ Paylis of rice	=	1	Phera.

In Madras,

8 Slaks	make	1	Puddi.
8 Puddis	...	1	Markal = 750 cub
5 Markals	...	1	Phera.

TABLE OF BENGAL TIME.

169.

60 Anupals	make	1	Vipal.
60 Vipals	...	1	Pal.
60 Pals	...	1	Ghari or Danda
$7\frac{1}{2}$ Gharis	...	1	Prahar = 3 hrs.
8 Prahars	...	1	Day.
7 Days	...	1	Hafta.
15 Days	...	1	Paksha.
2 Pakshas	...	1	Mas.
2 Mas	...	1	Ritu.
6 Ritus	...	1	Vatsar.
12 Vatsars	...	1	Yug.

The Hindu year of the Sakabda era begins from the 1st Baisakh, corresponding with about the 11th, 12th, or 13th of April. Its length = 365 days 15 Dandas 31 Pals 31 Vipals and 24 Anupals. The Samvat year begins from the first day after the new moon in Chaitra. The two Hindu eras in common use are the Samvat, which commenced from 57 B. C., and the Sakabda, which commenced from 78 A. D.

The following are the names of the Hindu months, with their astronomical lengths in days, hours, and minutes.

Months.			Days.	hrs.	min.
Baisakh	...	...	30	22	12·8
Jyaishtha	...	...	31	9	40·8
Ashadh	...	...	31	14	39·2
Sravan	...	...	31	11	16·8
Bhadra	...	...	31	0	52·0
Asvin	...	...	30	10	56·8

Months.	Days.	hrs	min.
Kartic	29	21	38·8
Agrahayan	29	12	9·6
Paush	29	8	21·2
Magh	29	10	54·4
Falgun	29	19	21·6
Chaitra	30	8	8·4

In ordinary reckoning, a fraction of a day belonging to a month is generally counted as one entire day, the difference between such fraction and a whole day being deducted from the length of the next month. Now the amount of this deduction is variable; and hence it is that the number of days in a month according to the Hindu Calendar is not always the same.

170. Act XXXI of 1871, which provides for the adoption of a uniform system of Weights and Measures of Capacity, enacts amongst other things, that the unit of weight shall be the Ser equal in weight to the French Kilogramme = 2 1lbs. 3oz. 4·3830 drams Avoirdupois, and the unit of the measure of capacity, the measure containing one such Ser of water at its maximum density weighed in a vacuum.

SECTION II. NOTATION OF CONCRETE INTEGERS. REDUCTION.

171. The Notation of concrete integers is extremely simple. We have only to write down the numbers of the successive denominations one after another, with the abbreviation for the denomination of each number, above or by the side of the same.

Thus, *five pounds six shillings eight pence* will be written

as £5. 6*s*. 8*d*. or

£.	*s*.	*d*.
5.	6.	8.

172. Def. Reduction is the method of expressing a number of one denomination in units of another.

173. Rule I. To reduce a quantity from higher denominations to a lower one, multiply the number of the highest denomination in the given quantity by the number connecting that denomination with the next lower, and to the product add the number of the next lower denomination, if any, in the given quantity; repeat this process for each succeeding denomination till the one required is arrived at.

Rule II. To reduce a quantity from a lower denomination to a higher, divide the given quantity by the number connecting its denomination with the next higher, and set down the remainder,

if any, under its own denomination, and the quotient under the next higher; repeat the process with the first and the successive quotients till the required denomination is arrived at.

Ex. 1. Reduce R28 13*a.* 3 pice to pies.

By the rule we have

$$\begin{array}{l}
\text{R}28 \quad 13a. \quad 3 \text{ pice.} \\
\underline{\quad 16} \\
168 \\
\underline{28} \\
448\ a. \\
\underline{\ \ 13\ a.} \\
461\ a. \\
\underline{\quad\ 4} \\
1844 \text{ pice.} \\
\underline{\quad\ 3}\ \ ,, \\
1847\ ,, \\
\underline{\quad\ 3} \\
5541 \text{ pies.}
\end{array}$$

Ex. 2. Reduce 1000 tolas to seers, &c.

By the Rule we have

$$\begin{array}{r|l}
5 & 1000 \text{ tolas.} \\ \hline
4 & \ \ 200 \text{ cht.} \\ \hline
4 & \quad 50 \text{ pow.} \\ \hline
 & \quad 12 \text{ seers. } 2 \text{ pows.}
\end{array}$$

The *reason for the Rules* will be seen below.

In Ex. 1,

$$\begin{aligned}
\text{R}28\ 13a.\ 3 \text{ pice} &= 28 \times 16a. + 13a. + 3 \text{ pice.} \\
&= 461a. + 3 \text{ pice} = 461 \times 4 \text{ pice} + 3 \text{ pice} \\
&= 1847 \text{ pice} = 1847 \times 3 \text{ pies} \\
&= 5541 \text{ pies}
\end{aligned}$$

In Ex. 2,

$$\begin{aligned}
1000 \text{ tolas} &= \tfrac{1000}{5} \text{ cht.} = 200 \text{ cht.} \\
&= \tfrac{200}{4} \text{ pows} = 50 \text{ pows.} \\
&= \tfrac{50}{4} \text{ seers} = 12 \text{ seers } 2 \text{ pows.}
\end{aligned}$$

174. Here may be noticed the advantages of the use of the Metric System.

In this system, Reduction from one denomination to another is performed by a simple transposition of the decimal point.

Thus, 3 kilom. 5 hectom. 6 dekam. 7 m. 9 decim.

$$=(3\times1000+5\times100+6\times10+7+\tfrac{9}{10})\text{m.}$$
$$=3567{\cdot}9 \text{ m.}$$
$$=356{\cdot}79 \text{ dekam.}$$
$$=35{\cdot}679 \text{ hectom.}$$
&c.

Examples XXIII.

1. Reduce

(1) R12 13*a.* 4*p.* to pies; and 1000 pies to rupees.

(2) R23 14*a.* 5*p.* to pies; and 250 annas to rupees.

(3) R159 0*a.* 7*p.* to pies; and 500 annas to rupees.

(4) R234 15*a.* to annas; and 196 annas to rupees.

(5) £21 2*s.* 3*d.* to pence; and 500*d.* to pounds.

(6) £32 3*s.* 4½*d.* to farthings; and 100*d.* to pounds.

(7) 225 half-sovereigns to pence; and 5633 pence to half-guineas.

(8) 529 half-crowns to pounds; and £52 5*s.* to crowns.

(9) 1 lb. 2 oz. 3 dwts. to grains; and 1234 grs. to ounces.

(10) 12 lbs. 13 oz. 14 dwts. to pennyweights; and 123414 grs. to pounds.

(11) 3 tons. 14 cwt. 3 qrs. to pounds; and 1000 oz. to pounds

(12) 5 tons. 15 cwt. 16 lbs. to grains Troy; and 10000 pounds to tons.

(13) 25 tolas 12 rattis to rattis; and 1000 rattis to tolas.

(14) 31 maunds 32 seers 3 chts. to kanchas; and 1000 tolas to seers.

(15) 100 maunds to tolas; and 10000 tolas to maunds.

(16) 1 mile 7 fur. 13 po. to yards; and 2000 yards to miles.

(17) 23 yds. 2 ft. 7 in. to inches; and 525 in. to yards.

(18) 1 ac. 2 ro. 3 sq. po. to square yds.; and 3000 sq. yds. to acres.

(19) 15 cub. ft. 27 cub. in. to cubic inches; and 500 cub ft. to cub. yds.

(20) 2 kroses 50 dhanus to cubits; and 15678 cubits to kroses.

(21) 5 kroses 20 rasis to cubits; and 1000 rasis to kroses.

(22) 28 bighas 15 kathas to kathas; and 525 kathas to bighas.

(23) 39 bighas 16 kathas to kathas; and 1650 chataks to bighas.

(24) 1 kilog. 2 hectog. 3 dekag. to grams; and 987654 millig. to grams.

(25) 4 kilom. 5 hectom. 6 dekam. to centimetres; and 78910 centim. to dekametres.

2. How many pence are there in £100, and how many pounds in 1600 pence?

3. How many acres are there in 640 sq. miles, and how many bighas in 144 kathas?

4. How many minutes are there in 1 Julian year, and how many dandas in 1 week?

SECTION III. COMPOUND ADDITION.

175. Def. The Addition of concrete numbers of different denominations is called **Compound Addition.**

It is evident that the numbers to be added together must be of the same *kind*. For otherwise, the Addition would be meaningless and absurd. Thus there can be no meaning in saying that 5 *rupees* are to be added to 6 *seers*, or the like.

176. Rule. Arrange the numbers so that units of the same denomination may be in the same column. Add up the column of the lowest denomination, divide the sum by the number of units of its denomination contained in one unit of the next higher denomination, set down the remainder under the column added, and carry the quotient to be added to the column of the next higher denomination. Repeat this process for every column.

If a column of any intermediate denomination be wholly wanting, indicate its existence by zeros.

Ex. 1. Add together £3. 13*s.* $4\frac{1}{2}d.$, £5. 12*s.* $6\frac{3}{4}d.$ and £8. 10*s.* $8\frac{1}{4}d.$

By the Rule we have

£.	*s.*	*d.*
3	13	$4\frac{1}{2}$
5	12	$6\frac{3}{4}$
8	10	$8\frac{1}{4}$
17	16	$7\frac{1}{2}$

Reason for the Rule.

$(2+3+1)$ farthings $=6$ farthings $=1d.+2$ farthings; so we put down 2 farthings or $\frac{1}{2}d.$ and carry $1d.$ to be added to $(4+6+8)d.$

thus getting in all 19*d*. which = 1*s*. 7*d*.; so we put down 7*d*. and carry 1*s*. to be added to (13+12+10)*s*. thus getting in all 36*s*. or £1. 16*s*.; we then put down 16*s*., and carry £1 which added to £(3+5+8) makes in all £17. The required sum ∴ = £17. 16*s*. 7½*d*.

Ex. 2. Add together 2 mds. 3 seers 3 chts, 5 mds. 4 seers 2 chts., 17 mds. 3 chts., 5 pus. 2 seers. 2 chts, and 3 mds. 7 pus. 3 chts.

By the Rule we have

mds.	pus.	seers.	pow.	chts.
2	0	3	0	3
5	0	4	0	2
17	0	0	0	3
0	5	2	0	2
3	7	0	0	3
28	5	4	3	1

Examples XXIV.

1. Add together

(1)

£.	s.	d.
1	2	3
4	5	6½
7	8	9¾
10	11	0¼

(2)

£.	s.	d.
20	19	11
21	18	10½
22	17	9
23	16	8¾

(3)

£.	s.	d.
39	15	7¼
38	14	6
37	13	5¾
36	12	4

(4)

R.	a.	p.
35	11	3
34	10	2
33	9	1
32	8	0

(5)

R.	a.	p.
43	7	1
44	6	2
45	5	3
46	4	4

(6)

R.	a.	p.
57	15	7
69	13	5
72	9	11
88	11	9

(7)

tons.	cwt.	qrs.	lbs.
20	15	3	16
31	16	2	17
42	17	1	18
53	18	0	19

(8)

tons.	cwt.	qrs.	lbs.
13	14	1	15
12	13	2	14
11	12	3	13
10	11	0	12

(9)

mds.	seers.	chts
32	33	9
33	34	10
34	35	11
35	36	13

(10)

mds.	seers.	chts.
45	23	3
47	21	5
53	25	7
63	35	9

(11)

yds.	ft.	in.
3	2	7
7	1	8
9	2	11
11	0	5

(12)

miles.	fur.	pol.
68	5	6
57	4	8
46	3	9
35	2	10

(13)

£.	s.	d.
5	7	9
11	13	4¼
23	15	7
28	19	11

(14)

£.	s.	d.
21	13	5
25	15	6
27	9	1½
28	14	3¾

(15)

£.	s.	d.
19	7	9
47	13	5½
68	9	4¼
73	16	6

(16)

R.	a.	p.
55	13	3
63	14	6
75	12	9
64	14	8

(17)

R.	a.	p.
79	11	6
8	12	3
9	15	9
11	7	3

(18)

R.	a.	p.
18	14	8
23	9	4
25	10	6
73	11	8

2. Find the sum of 15 bighas 16 kathas 14 chataks; 16 bighas 17 kathas 13 chataks; and 17 bighas 18 kathas 12 chataks.

3. Add together 26 sq. yds. 7 sq. ft. 9 sq. in.; 25 sq. yds. 8 sq. ft. 10 sq. in.; and 24 sq. yds. 9 sq. ft. 11 sq. in.

4. Find the sum 3 lbs. 4 oz. 5 dwts; 4 lbs. 5 oz. 6 dwts.; and 19 lbs. 7 oz. 13 dwts.

5. Add together R16 15*a.* 11*p.*; R28 13*a.*; R56 10*a.* 9*p.*; and R106 5*a.*

6. Add together 7 miles 5 fur. 6 po.; 9 miles 1 fur. 28 po.; and 91 miles 3 fur. 37 po.

SECTION IV. COMPOUND SUBTRACTION.

177. Def. Compound Subtraction is the method of finding the difference between two concrete numbers of different denominations.

As in Compound Addition, it is evident that the numbers between which the difference is to be found out must be of the same *kind*.

178. Rule. Place the less number below the greater so that numbers of the same denomination may be in the same column. Subtract the number of each denomination in the subtrahend from the corresponding number in the minuend, and put down the difference below. If a number of any denomination in the minuend

is less than the corresponding number in the subtrahend, increase it by the number of units of its own denomination contained in one unit of the next higher, and then perform the subtraction, taking care to add 1 to the number of the next higher denomination in the subtrahend.

The several partial differences taken together will be the difference required.

Ex. Subtract ₨5. 11*a* 3 pice from ₨18. 13*a*. 2 pice.

By the Rule we have

₨.	*a*.	*p*.
18	13	2
5	11	3
13	1	3

Reason for the Rule.

To subtract one concrete number from another is to subtract the number of each denomination in the subtrahend from the corresponding number in the minuend.

Now, as 3 pice cannot be subtracted from 2 pice, we increase the latter by 1 anna or 4 pice making it $4+2$ or 6 pice, and then $(6-3)$ pice $=3$ pice; so we put down 3 in the column of pice; and as we have added 1 anna to the minuend, to keep the difference unchanged, we add 1 anna to the 11 annas in the subtrahend (Art. 34 Prop. I) thus making it 12 annas; and 13 annas $-$ 12 annas $=$ 1 anna, which we put down in the annas' column; and lastly ₨18 $-$ ₨5 $=$ ₨13. Thus the difference required $=$ ₨13. 1*a*. 3 pice.

Examples XXV.

1. Subtract

(1) £15 15*s*. 6*d*. from £26 14*s*. $3\frac{3}{4}$*d*.

(2) £23 19*s*. 11*d*. from £70 13*s*. 9*d*.

(3) £36 5*s*. $7\frac{1}{2}$*d*. from £84 17*s*. 11*d*.

(4) £234 9*s*. 5*d*. from £500 0*s*. 6*d*.

(5) ₨7 8*a*. 9*p*. from ₨10 9*a*. 8*p*.

(6) ₨70 7*a*. 8*p*. from ₨88 6*a*. 5*p*.

(7) ₨56 13*a*. 11*p*. from ₨96 7*a*. 8*p*.

(8) ₨37 12*a*. 9*p*. from ₨59 13*a*. 3*p*.

(9) 3 lbs. 11 oz. 12 dwts. from 28 lbs. 8 oz. 10 dwts.

(10) 15lbs. 0 oz. 15drs. from 17 cwt. 3 qrs. 10 lbs.

(11) 1 md. 19 seers 7 chts. from 15 mds. 10 seers 3 chts.

(12) 17 mds. 28 seers 13 chts. from 29 mds. 8 seers.

(13) 1 mile 2 fur. 3 po. from 69 miles 5 fur. 37 po.

(14) 2 kroses 15 rasis from 5 kroses 10 rasis.

(15) 3 sq. yds. 7 sq. ft. 56 sq. in. from 20 sq. yds. 6 sq. ft. 20 sq. in.

(16) 15 cub. ft. 16 cub. in. from 24 cub. ft. 9 cub. in.

(17) 3 bighas 10 kathas 8 chts. from 12 bighas 8 kathas 6 chts.

(18) 7 kilog. 8 dekag. 9 gram. from 9 kilog. 8 hectog. 7 dekag.

2. Find the difference between

(1) 7 weeks 5 days 10 hrs. and 15 weeks 3 days 5 hrs.

(2) 6 prahars 2 dandas 56 pals and 7 prahars 1 danda 10 pals.

(3) 6 hrs. 7′ 8″ and 22 hrs. 2′ 5″.

(4) 5° 10′ 15″ and 18° 6′ 10″.

(5) 10 annas 13 gandas 1 cowry 1 krant and 5 annas 5 gandas 2 cowries 2 krants.

(6) 5*a.* 6 gan. 2 cowr. 2 kr. and 2*a.* 13 gan. 1 cowr. 1 kr.

SECTION V. COMPOUND MULTIPLICATION.

179. Def. The Multiplication of a concrete number of different denominations by an abstract number is called **Compound Multiplication.**

As we have already seen (Art. 38) the Multiplier must always be an abstract number. There are *apparent* exceptions to this rule, but they are *not real* exceptions. They will be noticed in Arts. 184 and 198.

180. Rule. Place the Multiplier below the lowest denomination of the multiplicand; multiply the number of the lowest denomination by the multiplier; divide the product by the number of units of its own denomination contained in one unit of the next higher; put down the remainder, if any, under the denomination in question; and carry the quotient to be added to the product of the multiplier by the number of the next higher denomination in the multiplicand. Repeat this process for each of the given denominations.

Ex. 1. Multiply ₹16 15*a.* 2*p.* by 7.

By the Rule we have

R.	a.	p.
16	15	2
		7
118	10	2

Reason for the Rule.

To multiply a concrete number of several denominations is to multiply the number of each separate denomination by the multiplier, and to take the sum total of all the partial products, after performing the necessary reductions.

Thus 2 pies $\times 7 = 14$ pies $= 1$ anna $+ 2$ pies; so we put down 2 pies in the pies' column. Next 15*a.* $\times 7 = 105$ annas, and to this must be added the 1 anna already obtained, and we thus get $105+1$ or 106 annas $=$ ₹6 10*a.*, and we put down 10 annas in the column of annas. Lastly we multiply ₹16 by 7 and get ₹112 to which must be added ₹6, thus making ₹118. Thus the product is ₹118 10*a.* 2 pies.

Examples XXVI.

1. Multiply

(1) £15. 10*s.* 6*d.* by 2, 3, and 5.

(2) £16. 11*s.* 7½*d.* by 4, 5, and 6.

(3) £717. 17*s.* 9¾*d.* by 7, 8, and 9.

(4) £28. 12*s.* 8*d.* by 10, 12, and 15.

(5) ₹20 12*a.* 9*p.* by 8, 12, and 16.

(6) ₹25 15*a.* 3*p.* by 5, 6, and 8.

(7) ₹32 8*a.* 6*p.* by 4, 6, and 12.

(8) ₹56 14*a.* 3*p.* by 3, 6, and 9.

(9) 10 lbs. 6 drs. 2 scrs. 15 grs. by 4, 5, and 10.

(10) 19 cwt. 2 qrs. 14 lbs. by 4, 8, and 12.

(11) 5 mds. 10 seers 3 powas by 6, 8, and 9.

(12) 17 mds. 15 seers 15 chts. by 15, 23, and 32.

(13) 2 wks. 5 days 15 hrs. 10′ by 15, 20, and 32.

(14) 5 days 21 hrs. 15′ by 31, 32, and 40.

(15) 6 kilog. 5 hectog. 6 grams 7 centig. by 5, 10, and 15.

(16) 8 dekam. 7 meters 6 decim. by 2, 4, and 8.

2. Find the product of

(1) £15. 10s. 10*d.* by 3 and 30.

(2) ₨18 10*a.* 6*p.* by 10 and 100.

(3) 15 cwt. 2 qrs. 24 lbs. by 50 and 100.

(4) 16 lbs. 15 dwts. 14 grs. by 20 and 200.

(5) 5 mds. 6 seers 7 chts. by 30 and 300.

(6) 30 mds. 20 seers 8 chts. by 10 and 15.

SECTION VI. COMPOUND DIVISION.

181. Def. The Division of a concrete number of different denominations by an abstract number or another concrete number is called **Compound Division.**

As we have already seen (Art. 51) the quotient in the former case will be a concrete number indicating the magnitude of each part after division, and in the latter, it will be an abstract number indicating the number of times that the divisor is contained in the dividend.

182. When the divisor is an abstract number proceed thus:

Rule. Place the numbers as in Simple Division. Divide the number of the highest denomination in the dividend by the divisor; the quotient will be of the same denomination; reduce the remainder, if any, to the next lower denomination, add to it the number of the corresponding denomination in the dividend, and then divide the sum by the divisor. Repeat this process down to the lowest denomination. The several partial quotients taken together will be the quotient required.

Ex. Divide ₨151 3*a.* 2 pies by 25.

By the Rule we have

```
       ₨.   a.  p.
  25)  151   3   2  (₨6.
       150
       ---
         1
        16
       ---
        16
         3
       ---
  25)   19  (0a.
        12
       ---
       228
         2
       ---
  25)  230  (9 5/25 pies.
       225
       ---
         5
```

$\therefore$ the quotient reqd. is R6 $0a.$ $9\frac{5}{25}$ pies.

Reason for the Rule.

The number of rupees to be divided is 151, and these rupees divided into 25 parts give 6 rupees for each part, with R1 over; and R1 = 16 annas; which together with the $3a.$ in the dividend give $19a.$ as the total number of annas to be divided; as $19a.$ cannot be divided into 25 parts, we reduce these to pies, getting 19×12 or 228 pies which together with the 2 pies in the dividend give 230 pies as the total number of pies to be divided, and 230 pies $\div$ 25 give $9\frac{5}{25}$ pies for each part; and nothing more remains to be divided. Thus the quotient is R6 $9\frac{5}{25}$ pies.

183. When the dividend and the divisor are both concrete numbers, proceed thus:

Rule. Reduce both the dividend and the divisor to the same denomination, and then proceed as in Simple Division.

Ex. Divide £57. 13*s.* by £5. 10*s.* 6*d.*

By the Rule we have

$$
\begin{aligned}
£57.\ 13s. &= (57 \times 20 + 13)s. &&= 1153s.\\
&= (1153 \times 12)d. &&= 13836d.\\
£5.\ 10s.\ 6d. &= (110s. + 6d.) &&= (110 \times 12 + 6)d.\\
& &&= 1326d.
\end{aligned}
$$

$$
\begin{array}{r}
1326)\ 13836\ (10\\
\underline{1326}\\
576
\end{array}
$$

$\therefore$ $10\frac{576}{1326}$ is the quotient reqd.

Reason for the Rule.

To divide £57. 13*s.* by £5. 10*s.* 6*d.* is to find how often the latter is contained in the former, *i. e*, how often the amount $1326d.$ is contained in $13836d.$, *i. e.*, how often the number 1326 is contained in 13836; so that we have only to divide 13836 by 1326 as in Simple Division.

Examples XXVII.

1. Divide—

(1) £9. 10*s.* 5*d.* by 2, 3 and 4.

(2) £55. 15*s.* 6*d.* by 5, 6 and 7.

(3) £67. 17*s.* 7*d.* by 8, 9 and 10.

(4) £226. 13*s*. 4*d*. by 72, 73 and 75.
(5) ₨56. 14*a*. 3*p*. by 10, 12 and 14.
(6) ₨150. 12*a*. 8*p*. by 15, 16 and 18.
(7) ₨225. 10*a*. 10*p*. by 19 and 31.
(8) ₨640. 8*a*. 6*p*. by 8 and 18.
(9) 56 gals. 3 qts. by 4 and 14.
(10) 72 qrs. 2 bus. 2 pks. by 8 and 16.
(11) 17 cwt. 2 qrs. 14 lbs. by 9 and 19.
(12) 1 ton 18 cwt. 16 lbs. by 15 and 20.

2. Divide—

(1) £15. 10*s*. 9*d*. by £1. 5*s*. 6*d*.
(2) £28. 9*s*. by £3. 3*s*. 3*d*.
(3) ₨52. 11*a*. by ₨3. 7*a*. 7*p*.
(4) ₨16. 15*a*. 11*p*. by ₨4. 3*a*. 2*p*.
(5) 22 sq miles by 3 sq. yds. 5 sq. ft.
(6) 22 sq. yds. 8 sq. ft. by 5 sq. yds. 2 sq. ft.
(7) 11 bighas 10 kathas by 2 bighas 11 kathas.
(8) 12 bighas 6 kathas by 4 bighas 2 kathas.
(9) 22 hrs. 55′ by 3 hrs. 10′.
(10) 16 hrs. 40′ by 3 hrs. 15′.
(11) 14 dwts. 10 grs. by 3 dwts. 12 grs.
(12) 42 mds. 10 seers by 8 seers 8 chts.

MISCELLANEOUS QUESTIONS AND EXAMPLES.

184. In working out Examples, the remarks in Art. 77 should be borne in mind.

Ex. 1. A person gives ₨5. 4*a*. to each of 13 men; how much does he spend altogether?

Here the total amount spent
= (amount given to one man) × (the number of men)
= (₨5. 4*a*.) × 13
= ₨68. 4*a*.

In the above process, it may appear at first sight that the multiplier 13 is a concrete number being 13 men; but in fact it is not so, for we have taken ₨5. 4*a*. 13 times simply, and not evidently 13 men times, which would be absurd.

Ex. 2. A person by giving R5. 4*a.* to each man, spent R68. 4*a.*; how many men were there?

Here the number of men reqd.
=the number of times that the sum of R5. 4*a.* is contained in R68. 4*a.*
=R68. 4*a.*÷R5. 4*a.*
=1092*a.*÷84*a.*=13.

Here it may appear at first sight that in dividing one concrete number R68. 4*a.* by another concrete number R5. 4*a.* we get a concrete number, *viz.*, 13 men for the quotient; but in fact that is not so. The quotient we get is not 13 men, but the abstract number 13, and it so happens only from the nature of the question that the *number* of men required to be found out is equal to this quotient 13, and has to be ascertained by means of the operation of Division indicated above.

Ex. 3. How many rupees, half-rupees, quarter-rupees, and two-anna pieces are there in R140. 10*a.*, supposing there to be an equal number of each?

Here, the number reqd.=the number of times that the sum of 1 rupee+1 half-rupee+1 quarter-rupee+1 two-anna piece is contained in R140 10*a.*

Now 1 rupee+1 half-rupee+1 quarter-rupee+1 two-anna piece
=15 two-anna pieces
and R140 10*a.* =2250*a.*
=1125 two-anna pieces;
∴ the number reqd. =the number of times that 15 two-anna pieces are contained in 1125 two-anna pieces,
$=\frac{1125}{15}=75.$

Ex. 4. A bag contains a certain number of sovereigns, twice as many half-sovereigns and five times as many crowns; and the whole sum in the bag amounts to £653. 5*s.* Find the number of coins of each kind.

Here, if we divide the coins in the bag into groups of 1 sovereign, 2 half-sovereigns and 5 crowns each, the number of such groups will be the same as the number of sovereigns contained.

Hence the no. of sovereigns in the bag
=the no. of times that one of these groups is contained in the given sum
=£653. 5*s.*÷(1 sovereign+2 half-sovereigns+5 crowns)
=13065*s.*÷65*s.*
$=\frac{13065}{65}=201.$
And the no. of half-sovereigns=2×201=402,
and .... crowns =5×201=1005.

Ex. 5. A milkman buys milk for ₨8 at 7 seers a rupee, and sells it adulterated with water at 8 seers a rupee, making a profit of 2 pice in the rupee; how much water has he added? and how much more water would have raised his whole profit to 1 rupee?

Here the quantity of milk bought $= 8 \times 7$ or 56 seers.

Now, in the first case, there is a profit of 2 pice in the rupee or 8×2 pice or 4 annas in all; and so there has been milk sold for ₨8. 4*a.* or ₨$8\frac{1}{4}$ at 8 seers a rupee;

$\therefore$ the quantity of milk sold $= (8 \times 8\frac{1}{4})$ seers $= 66$ seers.

and..............water added $= (66 - 56)$seers $= 10$ seers.

Similarly, in the second case,

the quantity of milk sold $= (8 \times 9)$ seers $= 72$ seers;

$\therefore$...............water added $= (72 - 56)$ seers $= 16$ seers, and $\therefore$ the additional quantity of water required in the second case $= (16 - 10)$ seers $= 6$ seers.

Ex. 6. A traveller who walks 20 miles a day, is followed in the same route by another traveller who walks 25 miles a day, and starts 3 days later. When will the latter overtake the former; and what will be the distance between them 10 days after the latter starts?

Here, the first traveller is 3×20 or 60 miles in advance of the second; and $\therefore$ before he is overtaken, the latter must walk 60 miles more than the former.

But he walks daily $(25 - 20)$ or 5 miles more than the first traveller;

$\therefore$ he will overtake the first traveller in $\frac{60}{5}$ or 12 days. Again, on the 10th day after starting, the second traveller is (10×25) miles or 250 miles from the starting point; and the first $(10 \times 20 + 60)$ miles or 260 miles from the same point; $\therefore$ the distance between them $= (260 - 250)$ miles
$= 10$ miles.

Ex. 7. A man buys 3 yds. of woollen cloth, 5 yds. of silk, and 7 yds. of linen, for ₨23. 10*a.* A yard of silk is worth 4 yards of linen, and a yard of woollen cloth is worth 3 yards of silk. What is the price of each per yard?

By the question,

5 yds. of silk are worth (4×5) yds. or 20 yds. of linen, and 3 yds. of woollen cloth worth (3×3) yds. of silk, *i. e.*, worth $(4 \times 3 \times 3)$ yds. or 36 yds. of linen; so that the man would have

spent the same amount of money if instead of buying 3 yds. of woollen cloth and 5 yds. of silk, he had bought (36+20) yds. of linen: so that (36+20) yds. +7 yds. or 63 yds. of linen are worth R23. 10*a*.

∴ the price of linen per yd. = R23. 10*a*. ÷ 63

$= \frac{378}{63}a. = 6a.$

and..................silk............ $= 4 \times 6a. =$ R1. 8*a*.

and............woollen cloth....... $= 3 \times$ R1 8*a*. = R4. 8*a*.

Examples XXVIII.

I.

1. Explain clearly how concrete quantities are numerically represented.

2. Give the different meanings of the term *carat*. What is the weight of shilling, and how many shillings are equal to a rupee in weight?

3. How many drams of Apothecaries' weight are there in 1000 dwts. and how many drams Avoirdupois are there in the same?

How many penny-weights are there in 1 cwt.?

4. What is Reduction?

Reduce 33 half-crowns to farthings, and 3300 pies to rupees.

5. How many barley-corns will reach round the Earth, supposing its circumference to be 25000 miles?

6. A room is 10 ft. 6 in. from the floor to the ceiling. How many copies of a book, 1 in. thick, must be piled upon the floor, one above another, so as to reach to the ceiling?

II.

1. What are the different uses of Troy weight and Avoirdupois weight?

Supposing the revenue of India to be 60 crores of rupees, and to be collected in rupees, what would be its weight in maunds and pounds Troy; and how many carts would be required to carry it, supposing each cart to carry 16 maunds?

2. Supposing the population of India to be 200 millions, what would be the amount raised, if every individual in India were to contribute 1 cowry (there being 80 cowries in a pice)? How much more money wonld be raised if each individual were to contribute 1 pie?

3. If each individual on an average consume half a seer of salt per month, what would be the total quantity of salt consumed in 1 year in India, supposing its population to be 200 millions; and what would be its price, at 2 annas a seer?

4. What is the relation between a linear foot and a square foot?

Shew how to find the number of square inches in 1 square foot.

5. England including Wales contains 58660 square miles. What is the area of England and Wales in bighas?

6. What is the Julian year, and what correction has been made in it by Pope Gregory XIII?

How many hours are there in an average Julian year?

III.

1. What is Compound Addition? State the Rule for Compound Addition.

Find the value of 1 crown + 1 mark + 1 pound + 1 guinea + 1 moidore.

2. How many pice are there in 50 rupees, 50 half-rupees, and 50 quarter-rupees taken together?

3. A gentleman's eldest child was born on the 5th of August 1869, his second child on the 13th of April 1872, his third child on the 7th of October 1874, and his fourth child on the 23rd of March 1877. What will be the sum of the ages of all these children on the 30th of September 1878; and what was the day and year of the father's birth, supposing that he had lived 8000 days on the birth day of his eldest child?

4. A person goes to the bazar with ₨86. 4*a.* in his pocket and spends ₨25. 4*a.* in the purchase of books, ₨26. 8*a.* in the purchase of cloth, and ₨4. 14*a.* in the purchase of stationery. How much money has he still left?

5. A milkman supplies a customer with milk at the rate of 3 seers a day for the whole of the year 1876. What is the total quantity of milk supplied, and what is its price, if milk sells at 2 annas a seer?

6. Between the execution of Louis XVI, January 21 of 1793, and the battle of Waterloo, June 18 of 1815, how many days intervened?

IV.

1. In Compound Subtraction, when a number of any denomination in the minuend is less than the number of the same

denomination in the subtrahend, how do you proceed, and what is your reason for proceeding in that way?

2. The first class railway fare from Howrah to Benares is R44. 8*a*. 6*p*., and from Howrah to Delhi is R89. 7*a*. Find the difference between the two fares, and the distance between Benares and Delhi by railway, supposing the fares to be at the rate of 1*a*. 6*p*. per mile.

3. A bag contains a certain number of rupees, twice as many half-rupees, three times as many four-anna pieces, and four times as many two-anna pieces; and the whole sum in the bag is R81. 4*a*. How many coins are there of each kind?

4. A grocer buys 7 mds. of sugar at R13. 4*a*. per maund, and 9 mds. at R12. 8*a*. per maund, and mixes the quantities together. At what price per maund must he sell the mixture to secure a profit of R20 on the whole?

5. A box with its contents weighs 7 mds. 8 seers, and the weight of the box is 1 md. 15 seers. Find the weight of its contents.

6. What is the relation between a linear and a square bigha?

The area of India is 1300000 square miles. How many bighas does it contain?

V.

1. What is Compound Multiplication? Can you multiply one concrete number by another?

Find the price of 58 mds. of rice at R4. 12*a*. a maund.

2. A dealer buys 150 mds. of rice at R4. 12*a*. a maund. At what price per maund must he sell it to secure a profit of R9. 6*a*. on the whole?

3. How much money must you lay out in the purchase of rice, when it is selling at R4. 14*a*. a maund, in order to be able to realize a profit of R25 by selling it at R4. 15*a*. a maund?

4. How often will a clock, which strikes the hours, strike in 1877?

5. In a certain manufactory, there are employed a certain number of men, each getting 5 annas a day, and twice as many women, each receiving 3 annas daily. Supposing the total amount of their wages per week to be R57. 12*a*., find the number of men employed.

6. A merchant bought 100 pieces of cloth at R4. 12*a*. per piece. Some pieces were damaged, and he sold the remaining pieces at R5 per piece, so as just to cover the amount of his outlay. How many pieces were damaged?

VI.

1. What is Compound Division? How do you proceed when the dividend and the divisor are both concrete numbers? How many copies of a book at £1. 15*s.* a copy can be had for £78. 15*s.*?

2. A certain number of men, twice as many women, and three times as many boys, together earned ₨37. 3*a.* a week, each man getting 5*a.*, each woman 3*a.*, and each boy 2*a.* a day. How many boys were there?

3. A pedestrian who walks 12 kroses a day, started from a certain place some time after another pedestrian, who walks 10 kroses a day, had left the same place. How long after the first did the second pedestrian start, supposing them to meet at the distance of 180 kroses from the place of starting?

4. Light travels at the rate of 192000 miles a second. How long does it take to come from the Sun to the Earth, a distance of 96 millions of miles?

5. Divide ₨1800 among *A*, *B* and *C*, so that as often as *A* gets ₨2 *B* may get ₨3 and *C* ₨4.

6. A contractor employs 1 foreman, 2 workmen, and 3 women to do a certain work. They work from the 1st of January to the 30th of April 1876. What is the amount due to them, if the daily wages of a foreman, a workman, and a woman be 8 annas, 6 annas, and 2 annas, respectively? And what would be the contractor's profit, supposing him to get 1 pice in the rupee?

CHAPTER IV.

THE FUNDAMENTAL OPERATIONS WITH CONCRETE FRACTIONS.

SECTION I. NOTATION AND REDUCTION OF CONCRETE FRACTIONS.

185. The method of notation for concrete fractions is the same as that for concrete integers. Thus, *two-thirds of a rupee, three-fifths of an anna, four-tenths of a pice,* will be written R$\frac{2}{3}$, $\frac{3}{5}a.$, ·4 pice respectively.

186. The Reduction of concrete fractions is effected by combining the Rules in Art. 173, with the Rules for the Multiplication and Division of fractions and decimals, as will be seen from the following examples.

Ex. 1. Reduce $\frac{5}{6}$ of a rupee to pies.

$$\text{R}\tfrac{5}{6} = \tfrac{5}{6} \times 16a. = 13\tfrac{1}{3}a.$$
$$= 13a. + \tfrac{1}{3} \times 12p.$$
$$= 13a.\ 4p. = 160p.$$

This Example might also have been worded thus:—

Find the value in pies of $\frac{5}{6}$ of a rupee.

Ex. 2. Find the value in pence of ·005 of £1.

$$\cdot005 \text{ of } £1 = \cdot005 \times 20s. = \cdot1s.$$
$$= \cdot1 \times 12d. = 1\cdot2d.$$

Ex. 3. Reduce $\frac{2}{3}$ of a pice to the denomination of a rupee.

$$\tfrac{2}{3} \text{ pice} = (\tfrac{2}{3} \div 4)a. = \tfrac{1}{6}a.$$
$$= \text{R}(\tfrac{1}{6} \div 16) = \text{R}\tfrac{1}{96}.$$

i. e., $\frac{2}{3}$ of 1 pice $= \frac{1}{96}$ of R1.

Ex. 4. Reduce 2·5*s.* to the decimal of £1.

$$2\cdot5s. = £(2\cdot5 \div 20) = £\cdot125.$$

Ex. 5. Find the value of $\frac{2}{3}$ of £5.

$$\tfrac{2}{3} \text{ of } £5 = 5 \text{ times } \tfrac{2}{3} \text{ of } £1 = 5 \times £\tfrac{2}{3} = £\tfrac{10}{3} = £3\tfrac{1}{3}$$
$$= £3 + \tfrac{20}{3}s. = £3.\ 6s. + \tfrac{1}{3}s.$$
$$= £3.\ 6s.\ 4d.$$

187. Rule I. To reduce one concrete number to the fraction of another concrete number of the same kind, reduce both to the same denomination, and put the former as the numerator and the latter as the denominator.

Ex. Reduce 11*a*. to the fraction of ₨5. 8*a*.

By the Rule we have

₨5. 8*a*. = 88*a*.,

∴ the fraction reqd. $= \frac{11}{88} = \frac{1}{8}$.

Reason for the Rule.

The question is in other words to find a fraction such that if ₨5. 8*a*, regarded as the unit, be divided into a number of parts as denoted by its denominator, and a number of these parts be taken as indicated by its numerator, the result will be equal to 11*a*. Now, our unit ₨5. 8*a*. *i. e.*, 88*a*. being divided into 88 parts, each part is 1 anna, and 11 such parts will be 11*a*.; ∴ $\frac{11}{88}$ *i. e.*, $\frac{1}{8}$ is the fraction required.

The same Example may be also worded thus :—

What part or fraction of ₨5. 8*a*. is 11*a*.?

The answer will be $\frac{11}{88}$, or $\frac{1}{8}$ for, evidently 11*a*. is $\frac{11}{88}$ths of ₨5. 8*a*. or 88*a*. regarded as the unit.

Rule II. To reduce one concrete number to the decimal of another concrete number of the same kind, reduce the former to the fraction of the latter and then reduce the fraction so obtained to its corresponding decimal.

Ex. Reduce 6*d*. to the decimal of £5.

By the Rule we have

£5 = 240 × 5*d*. = 1200*d*.

∴ the fraction $= \frac{6}{1200} = \frac{1}{200}$

and ∴ the decimal = ·005.

188. We can compare the values of concrete fractions by first reducing them to a common denomination and then comparing them as abstract fractions.

Ex. Compare the values of $\frac{5}{18}$ of ₨1 and $\frac{2}{3}$ of 8*a*.

We have ₨$\frac{5}{18} = \frac{5}{18} \times 16a. = \frac{40}{9}a.$

and $\frac{2}{3}$ of $8a. = \frac{2}{3} \times 8a. = \frac{16}{3}a. = \frac{48}{9}a.$

Hence $\frac{2}{3}$ of 8*a*. is greater than $\frac{5}{18}$ of ₨1.

Examples XXIX.

1. Find the values of

(1) $\frac{3}{8}$ of £1. 17*s*. 6*d*.; and ·5 of £6. 7*s*. 4*d*.

(2) $\frac{5}{7}$ of £15. 1*s*. 7*d*.; and ·25 of £10. 0*s*. 8*d*.

(3) $\frac{3}{8}$ of £10. 10*s*. 8*d*.; and ·6 of £56. 5*s*.

(4) $\frac{5}{6}$ of £9. 9*s*. 6*d*.; and ·4 of £66. 10*s*.

(5) $\frac{2}{3}$ of R4. 5*a*. 4*p*.; and ·8 of R6. 4*a*. 10*p*.

(6) $\frac{1}{10}$ of R8. 2*a*. 10*p*.; and ·2 of R7. 3*a*. 5*p*.

(7) $\frac{7}{8}$ of 15 cwt. 2 qrs. 9 lbs.; and ·3 of 5 lbs. 5 oz. 10 dwts.

(8) $\frac{4}{5}$ of 9 mds. 6 seers 9 chts.; and ·9 of 8 mds. 12 seers 8 chts.

(9) $\frac{4}{9}$ of 1 mile 6 fur. 7 po.; and ·12 of 15 yds. 2 ft. 11 in.

(10) $\frac{1}{15}$ of 31 days 11 hrs. 15′; and ·75 of 3 hrs. 32′.

2. Reduce

(1) 15*s*. to the fraction of £1; and 16*s*. to the decimal of £1.

(2) 9*s*. 6*d*. to the fraction of £19. 19*s*.; and 7*s*. 6*d*. to the decimal of £1. 10*s*.

(3) 10*a*. 8*p*. to the fraction of R1.; and 9*a*. 6*p*. to the decimal of R1. 3*a*.

(4) 5*a*. 6*p*. to the fraction of R6. 3*a*.; and 10*a*. to the decimal of R6. 4*a*.

(5) 15 hrs. 30 min. to the fraction of 31 days; and 5 hrs. 15 min. to the decimal of 1 week.

(6) 3 pts. to the fraction of 1 gal.; and 2 pks. to the decimal of 4 bus.

(7) 3 ft. 9 in. to the fraction of 4 yds.; and 9 in. to the decimal of 1 yd.

(8) 8 kathas to the fraction of a bigha; and 4 acres to the decimal of 1 sq. mile.

3. What fraction of £1 is 12*s*. 6*d*.; and what decimal of R1. is 5*a* 4*p*.?

4. What fraction of a month of 30 days is 18 hrs.; and what decimal of a week is 21 hrs.?

5. What part of R30 is R7. 8*a*.; and what part of R100 is R6. 4*a*.?

6. Compare the values of

(1) $\frac{1}{12}$ of R1., $\frac{2}{3}$ of 2*a*., and $\frac{1}{18}$ of R1. 8*a*.

(2) $\frac{1}{5}$ of £1., $\frac{3}{8}$ of 2 crowns, and $\frac{5}{6}$ of 1 florin.

(3) $\frac{1}{15}$ of 1 md., $\frac{2}{3}$ of 15 seers, and $\frac{7}{8}$ of 1 pusury.

(4) $\frac{1}{18}$ of 2 yds., $\frac{3}{8}$ of 4 ft., and $\frac{5}{6}$ of 3 ft.

SECTION II. ADDITION OF CONCRETE FRACTIONS.

189. The Addition of concrete fractions may be performed by either of two ways, *viz.*,

First, By finding the values of the several fractions and then adding these values; or

Secondly, By reducing the fractions to a common denomination, then adding them as abstract fractions, and lastly finding the value of the sum.

Sometimes the first method will be found convenient, and sometimes the second, as will be seen from the Examples given below.

Ex. 1. Add together $\frac{2}{5}$ of £10. 10*s*., $\frac{1}{3}$ of 6*s*. 9*d*., and $\frac{5}{7}$ of £14. 1*s*. 2*d*.

Here we follow the first method.

$\frac{2}{5}$ of £10. 10*s*. = £4. 4*s*.

$\frac{1}{3}$ of 6*s*. 9*d*. = 2*s*. 3*d*.

$\frac{5}{7}$ of £14. 1*s*. 2*d*.

= £10 + $\frac{5}{7}$ of 1*s*. 2*d*.

= £10 + $\frac{5}{7}$ of 14*d*. = £10. 0*s*. 10*d*.

∴ the sum reqd. = £14. 7*s*. 1*d*.

Ex. 2. Find the sum of $\frac{2}{27}$ of R3, $\frac{1}{2}$ of 8*a*., and $\frac{2}{21}$ of R1. 12*a*.

Here we follow the second method.

$\frac{2}{27}$ of R3 $= \frac{2}{27} \times 3 \times 16a. = \frac{32}{9}a.$

$\frac{1}{9}$ of 8*a*. $= \frac{8}{9}a.$

$\frac{2}{21}$ of R1. 12*a*. $= \frac{2}{21} \times 28a. = \frac{8}{3}a.$

∴ the sum $= (\frac{32}{9} + \frac{8}{9} + \frac{8}{3})a. = \frac{64}{9}a. = 7\frac{1}{9}a. = 7a.\ 1\frac{1}{3}p.$

Ex. 3. Find the sum of

$\cdot\dot{3}$ of £2., ·5 of 10*s*. and ·024 of £8.

$\cdot\dot{3}$ of £2 = $\frac{1}{3}$ of £2. = £$\frac{2}{3}$.

·5 of 10*s*. = $\frac{5}{10}$ of £$\frac{1}{2}$ = £$\frac{1}{4}$.

·025 of £1 = £$(\frac{25}{1000} \times 8)$ = £$\frac{1}{5}$.

∴ the sum reqd. = £$(\frac{2}{3} + \frac{1}{4} + \frac{1}{5})$ = £$\frac{40+15+12}{60}$

= £1 + $\frac{7}{60}$ × 20*s*.

= £1. 2*s*. 4*d*.

Examples XXX.

Find the value of

(1) £$\frac{1}{2}$ + $\frac{1}{3}$*s.* + $\frac{3}{7}$ guin. + $\frac{3}{5}$ crown.

(2) £$\frac{3}{5}$ + $\frac{7}{10}$*s.* + $\frac{7}{5}$ of $\frac{3}{7}$ of £1 + 3$\frac{1}{3}$*s.*

(3) R$\frac{2}{3}$ + $\frac{3}{15}$ of 5*a.* + $\frac{2}{5}$ of R7. (4) ·5*s.* + £·$\dot{3}$ + 3·$\dot{3}$*s.* + £$\frac{3}{5}$.

(5) £·$\dot{1}$ + ·2*s.* + ·3*d.* (6) R·3 + ·4*a.* + ·5 of R6. 10*a.*

(7) $\frac{1}{3}$ of 1 md. 2 seers 15 chts. + $\frac{1}{12}$ of 3 mds. + $\frac{7}{16}$ of 4 mds.

(8) $\frac{2}{3}$ of 1 ft. 6 in. + $\frac{3}{4}$ of 2 ft. 3 in. + $\frac{1}{5}$ of 1 ft. 3 in.

(9) $\frac{1}{7}$ of 2 wks. 21 hrs. + $\frac{1}{3}$ of 2 days + $\frac{1}{8}$ of 7 days.

(10) $\frac{1}{5}$ of 2 cwt. + $\frac{3}{5}$ of 15 lbs. + $\frac{3}{11}$ of 22 lbs.

SECTION III. SUBTRACTION OF CONCRETE FRACTIONS.

190. The Subtraction of concrete fractions may be performed in either of the two ways mentioned in Art. 189.

Ex. 1. Subtract $\frac{2}{5}$ of R10. 10*a.* from $\frac{3}{11}$ of R22.

$\frac{3}{11}$ of R22 = R6

$\frac{2}{5}$ of R10 10*a.* = R4. 4*a.*

∴ the difference = R1. 12*a.*

Ex. 2. Subtract $\frac{2}{7}$ of R2 from $\frac{3}{5}$ of R3.

Difference reqd. = R$(\frac{3}{5} \times 3 - \frac{2}{7} \times 2)$

= R$(\frac{9}{5} - \frac{4}{7})$ = R$\frac{43}{35}$

= R1 + $\frac{8}{35} \times 16$*a.*

= R1. 3*a.* + $\frac{23}{35} \times 4$ pice

= R1. 3*a.* 2$\frac{22}{35}$ pice.

Ex. 3. Subtract ·05 of £1 from ·5 of 4*s.*

·05 of £1 = (·05 × 20)*s.* = 1*s.*

·5 of 4*s.* = $(\frac{5}{10} \times 4)$*s.* = 2*s.*

∴ difference = (2 − 1)*s.* = 1*s.*

Examples XXXI.

Find the difference between

(1) $\frac{1}{2}$ of £3 and $\frac{3}{4}$ of £2.

(2) ·5 of £5. 10*s.* and ·25 of £10. 5*s.*

(3) $\frac{7}{8}$ of £3. 12*s.* and $\frac{2}{7}$ of £15. 1*s.*

(4) $\frac{2}{3}$ of R1. 4a and $\frac{3}{8}$ of R6. 9a.

(5) $\frac{1}{3}$ of R5. 4a. and $\frac{1}{12}$ of R9. 10a.

(6) ·2 of R5 and $\cdot\dot{3}$ of R6.

(7) $\frac{2}{3}$ of 1 cwt. 14lbs. and $\frac{1}{4}$ of 2 qrs. 12lbs.

(8) ·5 of 2 ft. 4 in. and $\frac{1}{8}$ of 6 ft. 8 in.

(9) ·24 of 3 mds. 5 seers and $\frac{1}{3}$ of 1 md. 2 seers.

(10) $\frac{1}{12}$ of 15 hrs. 16′ and $\cdot\dot{7}5$ of 5 hrs.

SECTION IV. MULTIPLICATION OF CONCRETE FRACTIONS.

191. The multiplier alone, or the multiplicand, or both may involve fractions.

Rule I. If the multiplier alone involves fractions, reduce it to the form of an improper fraction, if necessary, divide the multiplicand by the denominator of this fraction, and then multiply the quotient by the numerator.

Rule II. If the multiplicand or both the multiplicand and multiplier involve fractions, reduce the multiplicand to one denomination, then multiply the numbers as abstract fractions, and then find the value of the result.

Ex. 1. Multiply £6. 7s. 8d. by $\frac{2}{3}$.

By the Rule we have

	£	s.	d.
3)	£6.	7s.	8d.
	2.	2.	$6\frac{2}{3}$
			2
	£4.	5s.	$1\frac{1}{3}d$.

Reason for the Rule.

To multiply a number by a fraction is to divide it by the denominator and then multiply the quotient by the numerator (Art. 101). Therefore to multiply the given concrete number by $\frac{2}{3}$, we divide it by 3, and then multiply the quotient by 2.

Ex. 2. Multiply R$\frac{5}{48}$ + $\frac{3}{5}a$. by $3\frac{3}{4}$.

$$\text{R}\tfrac{5}{48} = \tfrac{5}{48} \times 16a. = \tfrac{5}{3}a.$$

∴ multiplicand $= (\frac{5}{3} + \frac{3}{5})a. = \frac{34}{15}a.$,

and multiplier $= \frac{15}{4}$;

∴ product $= \frac{34}{15} \times \frac{15}{4}a.$

$= \frac{17}{2}a.$

$= 8a.\ 6p.$

Ex. 3. Multiply $\frac{3}{5}$ of R1 by ·25.

Product $=(\frac{3}{5}\times$ R·25$)=$ R$\frac{3}{20}$

$=$ R·15.

Examples XXXII.

Find the value of

(1) £1. 5*s*. 7*d*. $\times \frac{3}{5}$.

(2) £15. 13*s*. 6*d*. $\times \frac{4}{9}$.

(3) £28. 7*s*. 7*d*. $\times$ ·25.

(4) R3. 5*a*. 6*p*. $\times \frac{3}{8}$.

(5) R57. 14*a*. 10*p*. $\times \frac{9}{10}$.

(6) R25. 15*a*. 9*p*. $\times$ ·75.

(7) ($\frac{3}{8}$ cwt. + $\frac{2}{7}$qrs.) $\times \frac{3}{4}$.

(8) ($\frac{17}{20}$md. + $\frac{3}{5}$seer) $\times \frac{7}{16}$.

(9) (£·5 + 5*s*) $\times$ ·025.

(10) (R·3 + 15*a*.) $\times \frac{5}{16}$.

(11) ($\frac{2}{3}$yd. + $\frac{1}{6}$ft) $\times \frac{5}{9}$.

(12) (£·02 + ·11*s*) $\times 33\frac{1}{3}$.

SECTION V. DIVISION OF CONCRETE FRACTIONS.

192. Rule I. If the dividend is wholly integral and the divisor is an abstract fraction, reduce it to the form of an improper fraction, if necessary, and then divide the dividend by the numerator, and multiply the quotient by the denominator.

Rule II. In other cases, reduce the dividend, and, if necessary, the divisor, to the same denomination; then divide as in the division of abstract fractions; and then find the value of the result, when it is a concrete number.

Ex 1. Divide £10. 6*s*. 6*d*. by $\frac{3}{4}$.

By Rule I, we have

3)	£10.	6*s*	6*d*.
	£3.	8*s*.	10*d*.
	£13.	15*s*.	4*d*.

Reason for the Rule

To divide a number by a fraction is to divide it by the numerator and then multiply the quotient by the denominator (Art. 109).

Therefore to divide the given concrete number by $\frac{3}{4}$, we divide it by 3 and then multiply the quotient by 4.

Ex. 2. Divide $\frac{2}{3}$ of £1 by $\frac{5}{6}$.

By Rule II, we have

of £1 $\div \frac{5}{6} =$ £$(\frac{2}{3}\div\frac{5}{6}) =$ £$\frac{4}{5}$

$=$ 16*s*.

Ex. 3. Divide £$\frac{2}{5}+\frac{3}{8}s.$ by $\frac{6}{7}d.$
By Rule II, we have

$$(£\tfrac{2}{5}+\tfrac{3}{8}s)\div\tfrac{6}{7}d.$$
$$=(8+\tfrac{3}{8})s.\ \div\frac{6}{7\times12}s.=(\tfrac{43}{8}\div\tfrac{1}{14})$$
$$=\tfrac{962}{8}=120\tfrac{2}{8}$$

Examples XXXIII

Find the value of

(1) £10. 19s. 11d ÷ $\frac{4}{5}$.

(2) £27. 10s. ÷ $\frac{2}{3}$ of 11s.

(3) ₨25. 4a. ÷ 33$\frac{2}{3}$.

(4) ₨17$\frac{3}{4}$ ÷ ₨4. 7a.

(5) ₨9. 15a. ÷ 6.

(6) 15 ft. 6 in. ÷ 3$\frac{1}{10}$.

(7) 12 ft. 8 in. ÷ ·9

(8) $\frac{2}{3}$ft. $\frac{4}{5}$in. ÷ 1 ft. 2 in.

(9) 11 mds. 10 srs. ÷ 1$\frac{1}{8}$ md.

(10) $\frac{2}{3}$ cwt. 1 qr. ÷ 2 cwt. 3 qrs.

(11) 10 bus. $\frac{1}{3}$ pk. ÷ 5$\frac{1}{2}$.

(12) ·5 hrs. 1 min. ÷ 15′. 30″.

SECTION VI. CONVERSION OF CONCRETE NUMBERS.

193. We shall here give some Examples of the conversion of concrete numbers expressed in units of one Table to their equivalents in units of another.

The Rules given in this and the preceding Chapters will be sufficient for the purpose.

Ex. 1. Convert £252. 9s. 9d. to Indian money, supposing ₨1 = 2s.

We have £1 = ₨10.
∴ £252 = ₨2520.
9s. = ₨4 8a.
9d. = $\frac{3}{4}$s. = ₨$\frac{3}{8}$ = 6a.
∴ £252. 9s. 9d. = ₨2524. 14a.

Ex. 2. Convert ₨194. 4a. to English money, supposing ₨1. = 2s.

₨194 = 388s. = £19. 8s.
4a. = ₨$\frac{1}{4}$ = $\frac{1}{2}$s. = 6d.
∴ ₨194. 4a = £19. 8s. 6d.

Ex. 3. Convert 325 Sicca R to current rupees.

Since 15 S. R $=$ R16

$\therefore$ 1 S. R $=$ R$\frac{16}{15}$;

and $\therefore$ 325 S. R. $=$ R$325 \times \frac{16}{15}$

$=$ R$\frac{65 \times 16}{3}$

$=$ R$\frac{1040}{3}$

$=$ R346. 10*a.* 8*p.*

Ex. 4. Convert R325 to Sicca rupees.

R1 $= \frac{15}{16}$ S. R.

$\therefore$ R325 $= \frac{15}{16} \times 325$ S. R.

$= \frac{4875}{16}$ S. R.

$= 304\frac{11}{16}$ S. R.

Ex. 5. Convert 8 cwt. 2qrs. 16 lbs. to Indian Bazar weight, and also to pounds Troy.

8 cwt. 2 qrs. 16lbs. $=$ 968 lbs.

Now, 1 seer $= \frac{72}{35}$ lbs. Avoir.

$\therefore$ 1 lb. Avoir. $= \frac{35}{72}$ seer,

and $\therefore$ 968 lbs. $= 968 \times \frac{35}{72}$ seers

$= \frac{121 \times 35}{9}$ seers

$=$ 11 mds. $30\frac{5}{9}$srs.

Again, 1 lb. Avoir. $= \frac{175}{144}$ lbs. Troy.

$\therefore$ 968 lbs. Avoir. $= 968 \times \frac{175}{144}$lbs. Troy

$= \frac{21175}{18}$lbs. Troy

$= 1176\frac{7}{18}$lbs. Troy.

Ex. 6. Convert 12 mds. 15 seers to Factory weight, and also to Troy weight.

12 mds. 15 srs. $= 12\frac{3}{8}$ mds.

$= \frac{99}{8}$mds.

Now, 1 md. $= \frac{54}{49}$ Factory md.

$\therefore$ $\frac{99}{8}$ mds. $= \frac{54}{49} \times \frac{99}{8}$ Fact. mds.

$= \frac{27 \times 99}{196}$ Fact. mds.

$= 13\frac{125}{196}$ Factory maunds.

Again, 1 md. $=$ 100 lbs. Troy.

$\therefore$ $\frac{99}{8}$mds. $= \frac{99}{8} \times 100$ lbs. Troy

$= \frac{99 \times 25}{2}$ lbs. Troy

$= 1237\frac{1}{2}$ lbs. Troy.

Ex. 7. Convert 2 sq. miles and 2 miles square to Bengal bighas.

$$2 \text{ sq. miles} = 2 \times 640 \text{ acres,}$$

and $$1 \text{ acre} = 3\tfrac{1}{40} \text{ bghs.}$$
$$= \tfrac{121}{40} \text{ bghs.}$$

$$\therefore\ 2 \text{ sq. miles} = 2 \times 640 \times \tfrac{121}{40} \text{ bghs.}$$
$$= 32 \times 121 \text{ bghs.}$$
$$= 3872 \text{ bghs.}$$

Again, $$2 \text{ miles square} = 2 \times 2 \text{ sq. miles}$$
$$= 2 \times 2 \times 640 \text{ acres}$$
$$= 2 \times 3872 \text{ bghs.}$$
$$= 7744 \text{ bghs.}$$

Ex 8. Convert 27 bghs. 15 kathas of Bengal measure to Benares bighas.

$$27 \text{ bghs. } 15 \text{ kths.} = 27\tfrac{3}{4} \text{ bghs.} = \tfrac{111}{4} \text{ bghs.}$$
$$= \tfrac{111}{4} \times 1600 \text{ sq. yds.}$$

Now, $$1 \text{ Benares bigha} = 3136 \text{ sq. yds.}$$

$$\therefore\ 27 \text{ bghs. } 15 \text{ kths.} = \tfrac{111}{4} \times 1600 \div 3136 \text{ Benares bghs.}$$
$$= \tfrac{111}{4} \times \tfrac{1600}{3136} \text{ Benares bghs.}$$
$$= \tfrac{111}{4} \times \tfrac{100}{196} \text{ Benares bghs.}$$
$$= \tfrac{111}{4} \times \tfrac{25}{49} \text{ Benares bghs.}$$
$$= \tfrac{2775}{196} \text{ Benares bghs.}$$
$$= 14\tfrac{31}{196} \text{ Benares bghs.}$$

Ex. 9. How many sq. bighas are there in 1 sq. kros?

Since $$1 \text{ kros} = 8000 \text{ hands}$$
$$= 100 \text{ linear bighas;}$$

$$\therefore\ 1 \text{ sq. kros} = 100 \times 100 \text{ sq. bighas}$$
$$= 10000 \text{ sq. bighas.}$$

Examples XXXIV.

Convert

1. 525 Sicca ₨ to current rupees.
2. 630 Sicca ₨ to current rupees.
3. 1000 Sicca ₨ to current rupees.
4. ₨100. 12*a*. to Sicca rupees.

5. R250. 10*a.* to English money, supposing R 1 = 2*s.*

6. £15. 10*s.* to Indian money, supposing £1 = R10. 8*a.*

7. 7 mds. 35 seers to Factory weight, and also to Avoirdupois weight.

8. 1 md. 2 seers to pounds Troy, and also to pounds Avoirdupois.

9. 14 lbs. 7 oz. Troy to lbs Avoirdupois.

10. 16 lbs. 8 oz. Avoir. to lbs. Troy.

11. 21 sq. miles to bighas.

12. 440 bighas 10 kathas to acres.

13. 12 acres 3 roods to bighas.

14. 15 hrs. 15 min. to dandas, and 15 dandas 15 pals to hours.

15. 2 dandas 8 pals to hours, and 9 hrs. 10 min. to dandas.

MISCLLLANEOUS QUESTIONS AND EXAMPLES.

194. We shall now give some Examples depending on the preceding Chapters.

Ex. 1. Find the value of

$\dfrac{1+\frac{2}{3}+\cdot 2}{\dot{7}}$ of R 6 $-\frac{7}{8}$ of 9*a.* $+\frac{10}{11}$ of R12. 6*a.*

The given quantity

$$\left\{\frac{\frac{5}{3}+\frac{2}{10}}{\frac{7}{9}}\times 6\times 16-\frac{63}{8}+\frac{10}{11}\times(12\times 16+6)\right\}a.$$

$$=\left(\frac{12}{5}\times 6\times 16-\frac{63}{8}+180\right)a.$$

$$=\left(\frac{1152}{5}-\frac{63}{8}+180\right)a.$$

$$=\left(230\tfrac{2}{5}-7\tfrac{7}{8}+180\right)a.$$

$$=402\tfrac{21}{40}a.$$

$$= \text{R}25.\ 2a.\ 6\cdot 3p.$$

Ex. 2. A man gives $\frac{1}{5}$ of what he has with him to *A* ; of what remains to *B* ; and 6 annas to *C*, and finds that he R1. 2*a.* left. How much had he at first, and how much did *A* and *B* each get ?

A gets $\frac{1}{5}$ of the whole ; ∴ there is left $1-\frac{1}{5}$ or $\frac{4}{5}$ of the whole ; and ∴ *B* gets $\frac{1}{4}$ of $\frac{4}{5}$ or $\frac{1}{5}$ of the whole ;

and then what is left $=1-(\frac{1}{5}+\frac{1}{5})=\frac{3}{5}$ of the whole ;

and this must be equal to what is given to C together with what is left after all;

i, e., 6*a*. + ₨1. 2*a*. $= \frac{1}{2}$ of the whole sum;

$\therefore$ the whole sum $= 2 \times$ (6*a*. + ₨1. 2*a*.)

$=$ ₨3.

Consequently A gets $\frac{1}{3}$ of ₨3 or ₨1,

and B gets $\frac{1}{4}$ of ₨2 or 8*a*.

Ex. 3. Divide £145. 4*s*. among A, B, and C in such a manner that as often as A gets £3, B shall get £4, and C, £5.

By the question,

for every sum of £(3+4+5) or £12 A gets £3, B gets £4, and C gets £5; $\therefore$ for every £1, A gets $\frac{3}{12}$ of £1, B gets $\frac{4}{12}$ of £1, and C gets $\frac{5}{12}$ of £1;

and $\therefore$ of the given sum, A gets $\frac{3}{12}$, B, $\frac{4}{12}$, and C, $\frac{5}{12}$; *i. e.*, A gets $\frac{3}{12}$ of £145. 4*s* or £36. 6*s*., B gets $\frac{4}{12}$ of £145. 4*s*. or £48. 8*s*., and C gets $\frac{5}{12}$ of £145. 4*s*., or £60. 10*s*.

Ex. 4. Divide ₨590 among A, B, and C in such a manner that as often as A gets ₨3, B shall get ₨4, and as often as B gets ₨5, C shall get ₨6.

For every ₨3 that A gets, B gets ₨4;

$\therefore$ .... ₨1 ...... A ...... B ......₨$\frac{4}{3}$;

$\therefore$ B's share $= \frac{4}{3}$ of A's share.

Similarly, C's......... $= \frac{6}{5}$ of B's share

$= \frac{6}{5}$ of $\frac{4}{3}$ of A's share

$= \frac{8}{5}$ of A's share.

$\therefore$ A's share + B's share + C's share

$=$ A's share + $\frac{4}{3}$ of A's share + $\frac{8}{5}$ of A's share

$= (1 + \frac{4}{3} + \frac{8}{5})$ of A's share $= \frac{59}{15}$ of A's share.

But A's share + B's share + C's share

$=$ the whole sum to be divided,

i. e., ₨590;

$\therefore$ $\frac{59}{15}$ of A's share $=$ ₨590,

and $\therefore$ A's share $=$ ₨590 $\div \frac{59}{15} =$ ₨590 $\times \frac{15}{59} =$ ₨150.

Consequently B's share

$= \frac{4}{3}$ of A's share $= \frac{4}{3}$ of ₨150 $=$ ₨200,

and C's share $= \frac{8}{5}$ of A's share $= \frac{8}{5}$ of ₨150 $=$ ₨240.

Ex. 5. *A* can do a piece of work in 2 days, *B* can do it in 3 days, and *C* in 4 days, working alone. In what time will they finish it working together?

In 2 days *A* does the whole work,

$\therefore$ in 1 day *A* does $\frac{1}{2}$ of the work.

Similarly, in 1 day *B* does $\frac{1}{3}$..................

and in 1 day *C* does $\frac{1}{4}$..................

$\therefore$ in 1 day *A*, *B*, *C*, together will do $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ of the work

i. e., $\frac{13}{12}$...............

$\therefore$ in 12 days......................................13 times the work,

and in $\frac{12}{13}$ of a day......................................the whole work.

Ex. 6. *A* can do half a piece of work in 3 hours, being twice as much as *B* can do; and *A*, *B* and *C* can together do the whole work in $2\frac{1}{2}$ hours. Shew that *C* can do in 5 hours as much work as *B* can do in 9 hours.

Here, in 1 hr. *A* can do $\frac{1}{3}$ of $\frac{1}{2}$ of the work

or $\frac{1}{6}$.................

and............*B*............ $\frac{1}{2}$ of $\frac{1}{6}$ or $\frac{1}{12}$.........

$\therefore$ in 1 hr. *A* and *B* together can do $\frac{1}{6}+\frac{1}{12}$ or $\frac{1}{4}$ of the work

and in 1 hr. *A*, *B* and *C* together can do

$\frac{1}{2\frac{1}{2}}$ or $\frac{2}{5}$ of the work

$\therefore$ .... *C* can do $\frac{2}{5}-\frac{1}{4}$.........

i. e., $\frac{3}{20}$........

$\therefore$ in 5 hrs. *C*....$5\times\frac{3}{20}$ or $\frac{3}{4}$.........

and in 9 hrs. *B*....$9\times\frac{1}{12}$ or $\frac{3}{4}$.........

$\therefore$ *C* can do in 5 hrs. as much work as *B* can do in 9 hrs.

Ex. 7. If an area of 1200 sq. ft. is to be plastered with a mixture of 3 parts of Portland cement, 3 parts of sand, and 1 part of lime, supposing 1 sq. ft. to require $1\frac{1}{3}$ seers of the mixture, how much of each ingredient will be required?

The total quantity of plastering mixture required $=1200\times1\frac{1}{3}$ seers $=1200\times\frac{4}{3}\times\frac{1}{40}$ mds. $=40$ mds.

Now, in the mixture, the whole no. of parts $=3+3+1=7$, and of these 3 are Portland cement, 3 sand, and 1 lime;

$\therefore$ the quantity of Portland cement

$=\frac{3}{7}$ of 40 mds. $=17$ mds. $5\frac{5}{7}$ seers;

the quantity of sand $= \frac{3}{7}$ of 40 mds.

$= 17$ mds. $5\frac{5}{7}$ seers.

and.................lime $= \frac{1}{7}$ of 40 mds.

$=$ 5 mds. $28\frac{4}{7}$ seers.

Ex. 8. An insolvent debtor owes three creditors, *A*, *B* and *C*, R4000, R5000, and R7000 respectively, and his assets amount to R2000 ; how much does each creditor get, and what do they get in the rupee ?

The debts amount to R(4000+5000+7000) or R16000.

$\therefore$ each creditor gets R$\frac{2000}{16000}$ or R$\frac{1}{8}$ or 2*a.* in the rupee ; and $\therefore$ *A* gets $\frac{1}{8}$ of R4000 or R500 ;

B gets $\frac{1}{8}$ of R5000 or R625 ;

and *C* gets $\frac{1}{8}$ of R7000 or R875.

Ex. 9. A corn-dealer has a mixture of 3 mds. of gram at R1 10*a.* a maund, 4 mds. at R1. 12*a.* a maund, and 5 mds. at R1. 14*a.* a maund. At what price must he sell the mixture (1) in order to secure a profit of R3 ; and (2) in order to secure the cost price after keeping 2 mds. for his own use ?

		R. *a.*		R.	
mds.	at	10	cost	4	14
		12		7	0
		14		9	6
12 mds.			cost	21	

(1) To secure a profit of R3, he must sell the 12 mds. for R21. 4*a.*+R3 or R24. 4*a.* ;

$\therefore$ selling price per maund = R24. 4*a.* $\div$ 12

= R2. 0*a.* 4 pies.

(2) After keeping 2 mds. for his own use, he must sell the remaining 10 mds. for R21. 4*a* ;

$\therefore$ selling price per maund = R21. 4*a.* $\div$ 10

= R2. 2*a.*

Ex. 10. A cistern has 3 pipes, *A*, *B*, and *C*. *A* and *B* can fill it in 3 and 4 hours respectively ; and *C* can empty it in 1 hour. If these pipes be opened in order at 3, 4, and 5 o'clock, when will the cistern be empty ?

In 1 hour *A* can fill $\frac{1}{3}$ of the cistern,

B......... $\frac{1}{4}$.....................

and *C* can empty the whole of the cistern ;

$\therefore$ at 5 o'clock,

A being open for 2 hours has filled $\frac{2}{3}$ of the cistern,

and *B*.................... 1 hr.................$\frac{1}{4}$.................... .

i. e., at 5 o'clock, $\frac{2}{3}+\frac{1}{4}$ or $\frac{11}{12}$ of the cistern is full.

Now after 5 o'clock *A* and *B* together tend to fill $\frac{1}{3}+\frac{1}{4}$ or $\frac{7}{12}$ of the cistern every hour, and *C* tends to empty the whole cistern per hour ;

$\therefore$ *A*, *B* and *C* being open at the same time, the rate of emptying is $1-\frac{7}{12}$ or $\frac{5}{12}$ of the cistern per hour ;

$\therefore$ $\frac{11}{12}$ of the cistern being the portion that was full, will be emptied in $(\frac{11}{12}\div\frac{5}{12})$ hrs.

i. e., in $\frac{11}{5}$ hrs. or 2 hrs. 12′.

Hence the cistern will be empty at 5 hrs. + 2 hrs. 12′ or 12′ past 7 o'clock.

Examples XXXV.

1. How far is the silver coinage a legal tender in England and in India, and what is its standard fineness in each of the two countries?

Find the value of a rupee in shillings, taking into account only the weight of pure silver contained in each.

2. A creditor in receiving payment of a certain amount of money in the silver coinage of India, finds that the weight of pure silver it contains is exactly $2\frac{3}{4}$ maunds. What is the amount received?

3. What are the weights of pure silver in 1000 rupees and in 2000 shillings?

4. Convert 525 Sicca rupees into current rupees, and also into English money, at 1*s.* 10*d.* per rupee.

5. A person inherits $\frac{2}{5}$ of an estate yielding R5250 a year. What is the annual income of his share, and what is his profit, if the Government revenue payable for the whole estate is R1050, and the cost of collection R10 for every R100 of income realised?

6. A person owns 5 annas 4 pies share of a zemindari yielding an income of R6525 a year, and he subsequently purchases a 2 annas, 13 gandas, 1 cowry, 1 krant share of the same. What fraction of the estate does he now own, and what is the income of his share?

II

1. What is the Imperial Standard Yard, and how may it be recovered if lost?

How many sq. yards are there in an acre, and how many in a bigha?

Convert 1 sq. mile into bighas, and 600 bighas into acres.

2. A person starts from a certain place and walks at the rate of 2 miles an hour. After $2\frac{1}{3}$ hours, another person starts from the same place, and walking at a certain rate overtakes the former at the distance of 25 miles from the starting place. At what rate does the second person walk?

3. *A* owns $\frac{3}{8}$ of a zemindari which contains 8225 bighas. He sells $\frac{4}{5}$ of his share to *B*, and ·5 of what remains to *C*. What share of the estate does he still own, and how much land does that share contain?

4. What fraction of a sq. mile is an acre, and what fraction of an acre is a bigha?

5. What decimal of a mile is a chain, and what fraction of a mile is a yard?

6. In the Thakbust scale, which is 16 inches to the mile, how many feet does an inch represent, and what fraction of an inch will represent a bigha?

III.

1. In a certain district the road cess is levied at the rate of one-half of an anna for every rupee of rent realized from an estate, less a deduction at one half of the said rate for every rupee of the revenue payable in respect thereof. What would be the amount of road cess leviable on an estate of which the gross rental is R8975, and for which the revenue payable is R3520?

2. An insolvent debtor, whose debts amount to R150000, has assets just sufficient to pay his creditors 7*a.* in the rupee. Find the amount of the assets.

3. Divide £3500 amongst *A*, *B*, and *C*, so that as often as *A* gets £2, *B* shall get £3, and as often as *B* gets £4, *C* shall get £5.

4. Divide R790 among *A*, *B*, and *C* in such a manner that *B*'s share shall be [illegible] of *A*'s, and *C*'s share ·3 of *B*'s.

5. What is the Imperial [illegible] may it be recovered if lost?

What fraction of a maund is a [illegible] Troy, and what fraction of a hundredweight is a maund?

6. A grocer bought sugar of a certain quality at R12. 4*a*. a maund, and twice the same quantity at R13 a maund, and by mixing the two together and selling the mixture at R12. 14*a*. per maund, made a profit of R12. 8*a*. How much sugar of each kind did he buy?

IV.

1. Find the value of

$$\frac{2+1\frac{1}{8}+\cdot 4}{\cdot 7} \text{ of } R12 + \cdot\dot{3} \text{ of } 2a. - \cdot 125 \text{ of } 1a.$$

2. *A* can do a piece of work in 4 days working 7 hrs. a day; *B* can do it in 4 days working 6 hrs. a day; and *C* and *A* can together do it in 2 days working 4 hrs. a day. In how many days will *A*, *B*, and *C* together finish the work by working 1 hr. a day?

3. How many revolutions will the wheel of a carriage 4 ft. 3 in. in diameter make in going over a mile, supposing the circumference of a circle to be 3·1416 × diameter?

4. How much land at R60 a katha must be given in exchange for 3 bighas 10 kathas of land at R85 a katha, together with a building on it worth R3500?

5. How many yards of cloth at 6*a*. per yard must be given in exchange for 16 yards of silk at R6. 4*a*. a yard?

6. How much of a maund is $(\cdot\dot{3}+\cdot 5+\frac{1}{12})$ of a cwt.?

V.

1. A cistern has three pipes, *A*, *B*, and *C*. *A* and *B* can fill it in 4 and 3 hours respectively, and *C* can empty it in 2 hours. When will the cistern be exactly full if the pipes are opened in order at 1, 2, and 3 o'clock respectively?

2. *A*, *B*, and *C* can do a piece of work in 12, 9, and 6 hours respectively. *A* and *B* together work for an hour, and then *B* goes away. How long will *A* and *C* take to finish the remaining portion of the work?

3. Give the length of the solar year and the average length of the Julian year; and find the difference between the two. In how many years would this difference amount to a day?

4. The 1st of January 1872 was a Monday. How many Sundays were there in that year, and how many Saturdays in the month of August of the same year?

5. What fraction of a kros is a mile, and how many jojans are there between the Moon and Earth, a distance of 240000 miles?

6. Light travels at the rate of 192000 miles a second, and sound, at the rate of 1142 feet a second. What time would intervene between the seeing of the flash and the hearing of the report of a gun fired at the distance of $2\frac{1}{2}$ miles?

VI.

1. What are the advantages of the Metric System of Weights and Measures?

How many grains are there in a seer of Bazar weight, and in a seer under the Indian Weights and Measures of Capacity Act 1871?

2. What decimal of an English mile is a kilometre?

3. A pedestrian who walks at the rate of $2\frac{1}{2}$ miles an hour, sets out from a place *A* for a place *B*, a distance of 66 miles, at the same time that another pedestrian who walks at the rate of 88 yards a minute sets out from *B* for *A*. Where will they meet, and how long after starting; and how much earlier ought the former to have started in order to meet the latter midway between *A* and *B*?

4. *A* and *B* can respectively finish a piece of work in 4 days of 9 working hours each and 3 days of 8 working hours each. In how many days will they finish the work if they work together 7 hours 12 minutes a day?

5. A person lays out £45 in spirits at 7*s.* 6*d.* a gallon. He adds water to it, and by selling the mixture at 7*s.* a gallon, finds that he has made a profit of one shilling for every pound of his outlay. How much water did he add?

6. The 1st of January fell on a Monday in 1877. When will it fall on a Monday again?

CHAPTER V.

SQUARE AND CUBIC MEASURE. DUODECIMALS.

SECTION I. SQUARE AND CUBIC MEASURE.

195. Def. The **Area** of any figure is the quantity of surface contained in it.

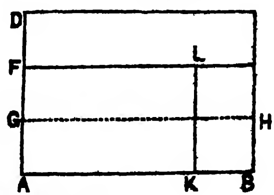
An area is measured by the number of times that it contains another known area which is taken as the *unit of area* We have seen in Art. 148, that the ordinary units of area of the several denominations, *viz.*, the square inch, the square foot, &c., are the areas of the squares on the ordinary units of length, *viz.*, the linear inch, linear foot, &c.

196. Def. A **Rectangle** is a four-sided figure having its opposite sides parallel and contiguous sides at right angles to each other.

Any side of a rectangle may be called its length and the contiguous side its breadth.

Prop. I. If two rectangles have the same length, the area of the one will be the same fraction of that of the other, that the breadth of the former is of the breadth of the latter.

Let $ABCD$ and $ABEF$ be two rectangles having the same length AB, and let $AF = \frac{2}{3} AD$. Divide AD into 3 equal parts AG, GF, FD; then AF will contain two of these parts. Now drawing the dotted line GH as in the figure, the rectangle $ABCD$ is divided into 3 equal rectangles whereof $ABEF$ contains two; and



$$\therefore ABCD = 3 \text{ times } ABHG,$$
$$\text{and } ABEF = 2 \text{ times } ABHG,$$
$$\therefore ABEF = \tfrac{2}{3}\, ABCD.$$

Similarly the Proposition can be proved in other cases.

Prop. II. If the length and the breadth of one rectangle be certain fractions of those of another, the product of these fractions indicates what fraction the area of the former is of that of the latter.

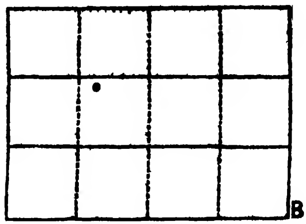
In the above figure, let $ABCD$ and $AKLF$ be two rectangles, and let $AK = \frac{3}{4} AB$, $AF = \frac{2}{3} AD$. Then by the preceding Proposition, $AKLF = \frac{3}{4} ABEF$

$= \frac{3}{4}$ of $\frac{2}{3} ABCD$

$= \frac{1}{2} ABCD$.

197. Prop. Taking the square on the linear unit as the superficial unit, the number of superficial units in a rectangle = the number of linear units in the length × the number of linear units in the breadth.

Let $ABDC$ be a rectangle.



First let the number of linear units in each of the sides be integral, say, 4 in AB and 3 in AC. Then drawing lines as in the figure, the rectangle $ABDC$ is divided into a number of equal horizontal slips equal to the number of linear units in AC; and each horizontal slip into a number of equal squares or units of superficial measure equal to the number of linear units in AB; and the number of superficial units in $ABDC$

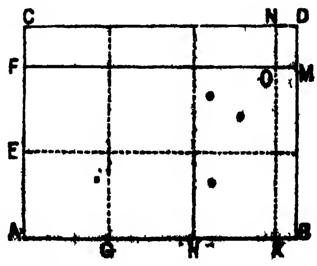
= the number of horizontal slips

× the number of squares in each slip

= the number of linear units in the breadth

× the number of linear units in the length

$= 3 \times 4$.



Next let the numbers involve fractions, and let AB contain $3\frac{1}{4}$ and AC, $2\frac{3}{8}$ linear units. In AB, AC measure off parts equal to the linear unit; and draw lines through the points of division as in the figure.

Then $AE = EF = AG = GH = HK =$ the linear unit,

$FC = \frac{3}{8}$ linear unit $= NO$,

$KB = \frac{1}{4}$ .... $= OM$;

and each of the 3 small rectangles in $CFON = \frac{2}{3}$ of superficial unit, as it has one side equal to a linear unit and the other to $\frac{2}{3}$ of linear unit (Art. 196, Prop. I). Similarly each of the 2 small rectangles in $BKOM = \frac{1}{4}$ of superficial unit. And the small rectangle $MOND = \frac{2}{3} \times \frac{1}{4}$ of superficial unit, as it has its side $ON = \frac{2}{3}$ linear unit, and the side $OM = \frac{1}{4}$ linear unit (Art. 196, Prop. II).

Hence, the no. of superficial units in $AFOK = 3 \times 2$,

.... $CFON = 3 \times \frac{2}{3}$,

.... $BKOM = 2 \times \frac{1}{4}$,

.... $MOND = \frac{2}{3} \times \frac{1}{4}$;

$\therefore$ adding, the no. of superficial units in

$AFOK + CFON + BKOM + MOND$

i. e., the no. of superficial units in $ABDC$

$= 3 \times 2 + 3 \times \frac{2}{3} + 2 \times \frac{1}{4} + \frac{2}{3} \times \frac{1}{4}$

$= 3 \times (2 + \frac{2}{3}) + \frac{1}{4} \times (2 + \frac{2}{3})$

$= (3 + \frac{1}{4}) \times (2 + \frac{2}{3}) = 3\frac{1}{4} \times 2\frac{2}{3}$.

Similarly the Proposition can be proved in other cases.

This Proposition is sometimes briefly stated thus :—

Area of a rectangle = length × breadth.

Thus, if the sides of a rectangle are 3 ft. and 4 ft.,

its area $= 3 \times 4$ sq. ft.

198. In the preceding Article it may appear that we have multiplied one concrete number 3 ft. by another 4 ft. and got 12 sq. ft. for the product. But in reality that is not so. What we have multiplied together are the abstract numbers 3 and 4; and it so happens only from the nature of things, that this product 3×4 represents the number of square feet in the rectangle whose length and breadth are 4 ft. and 3 ft. respectively.

It is only in this sense that we are to understand expressions such as these :—

Feet multiplied by feet give square feet.
Inches.....................inches.....................inches.
&c.............................&c...........................&c.

199. A rectangle 5 ft. by 4 inches $= \frac{1}{12}$ of a rectangle 5 ft. by 4 ft.,

$\because$ 4 inches the breadth of one $= \frac{1}{12}$ of 4 ft. the breadth of the other (Art. 196, Prop. I).

Hence 5 ft. × 4 in. $= \frac{1}{12}$ of 5×4 sq. ft. : and so in other cases.

200. In the Bengal superficial measure,

$\therefore$ 1 linear bigha $\times$ 1 linear bigha $=$ 1 sq. bigha.

1 .... bigha $\times$ 1 .... katha

$= 1$ .... bigha $\times \frac{1}{20}$ of 1...bigha $= \frac{1}{20}$ of 1 sq. bigha

$= 1$ sq. katha.

1 .... katha $\times$ 1..........katha

$= \frac{1}{20}$ of 1...bigha $\times$ 1..........katha $= \frac{1}{20}$ of 1 sq. katha

$= 1$ *dhool.*

The Bengal method of finding areas will be seen from the following Example :—

Ex. Find the area of a field 5 bighas 4 kathas by 4 bighas 8 kathas.

bghs.	kths.	
5	4	
4	8	
20	16	
2	1	12
22	17	12

The area is 22 sq. bghs. 17 sq. kths. 12 dhools.

The reason is obvious.

4 bghs. $\times$ 5 bghs. $=$ 20 sq. bghs.

4 bghs. $\times$ 4 kths. $=$ 16 sq. kths.

5 bghs. $\times$ 8 kths. $=$ 40 sq. kths. $=$ 2 sq. bghs.

8 kths. $\times$ 4 kths. $=$ 32 sq. dhools $=$ 1 sq. kth. 12 sq. dhools.

201. Def. The **Volume** of a solid is the quantity of space, having length, breadth, and thickness, that it contains.

The volume of a solid is measured by the number of times that it contains some other known volume which is taken as the unit of volume. We have seen in Art. 149 that the ordinary units of volume of the several denominations, *viz.*, the cubic inch, the cubic foot, &c., are the volumes of the cubes having for their edges the units of length, *viz.*, the linear inch, the linear foot, &c.

202. Def. A **Rectangular Parallelopiped** is a solid contained by six rectangles.

Any three contiguous edges of a rectangular parallelopiped may be called its length, breadth, and thickness, respectively.

Prop. The volume of a rectangular parallelopiped $=$ length $\times$ breadth $\times$ thickness.

This can be proved in a way similar to that given in Art. 149.

203. Areas and volumes are found by reducing the lengths, breadths, and thicknesses to one and the same denomination, and then performing the necessary multiplications, as will be seen from the subjoined Examples.

Ex. 1. Find the area of a rectangular court-yard 24 ft. 6 in. long and 15 ft. 3 in. broad.

The area $=(24 \text{ ft. } 6 \text{ in.}) \times (15 \text{ ft. } 3 \text{ in.})$

$=(24\frac{1}{2} \times 15\frac{1}{4})$ sq. ft. $=\frac{49}{2} \times \frac{61}{4}$ sq. ft.

$=\frac{2989}{8}$ sq. ft. $=373\frac{5}{8}$ sq. ft.

$=373$ sq. ft. 90 sq. in.

Ex. 2. Find the volume of a cube whose edge is 2 ft. 3 in.

2 ft. 3 in. $=2\frac{1}{4}$ ft. $=\frac{9}{4}$ ft.;

$\therefore$ the volume $=(\frac{9}{4} \times \frac{9}{4} \times \frac{9}{4})$ cub. ft.

$=\frac{729}{64}$ cub. ft.

$=11\frac{25}{64}$ cub. ft.

$=11$ cub. ft. 675 cub. in.

The above Example may also be worked thus :—

2 ft. 3 in. $=27$ in.;

$\therefore$ the volume $=(27 \times 27 \times 27)$ cub. in.

$=19683$ cub. in.

$=11$ cub. ft. 675 cub. in.

Ex. 3. What length must be cut off from a plank that is 1 ft. 3 in. broad, in order that it may contain 1 sq. yd.?

Since area = length × breadth,

$$\therefore \text{length} = \frac{\text{area}}{\text{breadth}};$$

$$\text{and } \therefore \text{ the reqd. length} = \frac{1 \text{ sq. yd.}}{1 \text{ ft. } 3 \text{ in.}} = \frac{9 \text{ sq. ft.}}{1\frac{1}{4} \text{ ft.}}$$

$$= 9 \times \tfrac{4}{5} \text{ ft.} = 7 \text{ ft. } 2\tfrac{2}{5} \text{ in.}$$

Examples XXXVI.

1. Find the areas of the following rectangles :—

(1) 2 ft. 3 in. by 1 ft. 6 in. (2) 3 ft. 6 in. by 2 ft. 9 in.

(3) 4 ft. 3 in. by 3 ft. 6 in. (4) 5 ft. 9 in. by 4 ft. 8 in.

(5) 6 ft. 4 in. by 5 ft. 6 in. (6) 7 ft. 8 in. by 1 ft. 4 in.

2. Find the volumes of the following rectangular parallelopipeds :—

(1) 2 ft. 3 in. × 1 ft. 6 in. × 9 in.

(2) 3 ft. 6 in. × 2 ft. 3 in. × 1 ft. 3 in.

(3) 4 ft. 3 in. × 2 ft. 8 in. × 1 ft.

(4) 4 ft. 9 in. × 3 ft. 6 in. × 2 ft.

3. A room is 18 ft. 6 in. by 15 ft. Find the area of its floor, and the length of carpet $1\frac{1}{4}$ yd. wide that will be required for carpeting the same.

4. A room is 21 ft. 3 in. by 12 ft. 6 in. Find the area of its floor. Find also the length of another room containing the same area, supposing its breadth to be 10 ft.

5. Find the solid content of a brick that is 10 in. long, 5 in. broad, $2\frac{1}{2}$ in. thick, and also that of a wall that is 20 ft. long, 10 ft. 6 in. high, and 2 ft. thick.

6. Find the areas of the following rectangular fields :—

(1) 2 bghs. 8 kths. by 1 bgh. 10 kths.

(2) 3 bghs. 10 kths. by 2 bghs. 5 kths.

(3) 4 bghs. by 3 bghs. 8 kths.

(4) 5 bghs. 6 kths. by 4 bghs. 15 kths.

(5) 10 bghs. 10 kths. by 1 bgh. 14 kths.

(6) 12 bghs. 10 kths. by 3 bghs. 8 kths.

SECTION II. DUODECIMALS.

204. Besides the method given in Art. 203, there is another method of finding areas and volumes, called the method of **Duodecimals** or **Cross Multiplication,** which is generally employed by painters, brick-layers, &c., in measuring work.

In this method, the successive linear units are Feet, Primes, Seconds, Thirds, &c., and are so connected that a unit of any denomination is $\frac{1}{12}$ of a unit of the next higher denomination.

Primes, seconds, thirds, &c., are indicated by the accents $'$, $''$, $'''$, &c., placed above the numbers, a little to the right. It is evident that primes are the same as inches.

The successive superficial units in this method are superficial feet, primes, seconds, thirds, &c., and are so connected that a unit of any denomination is $\frac{1}{12}$ of a unit of the next higher denomination. They are indicated in the same way as the linear units.

The successive solid units are solid feet, primes, seconds, thirds, &c.[c] They are connected and indicated in the same way as linear or superficial units.

205. Hence, by Art. 196,

$1 \text{ ft.} \times 1' = 1 \text{ ft.} \times \frac{1}{12} \text{ of } 1 \text{ ft.} = \frac{1}{12} \text{ of } 1 \text{ sq. ft.} = 1' \text{ of square measure}$

$1 \text{ ft.} \times 1'' = \frac{1}{144} \text{ of } 1 \text{ sq. ft.} = 1 \text{ sq. in} = 1'' \ldots\ldots\ldots\ldots$

$1 \text{ ft.} \times 1''' = \frac{1}{12} \text{ of } 1 \text{ sq. in.} = 1''' \ldots\ldots\ldots\ldots$

$1' \times 1' = 1 \text{ sq. in.} = 1'' \ldots\ldots\ldots\ldots$

$1' \times 1'' = \frac{1}{12} \text{ of } 1 \text{ sq. in.} = 1''' \ldots\ldots\ldots\ldots$

Similarly (by Art. 202),

$1 \text{ sq. ft.} \times 1' = \frac{1}{12} \text{ of } 1 \text{ cub. ft.} = 1' \text{ of solid} \ldots\ldots\ldots\ldots$

$1 \text{ sq. ft.} \times 1'' = \frac{1}{144} \text{ of } 1 \text{ cub. ft.} = 1'' \ldots\ldots\ldots\ldots$

$1 \text{ sq. ft} \times 1''' = 1 \text{ cub. in.} = 1''' \ldots\ldots\ldots\ldots$

$1' \text{ (superf.)} \times 1' = \frac{1}{144} \text{ of } 1 \text{ cub. ft.} = 1'' \ldots\ldots\ldots\ldots$

$1' \ldots\ldots \times 1'' = 1 \text{ cub. in.} = 1''' \ldots\ldots\ldots\ldots$

It will be seen that the denomination of every product is indicated by the sum of the accents of the factors, the number of accents in *feet* being regarded as 0.

206. We shall now give the Rule for finding areas and volumes by the above method.

Rule. Write the terms of the multiplier under the corresponding terms of the multiplicand. Multiply every term of the multiplicand beginning with the lowest by the highest term of the multiplier, divide each product (except where it is of the denomination of feet) by 12, carry the quotient to be added to the next product, and put down the remainder under the denomination indicated by the sum of the accents in the factors. Proceed in this way with every successive term of the multiplier, placing the partial product corresponding to each term, below the preceding partial product. Add up the several denominations of the partial products, carrying 1 for every 12. The sum will be the product required.

Ex. Multiply 3 ft. 9 in. by 2 ft. 7 in.

By the Rule we have

$$\begin{array}{rrr}
3. & 9' & \\
\underline{2.} & \underline{7'} & \\
7. & 6' & \\
2. & 2'. & 3'' \\
\hline
9. & 8'. & 3''
\end{array}$$

The *reason for the Rule* will be seen below.

It is evident from Art. 205,

that 2 ft. × 9′ = 18′ (superf.) = 1 sq. ft. + 6′

2 ft. × 3 ft. = 6 sq. ft.

∴ 3 ft. 9 in. × 2 ft. = 7 sq. ft. + 6′.

Again 7′ × 9′ = 63″ = (60″ + 3″) = 5′ + 3″

7′ × 3 ft. = 21′ = 1 sq. ft. + 9′

∴ 3 ft. 9′ × 7′ = 1 sq. ft. 9′ + 5′. 3″ = 2 sq. ft. 2′. 3″.

∴ the reqd. product = 7 sq. ft. 6′ + 2 sq. ft. 2′. 3″ = 9 sq. ft. 8′. 3″.

The result may be expressed in sq. feet and inches thus :—

9 sq. ft. 8′. 3″ $= 9$ sq. ft. $+ \left(\frac{8}{12} + \frac{3}{144}\right)$ sq. ft.

$= 9$ sq. ft. $+ \frac{99}{144}$ sq. ft.

$= 9$ sq. ft. 99 sq. in.

207. The above method is called the method of Duodecimals, because the number twelve (*duodecim* in Latin) forms the basis of connection between successive denominations ; and it is also called the method of Cross Multiplication, because the multiplication is performed in a cross order, *i. e.*, whereas we begin with the highest denomination in the multiplier, we begin with the lowest in the multiplicand.

Examples XXXVII.

Find by Cross Multiplication the areas of the following rectangles :—

1. 2 ft. 9 in. × 1 ft. 3 in.	**2.** 3 ft. 4 in. × 4 ft. 8 in.
3. 3 ft. 8 in. × 5 ft. 9 in.	**4.** 5 ft. 9 in. × 6 ft.
5. 6 ft. 7 in. × 7 ft. 8 in.	**6.** 6 ft. 8 in. × 7 ft. 7 in.
7. 8 ft. 9 in. × 6 ft. 11 in.	**8.** 12 ft. 11 in. × 4 ft. 6 in.
9. 9 ft. 9 in. × 3 ft. 5 in.	**10.** 10 ft. 10 in. × 5 ft. 5 in.

Miscellaneous Questions and Examples.

208. We shall here give some examples depending upon this and the preceding Chapters.

Ex. 1. What is the cost of building a wall 24 ft. long, 14 ft. high, and 2 ft. 3 in. thick for the first 4 ft. of its height, and 2 ft. thick for the remainder, at ₹22. 8*a*. per 100 cub. ft.? And how

many bricks will be required supposing each brick to be 9 in. × 4 in. × 3 in.?

The volume of the wall

$= 24$ ft. $\times 4$ ft. $\times 2$ ft. 3 in. $+ 24$ ft. $\times 10$ ft. $\times 2$ ft.

$= (24 \times 4 \times \frac{9}{4} + 24 \times 10 \times 2)$ cub. ft.

$= 696$ cub. ft.

Now, 100 cub. ft. cost R22. 8*a*. *i. e.*, R$\frac{45}{2}$,

$\therefore$ 1 cub. ft. will cost R$\frac{45}{2} \div 100$ or R$\frac{9}{40}$,

and $\therefore$ the wall will cost R$(696 \times \frac{9}{40})$,

i. e., R$(\frac{87 \times 9}{5})$ or R156. 9*a*. $7\frac{1}{5}$*p*.

Again, the no. of bricks reqd.

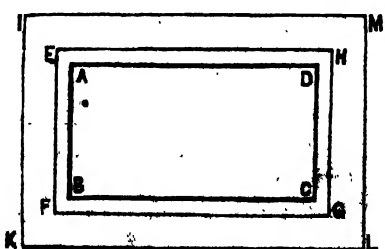
$$= \frac{\text{volume of the wall}}{\text{volume of a brick}}$$

$$= \frac{696}{\frac{9}{12} \times \frac{4}{12} \times \frac{3}{12}} = 696 \times 16$$

$$= 11136.$$

Ex. 2. A rectangular tank, 1 bigha 10 kathas in length and 1 bigha in breadth, has a rectangular gravel walk 4 cubits broad along its sides, at a distance of 2 cubits from each side. Find the cost of gravelling the walk, at the rate of R1 per 100 sq. ft.

Let *ABCD* be the tank, so that $AB = 1$ bgh., $BC = 1$ bgh. 10 kth.; and let the space between *EFGH* and *IKLM* be the gravel walk. Then area of the gravel walk
= rectangle *IKLM*—rectangle *EFGH*.



Now $EF = AB + 2 \times$ dist. of the walk from the tank

$= 80$ cubits $+ 2 \times 2$ cubits

$= 84$ cubits $= 126$ ft.;

and $FG = BC + 2 \times$ dist. of the walk from the tank

$= (120+4)$ cubits $= 186$ ft.

Again $IK = EF + 2 \times$ breadth of the walk

$= 126$ ft. $+ 12$ ft. $= 138$ ft.;

and $KL = FG + 2 \times$ breadth of the walk

$= 186$ ft. $+ 12$ft. $= 198$ft.

Therefore area of the walk

$= IK \times KL - EF \times FG$

$= (138 \times 198)$ sq. ft. $- (126 \times 186)$ sq. ft.

$= 3888$ sq. ft.

Now the cost of gravelling 100 sq. ft. $=$ R1,

$\therefore$ .... 1 sq. ft. $=$ R$\frac{1}{100}$,

and $\therefore$ .... 3888 sq. ft. $=$ R$\frac{1}{100} \times 3888$

$=$ R$\frac{972}{25}$

$=$ R$38\frac{22}{25}$

$=$ R38. $14a.\ \frac{24}{25}p.$

Ex. 3. Find the price of a beam 15 ft. long and 7 in. $\times$ 5 in. at its end, at 1 pice per 1 ft. $\times$ 1 in. $\times$ 1 in.

The volume of which the price is 1 pice

$= 1$ in. $\times 1$ in. $\times 1$ ft. $= 1 \times 1 \times 12$ cub. in. $= 12$ cub. in.

The volume of 1 foot of the beam

$= 7$ in. $\times 5$ in. $\times 1$ ft. $= 7 \times 5 \times 12$ cub. in.

$= 35 \times 12$ cub. in.

$= 35 \times$ the volume of which the price is 1 pice.

Therefore the price of 1 foot of the beam

$= 35$ pice $= 8a.$ 3 pice;

and $\therefore$ the price of the whole beam

$= 15 \times 8a.$ 3 pice

$=$ R8. $3a.$ 1 pice.

Ex. 4. What length must be cut off from a plank that is 9 in. wide in order that it may contain 1 sq. yard?

The area of the plank is required to be 1 sq. yd., and its breadth is 9 in.

Now, length × breadth = area,

$$\therefore \text{length} = \frac{\text{area}}{\text{breadth}},$$

and the reqd. length

$$= \frac{1 \text{ sq. yd.}}{9 \text{ in.}}$$

$$= \frac{9 \text{ sq. ft.}}{\frac{9}{12} \text{ ft.}}$$

$$= 12 \text{ ft.}$$

Ex. 5. Find the expense of painting the walls of a room 24 ft. in length, 15 ft. in breadth and 12 ft. in height, at 4*a*. per sq. yd.

The area of 2 of the walls = 2 × length × height
= 2 × 24 × 12 sq. ft.

............the other 2 walls = 2 × breadth × height
= 2 × 15 × 12 sq. ft.

∴ the whole area to be painted
= (576 + 360) sq. ft.
= 936 sq. ft.
= 104 sq. yd.;

and ∴ reqd. expense = 104 × 4*a*.
= R26.

Ex. 6. A piece of land measures 6 bghs. 5 kths. in length, and 3 bghs. 18 kths. in breadth: find its area in the Bengali method, and its price at R80 per katha.

bghs.	kths.	
6	5	
3	18	
18	15	
5	12	10 dhools.
area = 24 bghs.	7 kths.	10 dhools.

Now, price of 24 bghs. = 24 × 20 × R80 = R38400.
.... 7 kths. = 7 × R80 = R560.
.... 10 dhools = ½ × R80 = R40.
∴ total price = R39000.

Examples XXXVIII.

1. When one number is multiplied by another, the latter or the multiplier must be an abstract number. Show that exceptions to this rule are only apparent and not real.

Find the value of 2 mds. 16 seers of sugar at 5*a.* 6*p.* a seer, and explain clearly the steps of your process.

2. What do you mean by saying that the area of a rectangle is equal to the product of the base and the altitude? Illustrate your meaning by an example.

Find the area of a field 5 bghs. 6 kths. long and 4 bghs. 10 kths. broad.

3. What will be the cost of painting the four walls of a room whose length is 18 ft., breadth 9 ft. 6 in., and height 10 ft., at 1*s.* a sq. yard?

4. A house contains 6 rooms; each room has 3 windows; each window is 7 ft. in height and 3 ft. 6 in. in breadth. Find the expense of painting the windows, at 1 anna a sq. foot.

5. A rectangular piece of land 5 bghs. 10 kths. long, contains an area of 22 sq. bighas. What is its breadth?

6. A piece of land measures 18 kths. by 16 kths., and contains a small tank 90 ft. by 48 ft. What is its value, if the area covered by the tank is worth Rs60 a katha, and the remainder, Rs125 a katha?

7. Find the expense of paving a rectangular court 24 ft. long and 18 ft. broad, at Rs1. 4*a.* per sq. cubit.

8. A gentleman promenading in a veranda observes that in the time between his leaving an end of the veranda and coming back again to it, the minute hand of a clock passes exactly from one minute mark to the next. After pacing to and fro for 20 minutes, he finds that he has walked half a mile. What is the length of the veranda?

9. On buying a rectangular piece of land, the purchaser found that if the price he paid for it, were converted into eight-anna pieces, and the amount spread on the land in rows in contact with one another, it would have just covered the land. What was the price per katha, supposing an eight-anna piece to be 1 in. in diameter?

10. The cost of carpeting a room, 15 feet broad, with carpet $\frac{3}{4}$ yd. wide, at Rs1. 4*a.* a yard, is found to be Rs90. Find the length of the room.

11. A rectangular court-yard 160 cubits long and 80 cubits broad, has a walk within it along its border, 4 cubits wide. Find

the cost of paving the walk with Chunar stone at 9*a*. 6*p*. per sq. foot.

12. What time will a railway train, 180 yards long, and moving at the rate of 10 miles an hour, take in passing over a bridge 40 yards long?

13. What is the solid content of an Imperial Gallon? How many imperial gallons does a cistern contain, which is 4 feet long, 3 feet broad, and 2 feet deep?

14. The interior of a barrack, 200 feet long and 20 feet broad, accommodates 50 soldiers, giving 1000 cub. feet of air to each. Find the height of the barrack from the floor to the ceiling.

15. A hall is 24 ft. long and 18 ft. broad. How many chairs, each 1 cubit long and 1 cubit broad, will it contain, if arranged in rows along its length, so as to leave an open space of 2 cubits in the middle, and a passage 1 cubit wide in front of each row of chairs?

16. How many bricks, each 10 inches long, 5 inches broad, and 3 inches thick, will be required for building a wall 10 feet long, 5 feet high, and 2 feet thick; and what will be the cost of the bricks at ₨9 8*a*. per 1000?

17. How many beams each 12 feet long and measuring 7 inches by 5 inches, at the end, can you get out of a piece of timber 36 feet long and measuring 2 feet 11 inches by 1 foot 9 inches at its end?

18. Find the volume of a cube whose edge is 3 feet 6 inches.

19. In a railway train, there is a certain number of passengers in the first class, twice as many in the second class, and six times as many in the third. Each passenger in the first class has to pay 1*a*. 6*p*. per mile, each in the second class, 9*p*. per mile, and each in the third, 3*p*. per mile. How many passengers are there in each class, supposing the sum total of the fares to amount to ₨9 a mile?

20. A square whose side is 240 yards, has a road 40 feet broad along its sides. What is the cost of repairing the road, at ₨3. 4*a*. per 100 sq. feet?

CHAPTER VI.

PRACTICE.

209. Def. Practice is a short method of finding by means of aliquot parts the value of things from the given price of a unit of any denomination. It is called **Simple** or **Compound** according as the quantity of the things is or is not given wholly in the denomination of the unit.

Examples of Practice are worked out in the following manner :—

I. Simple Practice.

Ex. 1. Find the value of 372 things at 12*s.* $7\frac{1}{2}$*d.* each.

Supposing each thing to be worth £1,
the value = £372.

	£	*s.*	*d.*
∴ the value at 10*s.* each = $\frac{1}{2}$ of £372	= 186 .	0	0
..................2*s.*......... = $\frac{1}{5}$ of value at 10*s.*	= 37 .	4	0
..................6*d.*......... = $\frac{1}{4}$ ....2*s.*	= 9 .	6	0
..................$1\frac{1}{2}$*d.*......... = $\frac{1}{4}$ ....6*d.*	= 2 .	6	6
∴ value at 12*s.* $7\frac{1}{2}$*d.*	= 234 .	16	6

The operation is usually written thus :—

	£.	*s.*	*d.*	
10*s.* = $\frac{1}{2}$ of £1	372		0	= value at £1 each.
2*s.* = $\frac{1}{5}$ of 10*s.*	186		0	= ....10*s.*.......
6*d.* = $\frac{1}{4}$ of 2*s.*	37		0	= ....2*s.*.......
$1\frac{1}{2}$*d.* = $\frac{1}{4}$ of 6*d.*	9		0	= ....6*d.*......
	2		6	= ....$1\frac{1}{2}$*d.*......
	£234	16	6	= ....12*s.* $7\frac{1}{2}$*d.*

Ex. 2. Find the cost of 325 things at 9*a.* 2 pice each.

	R.	*a.*	*p.*	
8*a.* = $\frac{1}{2}$ of R1.	325	0 .	0	= cost at R1 each.
1*a.* = $\frac{1}{8}$ of 8*a.*	162	8 .	0	= ....8*a.*........
2 *pice* = $\frac{1}{2}$ of 1*a.*	20	5 .	0	= ....1*a.*........
	10	2 .	6	= ....2 *pice*...
	R192	15 .	6	= ....9*a.* 2 *pice.*

II. Compound Practice.

Ex. 1. Find the value of 18 cwt. 2 qrs. 16 lbs. of sugar at £2. 8*s*. 6*d*. per cwt.

	£	*s*.	*d*.
The value of 1 cwt. being £2. 8*s*. 6*d*,			
the value of 18 cwt.	43	13	0
the value of 2 qrs. = $\frac{1}{2}$ the value of 1 cwt.	1	4	3
................14 lbs. = $\frac{1}{4}$................ 2 qrs.	0	6	$0\frac{3}{4}$
.... 2 lbs. = $\frac{1}{7}$................14 lbs.	0		$10\frac{11}{28}$
....................18 cwt. 2 qrs. 16 lbs.	= £45 .	4*s*.	$2\frac{1}{7}$*d*.

The operation is usually written thus :—

	£.	*s*.	*d*.	
	2 .	8 .	6	value of 1 cwt.
			18	
2 qrs. = $\frac{1}{2}$ cwt.	43	13	0	= ......18 cwt.
14 lbs. = $\frac{1}{4}$ of 2 qrs.	1	4	3	= ...... 2 qrs.
2 lbs. = $\frac{1}{7}$ of 14 lbs.	0	6	$0\frac{3}{4}$	......14 lbs.
	0	0	$10\frac{11}{28}$	= .... 2 lbs.
	£45 .	4 .	$2\frac{1}{7}$	= value of 18 cwt. 2 qrs. 16 lbs.

Ex. 2. Find the value of 17 mds. 28 seers 3 powas at R3. 12*a*. per maund.

	R.	*a*.	*p*.	
	3 .	12 .	0	= value of 1 md.
			17	
20 seers md.	63 .	12 .	0	= ....17 mds.
8 seers md.	1 .	14 .	0	= ....20 seers.
2 powas = $\frac{1}{16}$ of 8 seers.	0 .	12 .	0	= .... 8 seers.
1 powa = $\frac{1}{2}$ of 2 powas.	0 .	0 .	9	= .... 2 powas.
	0 .	0 .	$4\frac{1}{2}$	= .... 1 powa.
	66 .	7 .	$1\frac{1}{2}$	= ....17 mds. 28 seers 3 powas.

210. From the above examples it will be seen that the operation will be simplified by the selection of the most convenient aliquot parts.

We shall here subjoin a Table of aliquot parts of some of the ordinary concrete units, English and Indian; and the student will do well to commit it to memory.

TABLE OF ALIQUOT PARTS.

of 1 Rupee.

$\frac{1}{2}$ = 8*a.*
$\frac{1}{3}$ = 5*a.* 6 gandas 2 cowries 2 krants
 = 5*a.* 4*p.*
$\frac{1}{4}$ = 4*a.*
$\frac{1}{5}$ = 3*a.* 4 gandas.
$\frac{1}{6}$ = 2*a.* 13 gandas 1 cowry 1 krant
 = 2*a.* 8*p.*
$\frac{1}{8}$ = 2*a.*
$\frac{1}{9}$ = 1*a.* 15 gandas 2 cowries 2 dantis.
$\frac{1}{10}$ = 1*a.* 12 gandas.
$\frac{1}{12}$ = 1*a* 4*p.*
$\frac{1}{16}$ = 1*a.*

of 1 anna.

$\frac{1}{2}$ = 2 pice.
$\frac{1}{3}$ = 4 pies
 = 6 gandas 2 cowries 2 krants.
$\frac{1}{4}$ = 1 pice.
$\frac{1}{5}$ = 4 gandas.
$\frac{1}{6}$ = 2 pies.
$\frac{1}{8}$ = 2 gandas 2 cowries.
$\frac{1}{10}$ = 2 gandas.
$\frac{1}{12}$ = 1 pie.

of 1 maund.

$\frac{1}{2}$ = 20 seers.
$\frac{1}{4}$ = 10 seers.
$\frac{1}{5}$ = 8 seers.
$\frac{1}{8}$ = 5 seers.
$\frac{1}{10}$ = 4 seers.
$\frac{1}{16}$ = $2\frac{1}{2}$ seers.
$\frac{1}{20}$ = 2 seers.

of 1 seer.

$\frac{1}{2}$ = 2 powas.
$\frac{1}{4}$ = 1 powa.
$\frac{1}{8}$ = 2 chts.
$\frac{1}{16}$ = 1 cht.

of £1.

$\frac{1}{2}$ = 10*s.*
$\frac{1}{3}$ = 6*s.* 8*d.*
$\frac{1}{4}$ = 5*s.*
$\frac{1}{5}$ = 4*s.*
$\frac{1}{6}$ = 3*s.* 4*d.*
$\frac{1}{8}$ = 2*s.* 6*d.*
$\frac{1}{10}$ = 2*s.*
$\frac{1}{12}$ = 1*s.* 8*d.*

of 1*s.*

$\frac{1}{2}$ = 6*d.*
$\frac{1}{3}$ = 4*d.*
$\frac{1}{4}$ = 3*d.*
$\frac{1}{6}$ = 2*d.*
$\frac{1}{8}$ = $1\frac{1}{2}$*d.*
$\frac{1}{12}$ = 1*d.*

of 1 cwt.

$\frac{1}{2}$ = 2 qrs.
$\frac{1}{4}$ = 1 qr.
$\frac{1}{7}$ = 16 lbs.
$\frac{1}{8}$ = 14 lbs.
$\frac{1}{14}$ = 8 lbs.

of 1 qr.

$\frac{1}{2}$ = 14 lbs.
$\frac{1}{4}$ = 7 lbs.
$\frac{1}{7}$ = 4 lbs.
$\frac{1}{8}$ = $3\frac{1}{2}$ lbs.

Examples like the following may be worked out in a way similar to the method of Practice.

Ex. 1. Find the cost of 6 seers at R2. 12*a.* per maund.

The cost of 5 seers = $\frac{1}{8}$ of R2. 12*a.* = 5*a.* 2 pice ;

∴ ....1 seer = $\frac{1}{5}$ of 5*a.* 2 pice = 1*a.* $\frac{2}{5}$ pice.

and ∴ ....6 seers = 6*a.* $2\frac{2}{5}$ pice.

Ex. 2. A man gets R7 a month; what does he get per day, supposing a month to contain 30 days?

The amount reqd. $= \frac{1}{30}$ of R7 $= \frac{1}{15}$ of $\frac{1}{2}$ of R7;

Now $\frac{1}{2}$ of R7 $= \frac{1}{2}$ of R6 + $\frac{1}{2}$ of R1

$=$ R2 + 5*a*. 4 pies;

$\therefore$ $\frac{1}{30}$ of R7 $= \frac{1}{15}$ of R2. 5*a*. 4 pies

$=$ 3*a*. $8\frac{1}{5}$ pies.

Ex. 3. A man supplies milk for a month of 31 days at 3 seers a day. What will be his charge, if milk sells at 8 seers a rupee?

The quantity of milk supplied

$= 3 \times 31 = 93$ seers.

	R.	*a.*	*p.*
The value of 88 seers =	11	0	0
..................5.........=	0	10	0
..................93.........=	11	10	0

Examples XXXIX.

Find the value of

1. 50 things at R2. 2*a*. each, and 64 things at R3. 5*a*. each.

2. 72 things at £1. 10*s*. each and 55 things at 15*s*. 6*d*. each.

3. 126 things at 13*s*. 4*d*. each, and 100 things at £2. 5*s*. each.

4. 30 things at R2. 6*a*. each and 31 things at 3*a*. 6*p*. each.

5. 40 things at R2. 5*a*. each, and 120 things at 5*a*. 6*p*. each.

6. 80 things at R3. 7*a*. each, and 90 things at R2. 9*a*. each.

7. 720 things at £3. 6*s*. each, and 885 things at 9*s*. 6*d*. each.

8. 144 things at £19. 17*s*. each, and 288 things at £7 6*s*. 5*d*. each.

9. 1000 things at £15. 16*s*. 11*d*. each, and 925 things at £5. 10*s*. each.

10. 1000 things at R2. 5*a*. each, and 1500 things at R3. 7*a*. each.

11. 1285 things at R5. 4*a*. each, and 725 things at R7. 6*a*. 4*p*. each.

12. 361 things at R16. 15*a*. each, and 2500 things at R9. 6*a*. each.

13. 15 cwt. 2qrs. 10 lbs. at 1*s*. 6*d*. per lb.

14. 17 cwt. 3qrs. 14 lbs. at £2. 9*s.* per cwt.
15. 110 cwt. 2qrs. 20 lbs. at £1. 13*s.* 4*d.* per cwt.
16. 7 mds. 35 seers of sugar at ₨13. 2*a.* per maund.
17. 225 mds. 33 seers of rice at ₨4. 14*a.* 6*p.* per maund.
18. 72 mds. 25 seers of rice at ₨5. 10*a.* per maund.
19. 16 yds. of silk at £1. 3*s.* per yard.
20. 170 yds. of linen at 2*s.* 6*d.* per yard.
21. 8 bghs. 11 kths. of land at ₨65. 8*a.* per katha.
22. 13 bghs. 7 kths. of land at ₨49. 8*a.* per katha.
23. 16 bghs. 16 kths. of land at ₨125 per katha.
24. 140 ft. 6 in. at ₨1. 4*a.* 6*p.* per foot.

CHAPTER VII.

PROPORTION AND VARIATION. RULE OF THREE.

SECTION I. PROPORTION AND VARIATION.

211. Def. Ratio is the relation which one number bears to another in respect of magnitude, the comparison being made by considering how many times or parts of a time the latter is contained in the former.

The two numbers are called the **Terms** of the ratio, the former being called the **Antecedent**, and the latter the **Consequent**.

A ratio is written by writing its terms one after the other and placing a colon (:) between them. Thus the ratio of 3 to 4 is written,

$$3 : 4.$$

It is evident from the definition, that the same ratio may also be represented by the fraction $\frac{3}{4}$; for by the definition, the ratio of $3 : 4$ denotes the number of times, or rather parts of a time, that 4 is contained in 3, and the fraction $\frac{3}{4}$ also denotes the same thing (Art. 84).

212. The terms of a ratio must either be both abstract numbers or both concrete numbers of the *same* kind; for otherwise there can be no comparison between them; thus we cannot compare 3 feet with 4 hours in respect of magnitude : and the ratio itself, as it indicates the *number* of *times* or *parts* of a *time* that the antecedent contains the consequent, must always be an abstract number.

Again the ratio of one concrete number to another, when both are expressed in the *same denomination*, will be the same as the ratio of the former number to the latter, both being regarded as abstract numbers. Thus, the ratio of 3 feet to 4 feet is the same as the ratio of the abstract number 3 to the abstract number 4, and is expressed by the fraction $\frac{3}{4}$. But the ratio of 3 feet to 4 inches, that is, of 36 inches to 4 inches, will not be the same as that of 3 to 4, but will be the same as that of 36 to 4, and will be expressed not as $\frac{3}{4}$ but as $\frac{36}{4}$.

213. Def. Four numbers are said to be **Proportionals**, or to constitute a **Proportion**, when the ratio of the first to the second is equal to that of the third to the fourth, or, in other words when the first contains the second as often as the third contains the fourth.*

A proportion is written by writing its ratios one after the other with a double colon (: :) between them. Thus, taking any four

numbers that are proportionals, for example, 3, 4, 12, 16, the proportion is written thus :—

$$3 : 4 :: 12 : 16;$$

and it is read thus :—

3 is to 4 as 12 to 16.

If any four numbers, 3, 4, 12, 16, constitute a proportion, then $\frac{3}{4} = \frac{12}{16}$.

For $\frac{3}{4}$ denotes the number of parts of a time that 3 contains 4, and $\frac{12}{16}$ denotes the number of parts of a time that 12 contains 16 ; and these are equal by the definition of proportion.

214. In the preceding Article we have seen four numbers constituting a proportion ; but we may also have three numbers constituting a proportion, and then they must be such that the first is to the second as the second is to the third.

Thus the three numbers 3, 6 and 12 constitute the proportion $3 : 6 :: 6 : 12$,

and $\frac{3}{6} = \frac{6}{12}$.

215. The terms of each of the two ratios constituting a proportion must satisfy the conditions mentioned in Art. 212. But it is not necessary that all the four terms should be simultaneously abstract numbers, or simultaneously concrete numbers. The terms of one ratio may be abstract numbers, and those of the other, concrete numbers. The ratio of two concrete numbers, which is always an abstract number, may be equal to that of two abstract numbers, or of two concrete numbers of another kind.

Thus R2 : R3 :: 12 ft. : 18 ft.

For $\frac{\text{R}2}{\text{R}3}$ = the abstract fraction $\frac{2}{3}$.

Similarly $\frac{12 \text{ ft.}}{18 \text{ ft.}} = \ldots\ldots\ldots\ldots\ldots\ldots \frac{12}{18} = \frac{2}{3}$.

216. Prop. I. If four numbers are proportionals when taken in a certain order, they are also proportionals when taken in the reverse order.

Thus, taking for example the numbers 3, 5, 9, 15, which constitute the proportion $3 : 5 :: 9 : 15$, so that $\frac{3}{5} = \frac{9}{15}$,

we have $1 \div \frac{3}{5} = 1 \div \frac{9}{15}$,

or $\frac{5}{3} = \frac{15}{9}$,

or $5 : 3 :: 15 : 9$, or $15 : 9 :: 5 : 3$.

Prop. II. When four numbers are proportionals, the product of the extremes = the product of the means.

For taking the same proportion

$$3 : 5 :: 9 : 15,$$

we have $\frac{3}{5} = \frac{9}{15}$,

or multiplying both sides by 5×15 we have

$$\frac{3}{5} \times 5 \times 15 = \frac{9}{15} \times 5 \times 15, \text{ or } 3 \times 15 = 5 \times 9.$$

217. Def. When four numbers are proportionals, the fourth is said to be a **Fourth Proportional** to the other three.

When three numbers are proportionals, the third is said to be a **Third Proportional** to the other two, and the second, a **Mean Proportional** between the other two.

The process for finding a fourth proportional to three numbers, or a third proportional to two numbers, consists merely in the application of Prop. II of the preceding Article; and is usually stated in the manner below.

Ex. 1. Find a fourth proportional to 7, 9 and 21.

Let x be the fourth proportional required.

Then $7 : 9 :: 21 : x$,

$\therefore 7 \times x = 9 \times 21$, or dividing by 7,

$$x = \frac{9 \times 21}{7} = 27.$$

Ex. 2. Find a third proportional to 7 and 9.

Let x be the third proportional required.

Then $7 : 9 :: 9 : x$,

$\therefore 7 \times x = 9 \times 9$, or dividing by 7,

$$x = \frac{9 \times 9}{7} = 11\tfrac{4}{7}.$$

218. Def. One quantity is said to **Vary Directly** as another, when either of them being *increased* or *decreased*, the other is *increased* or *decreased* in the same proportion.

Thus, at a given rate of walking, say, 2 miles an hour, the distance walked may be said to vary directly as the time; since in any time, say, 3 hours, the distance walked is 6 miles, and increasing the time to 4 hours, the distance is increased to 8 miles, so that

the former time 3 hours : the increased time 4 hours
:: the former distance 6 miles : the increased distance 8 miles.

Hence, it is evident that, when one quantity varies directly

as another, any two numerical values of the former expressed in the same denomination, and the corresponding numerical values of the latter expressed in the same denomination, and *taken in the same order*, will constitute a proportion.

One quantity is said to **Vary Inversely** as another when either of them being *increased* or *decreased*, the other is *decreased* or *increased* in the same proportion.

Thus, in walking a given distance, say, 24 miles, the time of walking may be said to vary inversely as the rate, since for any rate, say, 2 miles an hour, the time will be 12 hours, and *increasing* the rate to 3 miles an hour, the time is *decreased* to 8 hours, so that the former rate 2 miles : the increased rate 3 miles :: the latter or decreased time 8 hours : the former time 12 hours.

Hence it is evident that when one quantity varies inversely as another, any two numerical values of the former expressed in the same denomination, and the two corresponding numerical values of the latter expressed in the same denomination, and *taken in the reverse order*, will constitute a proportion.

219. When one quantity varies directly or inversely as another, and two values of the former and only one corresponding value of the latter are given, the other corresponding value of the latter can be determined as in the following Examples.

Ex. 1. If 6 yds. of cloth cost R13. 8*a*., find the price of 9 yds.

Let $x=$ the price of 9 yds. in rupees.

Then $\because$ R13. 8*a*. $=$ R$13\frac{1}{2}=$ R$\frac{27}{2}$,

and $\because$ the price of cloth varies directly as the quantity of cloth, we have by Art. 218,

$$6 : 9 :: \tfrac{27}{2} : x,$$

$$\therefore\ 6\times x=9\times\tfrac{27}{2},$$

$$\text{and } \therefore\ x=\tfrac{9\times 27}{2\times 6}=\tfrac{81}{4}=20\tfrac{1}{4}.$$

$\therefore$ price reqd. $=$ R20. 4*a*.

Ex. 2. If 6 men can do a piece of work in 9 days, in how many days will 15 men do the same work?

Here $\because$ the larger the no. of men employed, the shorter becomes the time for finishing the work, $\therefore$ the number of men varies inversely as the time.

Now let $x=$ the no. of days reqd.

Then $6 : 15 :: x : 9$;

$\therefore\ 15\times x=6\times 9$,

or $x=\frac{6\times 9}{15}=\frac{18}{5}=3\frac{3}{5}$, the no. of days required.

Ex. 3. If 9 men working 6 hours a day, can do a piece of work in 12 days, in how many days will 8 men do the same work, working 9 hours a day?

Let x = the no. of days reqd.

Then the time taken in doing the work in the one case $=6\times 12$ hours,

and in the other case $=9\times x$ hours.

Now, as in Ex. 2, the no. of men varies inversely as the time;

$\therefore$ the proportion will stand thus:—

$$9 : 8 :: 9\times x : 6\times 12,$$

and $\therefore$ $8\times 9\times x=9\times 6\times 12$,

and $\therefore$ $x=\frac{9\times 6\times 12}{9\times 8}=9$, the no. of days required.

Ex. 4. If 7 men can reap 18 bghs. in 12 hrs., how many men will be able to reap 45 bghs. in 14 hrs.?

Let x = the no. of men reqd.

Then in the former case we have

7 men working 12 hours,

which is the same as

7×12 men working 1 hour,

and the work done is the reaping of 18 bghs.

In the latter case, we have

x men working 14 hours,

which is the same as

$x\times 14$ men working 1 hour,

and the work done is the reaping of 45 bghs.

Now in the given time, 1 hour, the no. of bighas reaped will vary directly as the no. of men;

$$\therefore\ 18 : 45 :: 7\times 12 : x\times 14;$$

$$\therefore\ 18\times x\times 14=45\times 7\times 12,$$

and $\therefore$ $x=\frac{45\times 7\times 12}{18\times 14}=15$.

220. We may notice some of the ordinary instances in which one quantity varies as another.

I. With a given **area**, the **length** varies *inversely* as the **breadth.**

II. With a given **length**, the **breadth** varies *directly* as the **area.**

III. With a given **rate** of motion, the **distance** passed varies *directly* as the **time**, supposing the rate to be uniform.

IV. With a given **time**, the **distance** passed varies *directly* as the **rate** of motion, on the same supposition.

V. With a given **distance**, the **rate** of motion varies *inversely* as the **time**, on the same supposition.

VI. With a given **time**, the **work** done varies *directly* as the **agency**.

VII. With a given **agency**, the **work** done varies *directly* as the **time**.

VIII. With a given **work**, the **agency** varies *inversely* as the **time**.

IX. With a given **rate** of price, the **price** paid varies *directly* as the **quantity** of article purchased.

X. With a given **price**, the **quantity** of article purchased varies *inversely* as the **rate** of price.

The truth of the above statements will be made evident by one or two examples.

SECTION II. RULE OF THREE.

221. In Art. 219, we have already seen that when three quantities are given we can sometimes find out a fourth. The method of finding out the fourth is called the Rule of Three. It may be thus defined :—

Def. The Rule of Three is a method by which of four quantities which are proportionals, any three being given, the fourth can be determined.

It is called **Direct** or **Inverse** according as the variation upon which it depends is direct or inverse. Thus Ex. 1 of Art. 219 is an example of Rule of Three Direct; Ex. 2, one of Rule of Three Inverse. It is called **Single** or **Double** according as only three or more than three quantities are given from which the required one is to be ascertained. Examples 3 and 4 of Art. 219 are examples of Double Rule of Three.

It is called the Rule of Three because there are *three* quantities given from which we find the fourth or required quantity; and from its application to the solution of a large and important class of problems in the common affairs of life, it has sometimes been called the Golden Rule.

222. The Rule for working out Examples of Rule of Three may be gathered from Art. 219. We may state it thus :—

Rule. Put x for the required number or the required quantity expressed numerically. Make the necessary reductions to the same denomination. Then from the nature of the question ascertain what the numbers are that are proportionals, and state the proportion. Put the product of the extremes equal to the product of the means, and then divide the product which does not contain x by the product of all the factors of the other product except x. The quotient will be the required number or the required quantity expressed numerically.

The above Rule is quite general, and will apply to all cases of Rule of Three, whether Direct or Inverse, Single or Double, as will be seen from the Examples given below.

I. Single Rule of Three.

Ex. 1. Find the price of 5 yds. 9 in. of silk when 3 yds. cost R6. 12*a.*

Let x = the price reqd. in rupees

Then ∵ the quantity of silk in one case = 3 yds.,

and the quantity of silk in the other case = 5 yds. 9 in.
= $5\frac{1}{4}$ yds. ;

and the price in the former case = R6. 12*a.* = $6\frac{3}{4}$ R.,

and the price in the latter case = R x ;

and ∵ the price varies directly as the quantity of the article,

we have $3 : 5\frac{1}{4} :: 6\frac{3}{4} : x$,

$\therefore 3 \times x = 5\frac{1}{4} \times 6\frac{3}{4} = \frac{21}{4} \times \frac{27}{4}$,

and $\therefore x = \frac{21 \times 27}{3 \times 4 \times 4} = \frac{189}{16} = 11\frac{13}{16}$;

$\therefore$ the price reqd. = R$11\frac{13}{16}$ = R11. 13*a.*

Ex. 2. If 7 men reap a field 6 bghs. in length and 3 bghs. in breadth in 14 hrs., in how many hours will 8 men be able to reap the same field ?

Let x = the no. of hrs. reqd.

Then because the work, *viz.*, the reaping of the field, is the same in both cases, the no. of men will vary inversely as the time ; and

$\therefore 7 : 8 :: x : 14$;

$\therefore 8 \times x = 7 \times 14$, or $x = \frac{7 \times 14}{8} = \frac{49}{4} = 12\frac{1}{4}$.

Here it will be seen that the quantities 6 bghs. and 3 bghs.

are superfluous, as, being the same in both cases, they do not affect the question.

Ex. 3. If 7 men can reap a field 6 bghs. in length and 3 bghs. in breadth in 14 hrs., what is the area of the field that they can reap in 18 hrs.?

Let x = the area reqd. in sq. bighas.

The area in the former case $= 3 \times 6$ or 18 sq. bighas.

Now $\because$ the work done is measured by the no. of bighas reaped,

and $\because$ the no. of men working is the same in both cases,

$\therefore$ the time will vary directly as the no. of bighas;

and $\therefore$ $14 : 18 :: 18 : x$;

$\therefore x = \frac{18 \times 18}{14} = \frac{162}{7} = 23\frac{1}{7}$;

and $\therefore$ the area reqd. $= 23\frac{1}{7}$ bghs. $= 23$ bghs. $2\frac{6}{7}$ kths.

Here it will be seen that the number of men being the same in both cases does not affect the question.

Ex. 4. In the preceding Example, what will be the length of the field in the second case if its breadth is 9 bighas?

Let x = the reqd. length in bighas.

Then the area in the second case

$= 9 \times x$ bighas;

and $\therefore$ the proportion will stand thus :—

$14 : 18 :: 18 : 9 \times x$;

$\therefore 9 \times x \times 14 = 18 \times 18$,

$\therefore \quad x \quad = \frac{18 \times 18}{9 \times 14} = \frac{18}{7} = 2\frac{4}{7}$,

and $\therefore$ the length reqd. $= 2\frac{4}{7}$ bghs. $= 2$ bghs. $11\frac{3}{7}$ kths.

Ex. 5. If 3 yds. of silk cost as much as 7 yds. of linen, how many yards of silk can be given in exchange for 42 yds. of linen?

Let x = the no. of yds. reqd.

Then x must bear the same relation to 42 in respect of magnitude that the number 3 does to the number 7; in other words, the ratio of x to 42 = that of 3 to 7;

$\therefore x : 42 :: 3 : 7$,

and $\therefore x \times 7 = 42 \times 3$ or $x = \frac{42 \times 3}{7} = 18$.

$\therefore$ 18 is the no. of yds. of silk reqd.

Ex. 6. A watch is set right at 1 o'clock P. M. on Monday, and at 9 o'clock P. M. on Tuesday it is found to be 3′ too fast.

Supposing its rate regular, what will be the time by the watch at 6 o'clock A.M. on Saturday?

From 1 P. M. Monday to 9 P.M. Tuesday there are $24+8$ or 32 hrs., and in that time the watch gains $3'$.

From 1 P. M. Monday to 6 A. M. Saturday there are $4\times 24+17$ or 113 hrs., and in that time let the watch gain x'.

Then $\because$ the rate is regular, the gain by the watch will vary directly as the time;

$\therefore 32 : 113 :: 3 : x$,

and $\therefore\ 32\times x=3\times 113$ or $x=\frac{3\times 113}{32}=\frac{339}{32}=10\frac{19}{32}$.

$\therefore$ the watch has gained $10\frac{19}{32}'$ and the time by the watch is $10\frac{19}{32}'$ past 6 o'clock A. M.

II. Double Rule of Three.

Ex. 7. If 8 men can do a piece of work in 18 days, working 7 hours a day, how many hours a day must 12 men work to finish in 21 days a piece of work twice as great?

To finish the second piece of work, 8 men must work for 2×18 or 36 days of 7 working hours each.

Now let x = the no. of hours reqd.

Then the second piece of work is done by 8 men in 36×7 hrs.,

and by 12 men in $21\times x$ hrs.;

and as the no. of men must vary inversely as the time,

$\therefore 8 : 12 :: 21\times x : 36\times 7$;

$\therefore\ 12\times 21\times x=8\times 36\times 7$,

and $\therefore\ x=\frac{8\times 36\times 7}{12\times 21}=8$, the no. of hrs. reqd.

In this Example, we have three things involved, workmen, work done, and time, and all three are different in the two cases.

In the process given above, we have made the work done the same in both cases, by multiplying the time in the former case by 2; and thus the question is reduced to one in which the work done remains the same, and the time and the number of workmen are the only varying elements; and then as we know that the former varies inversely as the latter, we state the proportion accordingly. We might have worked out the Example by reducing the time to the same duration in the two cases by altering the number of workmen, and then having the number of workmen and work done for our varying elements. Thus,

8 men working 18×7 hours, is the same as
$8\times 18\times 7$ men working 1 hour;

Similarly, 12 men working $21 \times x$ hours, is the same as
$12 \times 21 \times x$ men working 1 hour ;

and in the former case the work done being 1, in the latter it is 2 ; and as in the given time 1 hour, the work done must vary directly as the number of men,

$$\therefore 8 \times 18 \times 7 : 12 \times 21 \times x :: 1 : 2 ;$$

and $\therefore$ $12 \times 21 \times x \times 1 = 2 \times 8 \times 18 \times 7$,

whence . $x = \frac{2 \times 8 \times 18 \times 7}{12 \times 21} = 8.$

In working out Examples of Double Rule of Three in which three varying elements occur, the chief art consists in reducing one of the three elements to a constant quantity ; and although the reduction of any one of the elements will give us a proportion from which the required quantity may be found, we must try in every case to make use of the most convenient reduction.

Ex. 8. If 9 masons, in 10 days of 8 working hours each, can build a wall 48 ft. long, 10 ft. high, and 2 ft. thick, how many masons will be required to build a wall 60 ft. long, 12 ft. high, and 3 ft. thick, in the same number of days, but working only 6 hours daily?

Let x = the number of masons reqd.

Then in the former case we have

9 masons working for 10×8 hours,

which is the same as

$9 \times 10 \times 8$ masons working for 1 hour ;

and in the latter case we have

x masons working for 10×6 hours,

which is the same as

$x \times 10 \times 6$ masons working for 1 hour.

Now in the former case,

the work done is measured by $2 \times 10 \times 48$ cub. ft. ;

and in the latter case,

it is measured by $3 \times 12 \times 60$ cub. ft. ;

and $\because$ in the given time 1 hour,

the work done will vary directly as the number of masons,

$$2 \times 10 \times 48 : 3 \times 12 \times 60 :: 9 \times 10 \times 8 : x \times 10 \times 6 ;$$

and $\therefore$ $2 \times 10 \times 48 \times x \times 10 \times 6 = 3 \times 12 \times 60 \times 9 \times 10 \times 8$,

whence $x = \frac{3 \times 12 \times 60 \times 9 \times 10 \times 8}{2 \times 10 \times 48 \times 10 \times 6} = 3 \times 9$

$= 27$, the no. of masons reqd.

In the foregoing process we have reduced the time to the same duration, 1 hour, in both cases. We can work out the Example by reducing the number of men to a constant thus :—

9 men working 10×8 hrs. is the same as

1 man working $9 \times 10 \times 8$ hrs. ;

so, x men working 10×6 hrs. is the same as

1 man working $x \times 10 \times 6$ hrs.

Now with the same 1 workman

the work done will vary directly as the time ;

$\therefore 2 \times 10 \times 48 : 3 \times 12 \times 60 :: 9 \times 10 \times 8 : x \times 10 \times 6$;

and $\therefore 2 \times 10 \times 48 \times x \times 10 \times 6 = 3 \times 12 \times 60 \times 9 \times 10 \times 8$,

whence $x = 27$.

Ex. 9. If the carriage of 25 mds. over 50 miles cost R18. 12*a.*, over how many miles can 30 mds. be carried for R20, supposing there to be a uniform rate for every 1 maund carried over one mile ?

Let x = the no. of miles reqd.

Then, the unit being 1 md. carried 1 mile,

the work in the former case

$= 25 \times 50$ units,

and the work in the latter case

$= 30 \times x$;

and the work will vary directly as its cost ;

$\therefore 25 \times 50 : 30 \times x :: 18\frac{3}{4} : 20$;

$\therefore 30 \times x \times \frac{75}{4} = 25 \times 50 \times 20$,

or $x = \frac{25 \times 50 \times 20 \times 4}{30 \times 75} = \frac{400}{9}$

$= 44\frac{4}{9}$, the no. of miles reqd.

Ex. 10. There is just sufficient rice in store to feed 50 men for 30 days, giving 15 chataks to each man daily. To how many chataks must the daily allowance be reduced to feed for 40 days these and 10 men more without any addition to the store ?

Let x = the no. of chataks reqd. Then to feed 50 men for 30 days will require as much food as to feed 30×50 men for 1 day ; and so to feed $(50 + 10)$ or 60 men for 40 days is the same as to feed 40×60 men for 1 day.

Now in 1 day, with a given store, the allowance to each man must vary inversely as the no. of men ;

$\therefore 30 \times 50 : 40 \times 60 :: x : 15,$

and $\therefore x \times 40 \times 60 = 30 \times 50 \times 15,$

or $x = \frac{30 \times 50 \times 15}{40 \times 60} = \frac{75}{8}$

$= 9\frac{3}{8}$, the no. of chataks reqd.

This Example may also be worked thus :—

Quantity consumed in the former case

$= 50 \times 30 \times 15$ chts.,

and quantity consumed in the latter case

$= 60 \times 40 \times x$ chts. ;

and these two quantities must evidently be equal ;

$\therefore 50 \times 30 \times 15 = 60 \times 40 \times x,$

whence $x = 9\frac{3}{8}$.

Ex. 11. If 7 seers of bread cost R2 when flour is R5 a maund, what is the price of flour per maund when 8 seers of bread cost R3, supposing the price of bread to vary directly as the price of flour ?

Let x = the price reqd. in rupees.

Now in the former case, we get 7 seers of bread for R2, *i. e.*, the price of 1 seer is R$\frac{2}{7}$.

Similarly, in the latter case,

the price of 1 seer is R$\frac{3}{8}$.

And as the price of bread varies directly as the price of flour,

$\therefore \frac{2}{7} : \frac{3}{8} :: 5 : x$, and $\therefore x \times \frac{2}{7} = 5 \times \frac{3}{8}$;

whence $x = \frac{5 \times 3 \times 7}{8 \times 2} = \frac{105}{16} = 6\frac{9}{16}$;

and $\therefore$ the reqd. price of flour per md. = R6. 9*a*.

Ex. 12. If 5 men in 12 days earn £2, in how many days will 6 men earn £5 ?

Let x = the number of days reqd.

Now the earnings of 5 men in 12 days will be the same as the earnings of 12×5 men in 1 day ;

and so the earnings of 6 men in x days will be the same as the earnings of $x \times 6$ men in 1 day.

And as in the given time 1 day, the earnings will vary directly as the number of men,

$\therefore 2 : 5 :: 12 \times 5 : x \times 6$;

$\therefore 2 \times x \times 6 = 5 \times 12 \times 5,$

and $\therefore x = \frac{5 \times 12 \times 5}{2 \times 6} = 25$, the no. of days reqd.

223. The applicability of the Rule of Three to the solution of any question depends upon the subsistence of the relation of proportion among the quantities involved; and when this relation does not exist, the Rule of Three will not apply. Thus in the question,

"If 8 mangoes are worth R1, what is the price of 15 pineapples?"
as evidently, the relation of proportion does not subsist among the quantities involved, *viz.*, the number of mangoes, the number of pineapples, the price given, and the price required, the Rule of Three will not apply.

But if the question be this,

"If 8 mangoes are worth R1, what is the price of 15 pineapples, supposing 2 mangoes to be worth as much as 3 pineapples?"

then as we can reduce the 8 mangoes to their equivalent number of pineapples in value by the proportion,

$2 : 8 :: 3 : x$, whence $x = \frac{8 \times 3}{2} = 12$,

we can state the question in other words thus :—

"If 12 pineapples are worth R1, what is the price of 15 pineapples?"

And to this clearly the Rule of Three applies. And since the price varies directly as the number of pineapples, the process as usual will stand thus :—

Let $x =$ the price reqd. in rupees.

Then $12 : 15 :: 1 : x$;

$\therefore x \times 12 = 15 \times 1$, and $\therefore x = \frac{15}{12} = \frac{5}{4} = 1\frac{1}{4}$;

and $\therefore$ the price reqd. $=$ R1. 4*a*.

Again, if the question be,

"What will the East Indian Railway Company charge for carrying 1 maund over 25 miles, when they charge 12 annas for carrying the same weight over 50 miles?"
although it may appear at first sight to be an ordinary question of Rule of Three, yet it may not always be really so. For the Railway Company may charge the same amount of 12 annas for every distance below 50 miles, or it may adjust its table of fares in any other way than in proportion to the distance, and in that case the Rule of Three will not apply. It will be a question of Rule of Three on the supposition that the Railway Company charges at a uniform rate per mile; and it is only in that case that the solution will be given in the usual way, by putting $x =$ the cost of carriage reqd. in annas, and stating the proportion $50 : 25 :: 12 : x$,

whence $x = 6$, the no. of annas reqd.

Before applying, therefore, the Rule of Three to the solution of any practical question, the student should enquire whether the relation of proportion subsists among the quantities involved.

In the theoretical questions set to be solved by the Rule of Three, it is always assumed either expressly or by implication that the rate involved in the question is uniform.

Thus in Example 1 of Art. 222, the rate or price per yard is assumed to be uniform. So in Examples 7 and 8 of Art. 222, each man is supposed to work at the same uniform rate per hour.

224. Sometimes again there may be the relation of proportion existing, but in a peculiar way. Thus in the question,

"What is the price of a diamond weighing 9 rattis, when another diamond weighing 3 rattis costs R90, supposing the value of diamonds to vary as the square of their weight?"

assuming x = the price required in rupees, the proportion will not be

$$3 : 9 :: 90 : x,$$

but will be

$$3^2 : 9^2 :: 90 : x,$$

i. e., $9 : 81 :: 90 : x$,

whence $x = \frac{81 \times 90}{9} = 810$;

and the price required is R810.

Examples XL.

1. Find a fourth proportional to

(1) 1, 2, and 3.

(2) 12, 14, and 16.

(3) £6, £9, and £12.

(4) £14, £12, and 7*s*.

(5) £10, 10*s*. and R25.

(6) 2 mds. 20 srs., 1 sr., and 100.

(7) 4, 5, and 5 bghs.

(8) R6. 4*a*., R1. 9*a*., and 16 yds. 2 ft.

2. Find a third proportional to

(1) 5 and 15. (2) 10 and 12.

(3) 192 and 24. (4) 1728 and 144.

(5) £1 and 5*s*. (6) R1 and 1*a*.

3. The lengths of three poles are such that the first contains the second as often as the second contains the third; and there are as many cubits in the second as there are yards in the first. Given that the length of the first is 60 ft., find the lengths of the other two.

4. Two rectangular fields, whose areas are equal, are such that the length of the one contains as many bighas as there are kathas in the breadth of the other. How often is the breadth of the former contained in the length of the latter?

5. If 16 yds. of cloth cost R15, how much will 20 yds. cost?

6. If 28 mds. of rice can be had for R91. 7*a.*, how much rice can be had for R52. 4*a.*?

7. If 18 mds. of sugar cost R225, find the value of 22 mds. 10 seers.

8. If 16 cwt. of sugar can be had for £20. 16*s.*, how much sugar can be had for £26?

9. For a certain sum, 15 yds. of silk can be had of a certain quality, or 25 yds. of silk of an inferior quality. Compare the prices of the two kinds of silk per yard, and find the price of the latter, supposing that of the former to be 10*s.* per yard.

10. If an ounce of quinine can be had for R5, what is the price of 3 drams?

11. If a tola of pure silver is worth R1. 1*a.* $5\frac{6}{11}$*p.*, and if pure gold is worth 16 times as much as pure silver, what is the value of a seer of pure gold?

12. A gentleman pays an income-tax of £17. 10*s.* a year when the tax is 7*d.* in the £. What is his annual income?

13. The assets of an insolvent debtor amount to R24000, and his creditors can get only 10*a.* in the rupee. Find the amount of his debts.

14. The assets of an insolvent amount to R15312. 8*a.*, and his debts amount to R35000. How much can his creditors get in the rupee?

15. Find the annual value of a revenue-free estate which pays R171. 14*a.* annually as road cess, such cess being levied at the rate of one-half of an anna in the rupee of the annual value.

16. The annual value of a certain revenue-paying estate is twice as much as the annual revenue payable for it, and the estate pays every year as road cess the sum of R2578. 2*a.* Find the annual value of the estate, supposing the road cess to be payable at the rate of one-half of an anna in the rupee of the annual value, less a deduction at one-half of the said rate for every rupee of the revenue.

17. If 5 boys earn as much as 3 men, what is the monthly earning of a boy, supposing a man to earn ₨10 a month?

18. If the wages of 5 carpenters amount to as much as the wages of 6 masons, what will 16 carpenters earn in 1 day, supposing the weekly earnings of 10 masons to be ₨21. 14*a*.?

19. If 6 men earn as much as 10 boys, how much will 15 boys earn per week, supposing a man's daily earning to be 5*a*.?

20. Supposing the value of diamonds to vary as the square of their weight, find the relation between the values of two diamonds weighing 3 and 5 rattis respectively.

21. The area of a circle varies as the square of its diameter. Find the area of a circle 8 ft. in diameter, supposing a circle 6 feet in diameter to contain 28·27 sq. ft.

22. The circumference of a circle varies as its radius. Find the length of the circumference when the radius is 3 ft., supposing the circumference of a circle whose radius is 2 ft. 6 in., to be 15·7080 feet.

23. The circumference of a circle is to its radius as 3·1416 : $\frac{1}{2}$. Find the radius of a circle whose circumference is 3 miles

24. The area of a square varies as the square of its diagonal. Find the area of a square field which measures 7 bighas along its diagonal, supposing the area of a square, whose diagonal is 5 ft., to be 12·5 square feet.

25. A rectangular plot of land measuring 4 bghs. 5 kths. by 3 bghs. 10 kths., is worth ₨15000. What is the value of another plot which is 5 times as long and 3 times as broad?

26. If an estate which contains 10000 acres yields an annual income of £25000, what would be the income of another estate which contains an area of 8 square miles at the same rate?

27. A zemindari containing 30000 bghs. of land yields an annual income of ₨28000. One-third of the whole area is waste-land, which yields nothing; and of the remainder, one-tenth consists of mulberry land, and the rest is paddy land. Supposing the former description of land to yield per bigha five times as much rent as the latter, find the rent per bigha of each.

28. One-tenth part of the land comprised in an estate which contains 15000 bghs., is homestead land, and the rest is arable land; and the rate of rent for the former is to that for the latter as 2·5 : 1. What is the rent per bigha for each description of land, supposing the whole rent realizable from the estate to be ₨17250?

29. If 3 men or 5 boys earn ₨6. 9*a*. per week, how much will 5 men and 3 boys earn in one year? (1 year = 52 weeks).

30. If 5 horses eat as much as 7 ponies, and if the feeding of 1 horse cost 18 rupees a month, what will be the monthly cost of feeding 3 horses and 2 ponies?

31. Divide ₨1600 between A and B so that their shares may be as 3 : 5.

32. Divide ₨2400 among A, B, and C, so that the shares of A and B may be as 1 : 2, and those of B and C as 2 : 5.

33. The Thakbust scale being 16 inches to the mile, how much is it to the bigha, and what is the length represented by an inch on the Thak map?

34. If the greatest length of India, which is 1880 miles, appears on a map to be 1 ft. 10·5 in., what would be the length on the map of the river Ganges, which is 1500 miles long?

35. Supposing the diameter of the Earth to be 8000 miles, and the height of the highest mountain upon it to be 28000 ft., what decimal of an inch would represent this height on a globe 2 ft. in diameter?

36. Two rectangular fields having the same length contain 140 bghs. and 240 bghs. respectively. Given that the breadth of the former is 3 bghs. 10 kths., find the breadth of the latter.

37. Two rectangular fields having the same area, measure along their lengths 300 poles and 1 mile respectively. Given that the breadth of the former is half a mile, find the breadth of the latter.

38. What must be the length of a plot of land that is 3 bghs. 15 kths. broad, in order that it may be given in exchange for a square plot measuring 4 bghs. 5 kths. along its side?

39. Two plots of land of the same length are worth respectively ₨5400 and ₨7200. What is the breadth of the former, if that of the latter be 4 bghs. 10 kths.?

40. A tank is to be given in exchange for a plot of land of equal length. The value of a katha of the latter is twice as much as that of a katha of the former. Find the breadth of the land supposing that of the tank to be 120ft.

41. A special train moving at a uniform rate, leaves Howrah for Allahabad at 6-30 P. M. on Wednesday, touches Burdwan at 51′ past 9 o'clock P. M. of the same day, and arrives at Allahabad at 42′ past 10 o'clock P. M. on Thursday. Given that Burdwan is 67 miles from Howrah, find the distance of Allahabad from Howrah by rail.

42. An express train proceeding at the rate of 30 miles an hour, after passing through two-thirds of its journey, meets with an accident which reduces its rate, and the train in consequence

takes the same time to complete the remainder of its journey that it took in travelling up to the place where the accident happened. Find the rate of the train after the accident.

43. An express train moving at the rate of 24 miles an hour, leaves Howrah for Allahabad, which is at a distance of 564 miles; and on reaching Mokameh, which is at a distance of 282 miles, it meets with an accident which compels it to reduce its rate, and the train is in consequence 7 hrs. 3′ late in reaching Allahabad. Find its rate after the accident.

44. Supposing that light takes 8′ to come from the Sun to the Earth, and that the Earth takes 365 days, 6 hrs. to describe a circle round the Sun, compare the velocity of the Earth with the velocity of light. (The circumference of a circle is to its radius as $3{\cdot}1416 : \frac{1}{2}$.)

45. A room 8 cubits broad is to be covered with coir mat 3 ft. wide. Find the length of matting required, supposing the length of the room to be 20 ft.

46. If 5 men can dig a trench 50 ft. long, 10 ft. broad, and 2 yds. deep, in a given time, how many men will be required to dig another trench twice as long, thrice as broad, and 3 yds. deep, in the same time?

47. If 35 men can reap a field in 6 days, in how many days will 42 men be able to reap the same?

48. If a certain quantity of rice is sufficient to feed 45 men for 16 days, how many men can be fed with it for 12 days?

49. If 5 men or 7 boys can reap a field in a certain time, in how many days will 8 boys be able to reap another field which 20 men take 4 days to reap?

50. If 6 men or 9 boys can finish a certain piece of work in a given time, how many men must be associated with 12 boys to finish in half the time a work twice as great?

51. A clock is set right at 8 o'clock P. M. on Monday, and at 1 o'clock P. M. on Wednesday, it is found to be 3′ too fast. Supposing its rate regular, what will be the true time when the clock strikes one on Sunday afternoon; and what will be the time by the clock at 1 o'clock P.M. of the same day?

52. Two watches, of which one gains 2′ and the other loses 30″ a day, are both set right at 1 o'clock P.M. on Monday. What will be the difference between the times indicated by them at 9 o'clock A.M. on Sunday, and when will that difference amount to half an hour?

53. If 4 yds. of silk cost as much as 9 yds. of linen, how many yards of linen can be given in exchange for 18 yds. of silk?

54. If 10 masons can build a wall 25 ft. long, 3 ft. high, and 2 ft. thick, in 1 day, in how many days will 15 masons be able to build another wall twice as long, thrice as high, and 3 feet thick ?

55. If 9 men can do a piece of work in 18 days, working 8 hours a day, how many men will be required to finish in 12 days of 6 working hours each a piece of work 3 times as great ?

56. If the carriage of 20 mds. over 5 miles cost ₨1. 4*a.*, how many maunds can be carried over 50 miles for ₨6. 4*a.* ?

57. If 10 men in 7 days consume 1 md. 30 seers of rice, what quantity of rice will be required to feed a company of 50 men for the year 1878 ? And how much more rice must be added to the stock if 12 more men join the company on the 8th of October ?

58. If 3 men or 5 boys earn ₨6. 9*a.* in 7 days, how much will 4 men and 6 boys earn in the year 1879 ?

59. If 18 mds. of gram be sufficient to feed 4 horses for 30 days, what quantity of gram will be required to feed 7 horses for the year 1878 ?

60. If 24 mangoes can be had for ₨3, how many mangosteens can be had for ₨5, supposing 2 mangoes to be worth as much as 5 mangosteens ?

61. If 500 men could be fed for ₨125, 30 years ago, what would the feeding of 600 men cost now, supposing the prices of articles of food to have risen three-fold ?

62. A town which is besieged and is defended by 1400 men with provisions enough to sustain them 42 days, supposing each man to receive 18 oz. a day, obtains an increase of 200 men to its garrison on the morning of the 11th day of the siege. What must now be the allowance to each man, in order that the remaining provisions may serve the whole garrison for a further period of 42 days ?

63. If 10 cannon which fire 3 rounds in 5 minutes kill 170 men in 1½ hours, how many cannon which fire 5 rounds in 6 minutes will kill 500 men in 1 hour at the same rate ?

64. A hare starts 40 yards before a greyhound, and is not perceived by him till she has been up 40 seconds : she gets away at the rate of 10 miles an hour, and the dog pursues her at the rate of 18 miles an hour ; how long will the course last, and what distance will the hare have run ?

65. If a tradesman with a capital of ₨1800 gain ₨252 in 7 months, how long will it take him with a capital of ₨5000 to gain ₨500 ?

66. If a tradesman with a capital of £2700 gain £216 in

6 months, what must be his capital in order that he may gain £1200 in 9 months ?

67. At what time between one and two o'clock do the hour and minute hands of a watch point in directions exactly opposite ?

68. At what time between twelve and two o'clock are the hands of a clock together again ?

69. If a sixpenny loaf weigh 4·35 lbs. when wheat is at 5·75*s.* a bushel, what must be paid for 49·3 lbs. of bread when wheat is at 18·4*s.* a bushel ?

70. If 15 men, 12 women, and 9 boys can do a piece of work in 11 days, in what time will 9 men, 12 women and 15 boys be able to finish a piece of work 3 times as great, supposing the parts done by each in the same time to be as the numbers 3, 2, and 1 ?

CHAPTER VIII.

DIVISION INTO PROPORTIONAL PARTS. PERCENTAGE, PROFIT AND LOSS, AND AVERAGE. FELLOWSHIP.

SECTION I. DIVISION INTO PROPORTIONAL PARTS

225. Prop. If there be any number of equal simple fractions, the sum of their numerators divided by the sum of their denominators will give another simple fraction equal to any one of them.

Thus take the simple fractions

$$\frac{2}{3},\ \frac{4}{6},\ \frac{6}{9}.$$

Then $\because\ \frac{2}{3}=\frac{4}{6}=\frac{6}{9}$,

$\therefore\ 2=3\times\frac{6}{9}$,

and $4=6\times\frac{6}{9}$,

and $2+4=(3+6)\times\frac{6}{9}$;

and dividing both sides by $3+6$,

$$\frac{2+4}{3+6}=\frac{6}{9}.$$

Again $\frac{2+4}{3+6}=\frac{2}{3}$,

$\therefore$ in the same way as above

$$\frac{2+4+6}{3+6+9}=\frac{2}{3}.$$

Similarly the Proposition may be proved in any other case.

226. Since $\frac{2+4+6}{3+6+9}=\frac{12}{3+6+9}=\frac{2}{3}=\frac{4}{6}=\frac{6}{9}$,

$\therefore\ 2:3::4:6::6:9$,

and $2=3\times\frac{12}{3+6+9},\ 4=6\times\frac{12}{3+6+9},\ 6=9\times\frac{12}{3+6+9}$.

Thus we see that if we divide any number 12 into parts 2, 4, 6, which are proportional to the numbers 3, 6, 9, these parts are equal to $3\times\frac{12}{3+6+9}$, $6\times\frac{12}{3+6+9}$, and $9\times\frac{12}{3+6+9}$ respectively.

Hence we can deduce the following general Rule :—

Rule. To divide a given number into parts proportional to certain other given numbers, divide the number to be divided by the sum of these latter, and multiply the quotient by each of them, and the product will be the required part corresponding to that number.

Ex. 1. Divide 27 into parts proportional to 2, 3, 4.

By the Rule,

$\because\ 2+3+4=9,$

the parts are $2\times\frac{27}{9}=6.$

$3\times\frac{27}{9}=9,$

and $4\times\frac{27}{9}=12.$

Ex. 2. Divide ₨4500 among *A*, *B*, and *C* so that their shares may be as 3, 5 and 7 respectively.

By the Rule,

$\because\ 3+5+7=15,$

the share of $A=3\times$ ₨$\frac{4500}{15}=$ ₨900 ;

..................$B=5\times$ ₨$\frac{4500}{15}=$ ₨1500 ;

and..................$C=7\times$ ₨$\frac{4500}{15}=$ ₨2100.

Examples XLI.

1. Divide

(1) 18 into 3 parts proportional to 1, 2 and 3.

(2) 27 into 3 parts ....2, 3 and 4.

(3) 36 into 3 parts ....3, 4 and 5.

(4) 128 into 4 parts ....1, 3, 5, and 7.

(5) 1000 into 4 parts ....2, 4, 6, and 8.

(6) 585 into 3 parts ....11, 13 and 15.

2. Divide ₨2700 among *A*, *B*, and *C* so that their shares may be as the numbers 1, 3 and 5.

3. Divide £2400 among *A*, *B*, and *C*, so that as often as *A* gets £5, *B* shall get £4, and as often as *B* gets £8, *C* shall get £6.

SECTION II. PERCENTAGE, PROFIT AND LOSS, AND AVERAGE.

227. The term *per cent.* means for every hundred.

Thus, if a dealer with a capital of ₨20 makes a profit of ₨1, he makes a profit at the rate of ₨5 for every ₨5 × 20 or ₨100, and he is said to make a profit of 5 *per cent.* on his outlay.

Tradesmen generally estimate their profit and loss by percentages of their capital.

We shall here define some allowances that are made at certain rates per cent. on certain other amounts.

Defs. Commission is an allowance made to an agent or factor for buying or selling goods for his employer.

Brokerage is an allowance made to a broker for effecting the sale of Government Promissory Notes, shares, and the like.

Premium is an allowance on the value of goods liable to risk, made to certain parties called *insurers*, who, in consideration thereof, undertake in case of loss to make good to the owner the value of the goods insured.

Examples of Profit and Loss and other Examples involving the term per cent. are worked out by the application of the Rule of Three, as will be seen below.

Ex. 1. A grocer buys sugar at R12 a maund and sells it R13. What does he gain per cent. on his outlay?

Let x = the gain per cent. required.

Then ∵ for R12 he gains R (13 − 12) or R1,

$$\therefore x : 100 :: 1 : 12,$$

whence $x = \frac{100}{12} = 8\frac{1}{3}$.

Ex. 2. How much per cent. is 6 of 15?

Let x = the no. reqd.

Then ∵ x must bear the same ratio to 100 that 6 bears to 15,

$$\therefore x : 100 :: 6 : 15,$$

whence $x = \frac{600}{15} = 40$.

Ex. 3. If a dealer gains 20 per cent. on his outlay by selling rice at R3 a maund, what was the cost price?

Let x = the cost price reqd. in rupees.

Then ∵ for every R100 of cost price, the sale brings R120,

$$\therefore x : 3 :: 100 : 120,$$

whence $x = \frac{100 \times 3}{120} = \frac{10}{4} = \frac{5}{2}$;

∴ the cost price = R2. 8*a*.

Ex. 4. A factor realizes R75 as commission on the selling price of grain at 2 per cent. What was that price?

Let x = the price reqd. in rupees.

Then $x : 100 :: 75 : 2$,

whence $x = \frac{75 \times 100}{2} = 3750$;

∴ the price reqd. = R3750.

Ex. 5. Goods worth £490 are to be insured at the rate of 2 per cent. To what amount must they be insured so that in case of loss the value of the goods and the premium paid may be recovered? And what will be the cost of insurance?

If the goods be insured for £490 only, then in case of loss the premium paid will be lost, as the party insured will get £490 only. If however every £100 − £2 or £98 of the value of the goods be insured for £100, then paying £2 as premium for the £100, the party insured will in case of loss recover £100, *i. e.*, £98 (the value of the goods) + £2 (the premium paid). Hence putting

x = the reqd. amount in pounds,

we have

$$98 : 100 :: 490 : x,$$

$$\text{whence } x = \frac{100 \times 490}{98} = 500;$$

$\therefore$ the amount reqd. = £500.

And the cost of insurance being 2 per cent. on the amount insured, *i. e.*, $\frac{2}{100}$ of £1 per £1,

$$= \frac{2}{100} \times £500 = £10.$$

228. Def. The **Average** of several quantities is a quantity which being taken as often as there are quantities will give a sum equal to the sum of the given quantities.

It is therefore found by dividing the sum of the given quantities by their number.

Examples involving the term average may be worked out in the manner given below.

Ex. 1. A tradesman in 3 months gains R625, R590 and R1020. What is his average monthly gain for these 3 months?

$$\text{The reqd. average} = \text{R}\frac{625 + 590 + 1020}{3}$$

$$= \text{R}\frac{2235}{3} = \text{R}745.$$

Ex. 2. In a class of 9 boys, there are 2 boys each 9 years old, 3 boys each 10 years old, and 4 boys each 11 years old. What is the average age of the boys in the class?

$$\text{The average reqd.} = \frac{2 \times 9 + 3 \times 10 + 4 \times 11}{9} \text{ years}$$

$$= \frac{92}{9} \text{ years}$$

$$= 10\tfrac{2}{9} \text{ years.}$$

Examples XLII.

1. If the annual value of a holding be R240, and the tax imposed upon it be R18 per annum, at what rate per cent. on the annual value is the tax levied?

2. A dealer buys gram at R1. 14*a.* a maund, and sells it at R2 per maund. What does he gain per cent. on his outlay? And at what price per maund must he sell it to secure a profit of 20 per cent?

3. A dealer by selling goods at R3. 11*a.* per maund makes a profit of 18 per cent. on his outlay. What was the cost price?

4. A factor realizes R120 as commission on the selling price of grain at $1\frac{1}{2}$ per cent. What was that price?

5. Goods worth £3900 are to be insured at $2\frac{1}{2}$ per cent. To what amount must they be insured so that in case of loss the value of the goods and the premium paid may be recovered?

6. A zemindar raises the rent of every ryot by 1 pice in the rupee. By how much per cent. is the income of the zemindari thereby increased?

7. In a class composed of 30 boys, there were present, on Monday 29, on Tuesday 26, on Wednesday 27, on Thursday 25, on Friday 28, and on Saturday 21. What was the average daily attendance during the week?

8. A gentleman earned R9500 from the 1st of January to the 30th of April; R10800 from the 1st of May to the 30th of September; and R4300 from the 1st of October to the 31st of December. What was his average monthly income during the year?

SECTION III. FELLOWSHIP.

229. Defs. Fellowship, called also **Partnership,** is a method by which the profits or losses of partners in any trade or business are determined.

It is called **Simple Fellowship** or **Compound Fellowship** according as the capitals of the several partners remain invested in the joint trade for the same period or for different periods of time.

Examples of Fellowship are only particular instances of Division into Proportional Parts, and are worked out in the manner given below.

I. Simple Fellowship.

1. Two partners *A* and *B* contribute R5000 and R6000 [respec]tively for their joint business, and make a profit of R990. [What s]hare of the profit will each get?

As the profit is to be divided in proportion to the capital contributed by each, the question is reduced to dividing 990 into parts proportional to 5000 and 6000.

Hence the share of $A = 5000 \times ₨\frac{990}{5000+6000}$

$= ₨450$;

and.....................$B = 6000 \times ₨\frac{990}{11000}$

$= ₨540$.

II. Compound Fellowship.

Ex. In a joint trade, A contributes the sum of ₨4000 which remains invested for 5 months, and B contributes ₨3000 which remains invested for 6 months. They make a profit of ₨570. What is the share of each?

The sum of ₨4000 invested for 5 months is the same as $5 \times$ ₨4000.....................1 month;

and similarly ₨3000.....................6 months is the same as $6 \times$ ₨3000.....................1 month;

and thus the question is reduced to one of Simple Fellowship where the capitals of the partners are $5 \times$ ₨4000 and $6 \times$ ₨3000, *i. e.*, ₨20000 and ₨18000.

Consequently, the share of $A = 20000 \times ₨\frac{570}{38000}$

$= ₨300$;

and ....$B = 18000 \times ₨\frac{570}{38000}$

$= ₨270$.

Examples XLIII.

1. Two partners in trade contribute respectively ₨4000 and ₨5000, and they gain ₨1350. How ought the profit to be divided between them?

2. A trading firm with a capital of £25000 is composed of three partners, A, B, and C. A owns £6000, B, £9000 and C, the remainder of the capital. If the profits of the firm amount to £3000, how much of it will each get?

3. A trading firm is composed of two partners A and B. For every rupee that A owns in the capital, B owns three rupees. How ought a profit of ₨2400 to be divided between them?

4. A opens a shop with a capital of ₨2000. Four months after, B joins with a capital of ₨3000, and two months after B's joining, C adds a capital of ₨4500. The profits of the shop at the end of a year amount to ₨900. How ought the amount to be divided?

5. In the preceding Example, if *A* puts in a further sum of ₨500 when *C* joins, and the profits amount to ₨910, how much of the amount will each get?

6. *A* buys an estate yielding an income of ₨12000 per annum and 4 months after his purchase, sells a five annas share of it to *B*. How ought the income to be divided at the end of the year?

CHAPTER IX.

INTEREST.

230. Defs. Interest is money paid for the use of money.

The sum lent is called the **Principal.**

The principal together with the interest for any period is called the **Amount.**

Interest is generally reckoned at a certain *rate* per cent. *per annum*, *i. e.*, for a year. In this country interest is often reckoned at a certain rate per cent. per *mensem*, *i. e.*, for a month.

When interest is charged on the principal alone, it is called **Simple Interest.**

When interest remains unpaid and is added to the principal as soon as it is due, and then interest is charged on the whole, it is called **Compound Interest.**

231. When interest has to be calculated from one given day to another, as, for instance, from the 15th of August to the 2nd of December, the first day, *i. e.*, the 15th of August is left out, but the last day, *i. e.*, the 2nd of December, is taken into account. This is the rule given in English books on Arithmetic. But in this country the practice is just the reverse, interest being charged for the day of borrowing and not for the day of repayment. The result, however, would be the same in both cases.

232. Since Amount = Principal + Interest,
$\therefore$ Interest = Amount—Principal,
and Principal = Amount—Interest.

SECTION I. SIMPLE INTEREST.

233. *Given the principal, the rate, and the time, to find the interest.*

Ex. 1. Find the interest on £325 for 3 years at 6 per cent. per annum.

Let x = the interest reqd. in pounds.

Then $\because$ £100 give £6 in one year,

$\therefore$ £100 will give £ (3×6) in 3 years;

and as £325 .... £x in the same period of 3 years, at the same rate,

$\therefore$ $100 : 325 :: 3\times 6 : x$;

and $\therefore$ $x = \frac{325\times 3\times 6}{100} = \frac{117}{2} = 58\frac{1}{2}$;

$\therefore$ the interest reqd. = £58. 10s.

Ex. 2. The sum of R650 is lent on the 2nd of December 1872. Find the interest due on the 16th of August 1875, at 5 per cent. per annum.

Here the time for which interest is due is the time from 2nd December 1872 to 16th August 1875, and in reckoning that time, the 2nd of December is to be included and the 16th of August is to be excluded. Now the time from 2nd December 1872 to 1st December 1874 is 2 years, and that from 2nd December 1874 to 15th August 1875, both days inclusive, is

$(30+31+28+31+30+31+30+31+15)$ days or 257 days;

$\therefore$ the whole time is 2 years and 257 days or $2\frac{257}{365}$ years.

Hence as in Ex. 1,

the interest reqd. $= \text{R}\dfrac{650\times2\frac{257}{365}\times5}{100}$.

In practice it is usual to calculate the interest for the year and that for the days separately.

Thus in the above Example,

the interest for 2 years $= \text{R}\frac{650\times2\times5}{100} = \text{R}65$;

and...............257 days $= \text{R}\dfrac{650\times\frac{257}{365}\times5}{100} = \text{R}\dfrac{650\times257\times5}{100\times365}$

$= \text{R}\frac{13\times257}{2\times73} = \text{R}\frac{3341}{146}$

$= \text{R}22\frac{129}{146}$;

$\therefore$ the whole interest $= \text{R}87\frac{129}{146}$.

Ex. 3. Find the interest on R750. 8*a.* for 2 years and 10 months at $7\frac{1}{2}$ per cent. per annum.

In Examples like these, where the months are not named, 1 month is taken to be $\frac{1}{12}$ of a year.

Hence in this case, the no. of years $= 2\frac{10}{12} = 2\frac{5}{6}$;

and the principal $=$ R750. 8*a.* $= \text{R}750\frac{1}{2}$.

Therefore as in Ex. 1,

the interest reqd. $= \text{R}\dfrac{750\frac{1}{2}\times2\frac{5}{6}\times7\frac{1}{2}}{100}$

$= \text{R}\frac{1501\times17\times15}{2\times6\times2\times100} = \text{R}\frac{25517}{160}$

$=$ R159. 7*a.* $8\frac{2}{5}$*p.*

From the above we deduce the following Rules:

Rule I. To calculate the interest for any number of years integral or fractional, multiply the principal by the number of years, and the product by the rate per cent. per annum, and divide the result by 100; the quotient will be the interest expressed in the same denomination as the principal.

Rule II.* To calculate the interest for any number of days, multiply the principal by the number of days, and the product by the rate per cent. per annum and divide the result by 100×365; and the quotient will be the interest required.

234. From the preceding Article we see that if the rate be a rate per cent. per annum, and the time be expressed in years,

$$\text{Interest} = \frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100} \ldots\ldots\ldots\ldots(1)$$

$$\therefore \quad 100 \times \text{Interest} = \text{Principal} \times \text{Time} \times \text{Rate},$$

and $\therefore$ dividing both sides by Principal $\times$ Rate, we have

$$\text{Time} = \frac{100 \times \text{Interest}}{\text{Principal} \times \text{Rate}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Again, dividing both sides by Principal $\times$ Time, we have

$$\text{Rate} = \frac{100 \times \text{Interest}}{\text{Principal} \times \text{Time}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Lastly, dividing both sides by Time $\times$ Rate, we have

$$\text{Principal} = \frac{100 \times \text{Interest}}{\text{Time} \times \text{Rate}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

An equation like any of the preceding, (1), (2), (3), or (4), is called a *Formula*.

From the four formulæ given above, we see that when any three of the four quantities, principal, interest, rate and time, are given, the fourth can be found.

235. We can obtain the above formulæ independently of Art. 233. Thus take formula (2).

Let it be required to find in how many years £550 will amount to £616 at 6 per cent. per annum.

Let x = the no. of years reqd.

Then in x years £100 will give $x \times$ £6 as interest,

and £550............£616 − £550 or £66

at the same rate;

$$\therefore 100 : 550 :: x \times 6 : 66,$$

$$\text{and } \therefore x \times 6 \times 550 = 100 \times 66,$$

$$\text{whence } x = \frac{100 \times 66}{550 \times 6},$$

$$\textit{i. e.}, \text{ the time reqd.} = \frac{100 \times \text{interest}}{\text{principal} \times \text{rate}}$$

Similarly by the application of the principle of proportion, the other two formulæ (3) and (4) can be obtained independently of Art. 233.

236. *Given the amount, the rate and the time, to find the principal.*

Let it be required to find what sum will amount to R616 in 2 years at 6 per cent. per annum.

Let x = the sum reqd.

Then ∵ R100 in 2 years amount to R$(100+2\times6)$,

and Rx..................................R616

at the same rate of interest, *i. e.*, at the same rate of increase,

$$\therefore\ 100 : x :: 100+2\times6 : 616,$$

$$\text{whence } x=\frac{616\times100}{100+2\times6},$$

$$i.\ e.,\ \text{Principal}=\frac{\text{Amount}\times100}{100+\text{Time}\times\text{Rate}}.$$

237. When partial payments are made on different dates, the interest for each interval is calculated on the portion of the principal that remains due during it, as will be seen from the Example given below.

Ex. The sum of R600 is borrowed on the 3rd of January 1874; and on the 7th of February and on the 21st of April following, payments of R50 and R100 respectively are made in liquidation of the principal. Find the amount due on the 21st of May following, at 5 per cent. per annum simple interest.

The process may be stated thus :—

	R	R
From 3rd Jan. to 6th Feb. both days inclusive *i. e.*, for 35 days	Principal = 600	Int. = $2\frac{64}{73}$
From 7th Feb. to 20th Apr. both days inclusive *i. e.*, 75 days	.... = 600 − 50 = 550	... = $5\frac{1}{2}$
From 21st Apr. to 20th May both days inclusive *i. e.*, for 30 days	.... = 550 − 100 = 450	... = $1\frac{62}{73}$
		Total $10\frac{33}{146}$

Thus the reqd. amount due = R450 + R$10\frac{33}{146}$ = R$460\frac{33}{146}$.

We have supposed the payments to be made in liquidation of the principal only. If however the creditor has the choice of

appropriating the payments, and he takes them in liquidation of the interest due as well as the principal, the process will stand thus :—

For the 1st period, Principal $=$ R600, Int. $=$ R$2\frac{64}{73}$.

..........2nd...................... $=$ R$600-(50-$ R$2\frac{64}{73})$

$=$ R$552\frac{64}{73}$, Int. $=$ R$5\frac{193}{365}$.

..........3rd...................... $=$ R$552\frac{64}{73}-$ R$(100-5\frac{193}{365})$

$=$ R$458\frac{148}{365}$, Int. $=$ R$1\frac{235504}{266450}$.

Here we need not add the interest, as the last interest is the only one that is due, the others having been paid up.

Thus, the amount due $=$ R$(458\frac{148}{365}+1\frac{235504}{266450})=$ R$460\frac{77094}{266450}$.

238. When there is a mutual account current between two persons, so that each may be regarded as a lender and a borrower in respect of the sums paid and received by him respectively, the payments and receipts by either of them, (which will be the same as the receipts and payments by the other), are kept separate, and on opposite sides of the account, and interest runs on each side, for each period on the sum that remains due for that period, as will be seen from the Example given below.

Ex. *A* receives from *B*	*B* receives from *A*
on Mar. 3, 1874, R10000	on Apr. 2, 1874, R100
... May 12,.........R2000.	... Apr. 22,................,R9100

Find the balance due to *B* on May 17, allowing interest at 18¼ per cent. per annum.

The calculation will stand thus :—

On the left side of the account—

From Mar. 3 to May 11 both days inclusive *i. e.*. for 70 days	Principal $=$ R10000, Int. $=$ R350.
From May 12 to May 16 both days inclusive, *i. e.*, for 5 days	Principal $=$ R12000, Int. $=$ R30
	Total R380.

Thus on May 17, the total sum due from *A* to *B*

$=$ R $(12000+380)=$ R12380.

On the right side of the account—

From Apr. 2 to Apr. 21 both days inclusive *i. e.*, for 20 days	Principal $=$ R100, Int. $=$ R1.
From Apr. 22 to May 16 both days inclusive *i. e.*, for 25 days	Principal $=$ R9200, Int. $=$ R115
	Total R116.

∴ Thus on May 17 the total sum due from B to A

$$=R(9200+116)=R9316.$$

Therefore on May 17, the balance due to B

$$=R(12380-9316)=R3064.$$

The above mode of calculating interest is called in this country the *Ganga-Jamuna* mode, because interest on sums paid by B, and that on sums paid by A, run side by side like the Ganges and the Jamuna.

If we regard A alone as the borrower and B the lender having the option of appropriating payments in his own way, the account will stand differently thus :

From Mar. 3 to Apr. 1 both days inclusive *i. e.*, for 30 days } Principal = R10000 ; Int. = R150.

On Apr. 2, after payment of R100 Int. due = R50.

From Apr. 2 to Apr. 21 both days inclusive *i. e*, for 20 days } Principal R10000 ; Int. = R100.

On Apr. 22 Int. due = R150.

From Apr 22 to May 11 both days inclusive *i. e.*, for 20 days } $= R10000 - R(9100-150)$

$= R1050$; Int. $= R10\frac{1}{2}$.

From May 12 to May 16 both days inclusive *i. e.*, for 5 days } .. $= R1050 + R2000$

$= R3050$; Int. $= R7\frac{5}{8}$.

Therefore on May 17, the amount due to B

$$= R(3050+10\tfrac{1}{2}+7\tfrac{5}{8})$$
$$= R3068\tfrac{1}{8}.$$

Hence it will be seen that the Ganga-Jamuna mode is more advantageous to the borrower than the latter mode ; and the reason is obvious. For whereas, in the former mode, the sum of R100 paid by A on Apr. 2, and other sums paid by him all carry interest in his favour, in the latter mode, being taken in whole or in part in liquidation of interest due, they do not in the same way carry interest in his favour. For convenience of calculation the former mode is often adopted instead of the latter.

Examples XLIV.

1. Find the simple interest and the amount

(1) Of R75 for 1 year at 6 per cent. per annum.

(2) Of R80 for 2 years at 9 ....

(3) Of R125 for $2\frac{1}{2}$ years at $7\frac{1}{2}$ per cent. per annum.

(4) Of R2560 for 4 years at 12..............................

(5) Of £1050. 10*s*. for 5 years at 4......................

(6) Of £5500 for $3\frac{1}{2}$ years at $4\frac{1}{2}$.........................

(7) Of R1050 for $2\frac{1}{4}$ years at $1\frac{1}{2}$ per cent per mensem.

(8) Of R750 for 2 years at $1\frac{1}{4}$...............................

2. Find the simple interest

(1) On R60 for 4 months at 2 per cent. per mensem.

(2) On R85 for 9 months at $1\frac{9}{16}$ ....

(3) On R225 for 10 months at 10 per cent. per annum.

(4) On R520 for 8 months at 9 ....

(5) On R120 for 7 months at 1 pice per rupee per month.

(6) On R56 for 6 months at 2 ....

(7) On R1200 from October 18, 1878 to March 21, 1879, at 6 per cent. per annum.

(8) On R1560 from July 21, 1870 to June 20, 1871 at 9 per cent. per annum.

(9) On £500 from March 20 to August 31, at 5 per cent. per annum.

3. In what time will

(1) £1000 amount to £1500 at 5 per cent. per annum ?

(2) £625 amount to £800 at 4 .... ?

(3) R120 amount to R200 at 10 .... ?

(4) R40000 amount to R50000 at 4 .... ?

(5) R80, amount to R100 at $1\frac{9}{16}$ per cent. per mensem ?

(6) R125 amount to R200 at $1\frac{1}{4}$ per cent. per mensem ?

4. At what rate of simple interest will

(1) R60 amount to R80 in 2 years ?

(2) R75...............R100 in $2\frac{1}{2}$ years ?

(3) £4000............£5000 in 8 years ?

(4) £640 amout to £700 in 5 years ?

(5) R1000............R1250 in 4 years ?

(6) R2225.............R2670 in 2 years ?

5. Find the sum of which the interest is

(1) R60 in 3 years at 4 per cent. per annum.

(2) £80 in 2 years at 5 per cent. per annum.
(3) £100 in 5 years at 4........................
(4) R600 in 2 years at 4........................
(5) R10000 in 5 years at 6........................
(6) R1664 in 8 years at 13........................

6. What sum will amount to

(1) £100 in 3 years at 4 per cent. per annum?
(2) £625 in 5 years at 5........................?
(3) R1000 in 4 years at 12........................?
(4) R25000 in 6 years at 10........................?

7. In what time will a sum double itself at 4 per cent. per annum?

8. At what rate will a sum double itself in 5 years?

SECTION II. COMPOUND INTEREST.

239. Rule. To find the compound interest of a given sum for a given time at a given rate, find the interest of the given sum for the first year, and add it to the principal for that year; the sum will be the principal for the second year. Find the interest of this for the second year, and add it to the principal for that year; the sum will be the principal for the third year. Proceed in this way, and the sum of the interests for the several years will be the compound interest required.

The reason for this Rule is obvious from the definition of compound interest.

Ex. Find the compound interest and the amount of R325 for 3 years at 5 per cent. per annum.

We have,

principal for 1st year =R325,

int..................$=\frac{\quad}{100}$ = R16·25

principal......2nd..... =R341·25,

int..................$=\frac{341\cdot25\times5}{100}$ = R17·0625;

principal for 3rd year =R358·3125,

int..................$=\frac{358\cdot3125\times5}{100}$ =R17·915625

∴ amount at the end of the 3rd year = R376·228125 ;

and the int. reqd. = R(16·25 + 17·0625 ÷ 17·915625)

= R51·228125

= R51. 3*a*. $2\frac{2}{8}$ pice.

To avoid fractions, the calculation is usually made in decimals.

240. It is customary, when compound interest for a number of entire years and for a part of a year, as for instance for $2\frac{3}{4}$ years, is required, to find the interest for the last or the 3rd year, and then to take $\frac{3}{4}$ of it as the interest for the $\frac{3}{4}$ths of the 3rd year.

When interest is payable oftener than once a year, the interest due at the end of each interval will have to be added to the principal for that interval, as will be seen from the Example given below.

Ex. Find the compound interest of £250 for 2 years at 4 per cent. per annum, interest being payable half-yearly.

We have

principal for 1st half-year		= £250.
int..............................	$=\frac{250 \times 4}{2 \times 100}$	= £5·00
principal......2nd............		= £255.
int..............................	$=\frac{255 \times 4}{2 \times 100}$	= £5·1
principal......3rd............		= £260·1.
int..............................	$=\frac{260 \cdot 1 \times 4}{2 \times 100}$	= £5·202
principal......4th............		= £265·302.
int..............................	$=\frac{265 \cdot 302 \times 4}{2 \times 100}$	= £5·30604

the compound interest reqd. = £(5 + 5·1 + 5·202 + 5·30604)

= £20·60804.

Examples XLV.

Find the compound interest and the amount

1. Of R80 for 2 years at 10 per cent. per annum.
2. Of R75 for $2\frac{1}{2}$ years at 8..............................
3. Of £125 for 2 years at 4..............................
4. Of £100 for 3 years at 10..............................
5. Of R2000 for 2 years at 4..............................
6. Of R25000 for 3 years at 10..............................

CHAPTER X.

PRESENT WORTH AND DISCOUNT.

241. When a sum of money is payable at the end of a given time, and the debtor, instead of waiting for that time, wishes to pay off the debt immediately, it is clear that he ought to pay, not the whole amount due, but something less, as he will not have the use of the money for the given time, and the creditor, who will have that use, may, by investing the money at the current rate of interest, get at the end of the given time an amount equal to what was then payable originally.

Defs. The amount which the creditor is entitled to receive when payment is made before it is due, is called the **Present Value** or the **Present Worth** of the amount due.

The allowance made on any sum payable after some time, when it is paid before it becomes due, is called **Discount.**

Hence,

Present Worth = Given sum − Discount,

and Discount = Given sum − Present Worth

= Int. on Present Worth for the given time at the assumed rate.

242 *To find the present value of a given sum due at the end of a given time, at a given rate of simple interest.*

Let it be required to find the present value of £750 due 3 years hence, at 5 per cent. per annum simple interest.

Let x = the present value reqd. in pounds.

Then ∵ £100 in 3 years at 5 per cent. will amount to £100 + £(3×5)

∴ £100 is the present value of £$(100+3\times5)$ due 3 years hence at 5 per cent.

Similarly £200 is the present value of £$(200+3\times5\times2)$ that is £$(100+3\times5)\times2$ due 3 years hence at 5 per cent.;

£300 is the present value of £$(300+3\times5\times3)$ that is

£$(100+3\times5)\times3$ due 3 years hence at 5 per cent.; and so on.

That is, the present values of different sums due at the end of a given time, at a given rate of simple interest vary as the sums due.

Therefore as £ x is the present value of £750 due 3 years hence at the same rate,

$100 : x :: 100+3\times5 : 750,$

whence $x = \frac{100 \times 750}{100 + 3 \times 5}$,

i. e., Present Value $= \dfrac{100 \times \text{Given sum}}{100 + \text{Time} \times \text{Rate}}$.

Ex. A tenant has to pay a rent of ₨240 at the end of the year. If he pays at the beginning of the year, what sum will suffice, supposing the rate of simple interest to be 12 per cent. per annum?

By the formula,

the amount reqd. $= ₨\frac{100 \times 240}{100 + 1 \times 12}$

$= ₨\frac{100 \times 240}{112}$

$= ₨\frac{1500}{7}$

$=$ ₨214. 4*a*. $2\frac{2}{7}$ pice.

243. *To find the discount on a given sum due at the end of a given time at a given rate of simple interest.*

Since discount $=$ given sum $-$ present value,

$\therefore$ the discount is found by first finding the present worth (Art. 242) and then subtracting it from the given sum.

It may also be found independently thus:

Let it be required to find the discount on £250 due 2 years hence, at 5 per cent. per annum simple interest.

Let $x =$ the discount reqd.

Then $\because$ £100 in 3 years will amount to £$(100 + 3 \times 5)$

$\therefore$ £(3×5) is the discount on £$(100 + 3 \times 5)$ due 3 years hence at 5 per cent.

Similarly £$(3 \times 5) \times 2$ is the discount on £$(200 + 3 \times 5 \times 2)$ that is £$(100 + 3 \times 5) \times 2$ due 2 years hence at 5 per cent.;

£$(3 \times 5) \times 3$ is the discount on £$(100 + 3 \times 5) \times 3$; and so on. That is the discount on different sums due at the end of a given time at a given rate of simple interest vary as the sums due.

Therefore as £ x is the discount on £250 due 3 years hence at 5 per cent.,

$$x : 3 \times 5 :: 250 : 100 + 3 \times 5, \text{ and } \therefore x = \frac{250 \times 3 \times 5}{100 + 3 \times 5},$$

i. e., Discount $= \dfrac{\text{Given sum} \times \text{Time} \times \text{Rate}}{100 + \text{Time} \times \text{Rate}}$.

Ex. If the credit price of a set of books to be paid after 6 months be ₨150, what deduction will be made when the price is paid in cash, supposing the rate of simple interest to be 18 per cent. per annum?

By the formula,

the deduction or discount reqd. $= \text{R}\frac{150 \times \frac{1}{2} \times 18}{100 + \frac{1}{2} \times 18}$

$= \text{R}\frac{150 \times 9}{109}$

$= \text{R}\frac{1350}{109}$

$= \text{R}12.\ 6a.\ 1\frac{107}{109}p.$

244. Since the present value of any sum is less than that sum, and since the discount on any sum is the interest on its present value,

$\therefore$ the discount on any sum payable after any given time is less than the interest on that sum for that time at the given rate.

The distinction between discount and interest will be made clear by an Example.

Ex. If 5 copies of a certain book can be had for a certain sum payable at the end of a year, and 6 copies for the same sum paid immediately, find the rates of discount and interest.

Since cash price of 6 copies = credit price of 5 copies,

$\therefore$ 6 × cash price of 1 copy = 5 × credit price of 1 copy,

and $\therefore$ cash price of 1 copy $= \frac{5}{6}$ of credit price of 1 copy

= credit price of 1 copy

$- \frac{1}{6}$ of credit price of 1 copy.

Hence in 1 year, discount on credit price $= \frac{1}{6}$ of that price;

and $\therefore$ .... R100 $= \frac{1}{6}$ of R100 = R$16\frac{2}{3}$,

or $16\frac{2}{3}$ per cent. is the rate of discount.

Again,

$\because$ 5 × credit price of 1 copy = 6 × cash price of 1 copy

$\therefore$ credit price of 1 copy $= \frac{6}{5}$ of cash price of 1 copy

= cash price of 1 copy

$+ \frac{1}{5}$ of cash price of 1 copy.

Hence in 1 year,

interest on the cash price $= \frac{1}{5}$ of that price;

and $\therefore$ .... R100 $= \frac{1}{5}$ of R100 = R20,

or 20 per cent. per annum is the rate of interest.

245. Defs. A **Bill of Exchange** or a **Hundi** is a writing by which one person directs another to pay a certain sum of money to a third at a certain time.

A **Promissory Note** is a writing by which a person promises to pay a certain sum of money at a certain time.

Bills of exchange, promissory notes, and hundis are instances in which money is payable at a future period. In cases of bills of exchange and promissory notes, except those payable on demand, after due date, three additional days called *days of grace* are allowed by the law of England, so that a bill or a note becomes *legally* due three days after it becomes *nominally* due. In cases of hundis in this country, merchants and bankers usually allow three days of grace.

A bill or a note drawn on the last day of any month and made payable a certain number of months after date, will become nominally due on the last day of the last month whether it be the 28th, 29th, 30th or 31st, and legally due on the 3rd of the next month. When the day on which a bill with or without the days of grace is due falls on a Sunday, Good Friday, or Christmas day in England, the bill or note becomes due on the previous day.

246. When a banker makes cash payment to the holder of a bill for a given sum payable after a given time, it is customary to deduct the interest on that sum for the given time instead of the discount ; and as by Art. 244, the interest is always greater than the discount, the transaction is always advantageous to the banker.

Ex. A bill of £500 is drawn on April 3, 1874 at 5 months date, and is discounted on June 25, at 5 per cent. What does the banker gain by the transaction ?

Adding the three days of grace the bill falls due on the 6th of September 1874 ; and from the 25th of June to the 6th of September, there are

$$6+31+31+5 \text{ or } 73 \text{ days ;}$$

$$\therefore \text{ interest deducted} = £\tfrac{500 \times 73 \times 5}{100 \times 365} = £5,$$

$$\text{and the true discount} = £\frac{500 \times \frac{73}{365} \times 5}{100 + \frac{73}{365} \times 5} = £\frac{500}{101} = £4\frac{96}{101};$$

$$\therefore \text{ the banker's gain} = £\left(5 - 4\frac{96}{101}\right) = £\frac{5}{101} = 11\frac{89}{101}d.$$

Examples XLVI.

1. Find the present worth of

(1) R100 due 1 year hence at 12 per cent. per ann. simp. int.

(2) £200 due 2 years hence at 5..................................

(3) £784 due 3 years hence at 4...................................

(4) Rs1020 due 4 years hence at 9 per cent. per ann. simp. int.
(5) Rs575 due 2 years hence at $7\frac{1}{2}$..
(6) Rs2623 due 12 years hence at 6..

2. Find the discount on
(1) £260 due 1 year hence at 4 per cent. per ann. simp. int.
(2) £1045 due 2 years hence at 5..
(3) Rs1239 due 3 years hence at 6..
(4) Rs1560 due 4 years hence at $7\frac{1}{2}$..
(5) Rs1000 due 5 years hence at 10..
(6) Rs1250 due 6 years hence at $12\frac{1}{2}$.. ....

3. A bill of £500 drawn on the 13th of March, and payable 6 months after date, is discounted on June 30th at 4 per cent. What does the banker gain by the transaction?

4. *A* grants a lease of his zemindary to *B* for 3 years. *B* after paying all expenses and the rent reserved by the lease, has a clear profit of Rs1200 a year. If at the end of a year, *B* agrees to give back the estate to *A* upon receipt of proper compensation, what ought the amount of such compensation to be, supposing the ordinary rate of interest to be 6 per cent. per annum?

5. The Prem Chand Roy Chand Studentship is worth Rs2000 a year, and is tenable for 5 years For what sum paid down immediately, ought a successful candidate to commute it, if the rate of interest be 4 per cent. per annum?

6. If the cash price of a book be Rs5. 5*a.*, and its credit price payable at the end of a year, Rs6. 4*a.*, find the rates of interest and discount.

CHAPTER XI.

EQUATION OF PAYMENTS.

247. Defs. When several debts are due from one person to another after different periods of time, the time after which all the debts may be paid at once without loss to either party is called the **Equated Time of Payment.**

To satisfy the condition of fairness to both parties, the equated time ought to be such that the sum of the present values of the several debts due after their respective periods is equal to the present value of the sum of those debts due after the equated time. In practice, however, the condition of fairness is supposed to be satisfied if the sum of the interests on the several debts for their respective times is equal to the interest on the sum of those debts for the equated time. On this supposition the rule for finding the equated time may be deduced from the following Example.

Ex. Three sums ₨100, ₨200 and ₨300 are due after 2, 3 and 4 years respectively. Find the equated time of payment.

Let x = the equated time reqd. in years.

Then by the supposition,

int. on ₨100 for 2 years + int. on ₨200 for 3 years + int. on ₨300 for 4 years = int. on ₨(100 + 200 + 300) for x years ;

$$\therefore \frac{100 \times 2 \times \text{Rate}}{100} + \frac{200 \times 3 \times \text{Rate}}{100} + \frac{300 \times 4 \times \text{Rate}}{100} = \frac{(100+200+300) \times x \times \text{Rate}}{100};$$

or multiplying both sides by 100 and dividing by the Rate,

$$100 \times 2 + 200 \times 3 + 300 \times 4 = (100 + 200 + 300) \times x ;$$

$$\therefore x = \frac{100 \times 2 + 200 \times 3 + 300 \times 4}{100 + 200 + 300}.$$

Hence we get the following Rule :—

Rule. Multiply each debt by the time after which it is due, and divide the sum of these products by the sum of the debts. The quotient will be the equated time required.

Examples XLVII.

1. A owes B ₨100 whereof the sum of ₨20 is to be paid in 3 months, and the remainder in 4 months. Find the equated time.

2. A debt of £1000 is payable at the end of a year. The debtor however pays £200 after 3 months, and £300 after 4 months. When ought the remainder to be paid?

3. A tenant pays a yearly rent of R500 according to the following instalments :—R100 at the end of 3 months; R100 at the end of 6 months; R200 at the end of 7 months; and R100 at the end of the year. When ought he to pay the whole rent if he pays it in one sum?

4. *A* owes *B* a debt of R4800, one-half of which is due in 3 months, one-third in 4 months, and the remainder in 8 months. Find the equated time.

5. Find the equated time of payment of £960, $\frac{1}{8}$ of which is due in 7 months, $\frac{1}{6}$ in 9 months, and the rest in 18 months.

6. *A* owes *B* a debt payable in 8 months, but he pays $\frac{1}{3}$ of the debt in 4 months, and $\frac{1}{4}$ in 5 months; when ought the remainder to be paid?

CHAPTER XII.

STOCKS.

248. Def. Stock means the capital of banks or trading companies ; or the capital borrowed by any Government to meet national expenses.

In the latter case, it is called the *Funds* or the *National Debt*, and is also called in England *consols*, a contraction for *consolidated annuities*, and in India, Government Securities or Government Promissory Notes.

When any Government raises capital by borrowing, it generally reserves to itself the option of paying off the principal at any future time, promising however to pay interest regularly at fixed periods. In India, the interest on Government Securities is generally paid half-yearly.

Banks and trading companies make periodical distributions of their profits amongst their shareholders, the portions of the profit given to the shareholders being called *dividends*.

249. Stock is transferable by sale, and at any time can be converted into money. But its price is continually fluctuating, depending upon a variety of causes, and amongst others, upon the amount of money available in the market for investment in Stock.

Stock is said to be at *a premium*, at *par* or at *a discount* according as the price of R100 stock is greater than, equal to, or less than R100 in money.

When R100 stock at 4 per cent. is sold for any sum, such as R105 or R100 or R98 as the case may be, for every R105 or R100 or R98, the purchaser will get R4 per annum as interest from Government.

Stock is bought and sold through brokers who generally charge R$\frac{1}{8}$ on every R100 stock bought or sold. Thus, the purchaser of the 4 per cents. at 102 will have to pay R102 + R$\frac{1}{8}$ or R102$\frac{1}{8}$ for R100 stock purchased.

In working out Examples however, the brokerage, if not mentioned, need not be taken into consideration.

250. The different classes of Examples in Stocks can be worked out by the aid of the principles of proportion. We shall give one or two instances of each.

I. Value of Stock.

Ex. 1. What amount of stock in the 4 per cents. at 95 can be bought for ₹19000?

Let x = amount reqd. in rupees.

Then ∵ ₹95 is the price of ₹100 stock,

$\therefore 95 : 19000 :: 100 : x,$

and $\therefore x = \frac{19000 \times 100}{95} = 20000,$

and the amount of stock reqd. = ₹20000.

Ex. 2. When the 5 per cents. are at 102, what is the cost of purchasing £2720 in the 5 per cents, brokerage being $\frac{1}{8}$ per cent?

Let x = the cost reqd. in pounds.

Then ∵ £100 stock cost £$102\frac{1}{8}$,

$\therefore 100 : 2720 :: 102\frac{1}{8} : x,$

and $\therefore x = \frac{2720 \times 817}{8 \times 100} = \frac{17 \times 817}{5} = 2777\frac{4}{5},$

and the price reqd. = £2777. 16*s*.

II. Interest on Stock.

Ex. 3. What is the interest on Government Securities for ₹19500 at $4\frac{1}{2}$ per cent. for 2 years?

The int. reqd. = ₹$\frac{19500 \times 4\frac{1}{2} \times 2}{100}$ = ₹1755.

Ex. 4. What annual income can be secured by investing ₹17000 in the 4 per cents. at 102?

Let x = income reqd. in rupees.

Then ∵ ₹102 give ₹4,

$\therefore 102 : 17000 :: 4 : x,$

whence $x = \frac{17000 \times 4}{102} = 666\frac{2}{3},$

and ∴ income reqd. = ₹$666\frac{2}{3}$.

III. Comparison and Transfer of Stock.

Ex. 5. If the 4 per cents. be at 92 and the 5 per cents. at 105, which is the more profitable investment of the two?

In the former case ₹92 give ₹4,

or ₹1 gives ₹$\frac{4}{92}$.

In the latter case ₹105 give ₹5,

or ₹1 gives ₹$\frac{5}{105}$.

And as $\frac{5}{105}$ is greater than $\frac{4}{92}$,

∴ the latter is the more profitable investment of the two.

Ex. 6. What change in income is produced by the transfer of ₨7500 stock from the 4 per cents. at 98 to the 6 per cents. at 105 ?

The income before transfer $=\frac{7500\times4}{100}=$ ₨300.

Now let $x=$ the amount in rupees of the 6 per cents. that can be bought for ₨7500 in the 4 per cents.

Then ∵ every ₨100 stock in the 6 per cents. costs ₨105,

and ....4 .... ₨98,

and ∵ the quantity of stock that can be bought for a given amount varies inversely as the price per ₨100 stock,

∴ $98 : 105 :: x : 7500$;

whence $x=\frac{7500\times98}{105}=7000$;

∴ the transfer gives ₨7000 in the 6 per cents. ;

and the income $=\frac{7000\times6}{100}=$ ₨420.

The transfer therefore increases the income by ₨120.

Examples XLVIII.

1. Find the amount of stock that can be purchased by investing

(1) ₨4818. 12*a.* in the 4 per cents. at $96\frac{3}{8}$.

(2) ₨4650 in the 4 per cents. at 93.

(3) ₨63000 in the $5\frac{1}{2}$ per cents. at 105.

(4) £3380 in the 3 per cents. at $84\frac{1}{2}$.

(5) £9072. 10*s.* in the 4 per cents. at $95\frac{1}{2}$.

(6) ₨27000 in the 6 per cents. at 108.

2. Find what sum will purchase

(1) ₨10000 in the 4 per cents. at 97.

(2) ₨12000 in the 5 per cents. at 105.

(3) ₨200 in the $4\frac{1}{2}$ per cents. at 101.

(4) £600 in the 3 per cents. at 88.

(5) £1800 in the 4 per cents. at 96.

(6) ₨25000 in the 6 per cents. at 106.

3. Find the yearly income arising from the investment of

(1) ₨9637. 8*a.* in the 4 per cents. at $96\frac{3}{8}$.

(2) £13950 in the 3 per cents. at 93.

(3) £13520 in the $3\frac{1}{2}$ per cents. at $84\frac{1}{2}$.

(4) ₨31500 in the 6 per cents. at 105.

(5) R45362. 8*a*. in the 4 per cents. at $95\frac{1}{2}$.

(6) R13500 in the 6 per cents. at 108.

4. Find which of the two is the more profitable investment in each of the following Examples :—

(1) The 4 per cents. at 96 and the 5 per cents. at 108.

(2) The 3 per cents. at 84 and the 4 per cents. at 96.

(3) The 4 per cents. at 95 and the 6 per cents. at 115.

(4) The $4\frac{1}{2}$ per cents. at 105 and the 4 per cents. at 96.

(5) The $3\frac{1}{2}$ per cents. at 90 and the 4 per cents. at 98.

(6) The 6 per cents. at 112 and the $4\frac{1}{2}$ per cents. at 92.

5. Find the change in income resulting from the transfer of

(1) R15000 stock from the 4 per cents. at 98 to the 5 per cents. at 105.

(2) R12000 stock from the 5 per cents. at par to the $4\frac{1}{2}$ per cents. at 90.

(3) £6300 stock from the 3 per cents. at 84 to the 5 per cents. at 105.

6. At what rate per cent. does a person receive interest who invests his capital

(1) In the 4 per cents. at 94 ?

(2) In the 3 per cents at 84 ?

(3) In the $4\frac{1}{2}$ per cents. at 102 ?

(4) In the 6 per cents. at 112 ?

7. If R100 stock in the 4 per cents. can be purchased for R96. 10*a*., for what sum may the same quantity of stock be purchased in the $4\frac{1}{2}$ per cents. with equal advantage ?

8. If a person invest R14456. 4*a*. in the 4 per cents. when they are at $96\frac{3}{8}$, what will be his loss of property when they fall to 96 ?

CHAPTER XIII.

ALLIGATION.

251. Defs. Alligation is a method of finding the price of a mixture when the prices and quantities of the ingredients are given, or of finding the quantities of the ingredients when the prices of the ingredients and of the mixture are given. In the former case it is called **Alligation Medial**, and in the latter, **Alligation Alternate.**

252. In Examples of Alligation Medial, proceed thus :—

Rule. Multiply each price by the corresponding quantity, and divide the sum of the products so obtained by the sum of the quantities. The quotient will be the price of the mixture.

The *reason for the Rule* will be evident from the Example given below.

Ex. Find the price per maund of a mixture of 2 mds. of rice at R4 a maund, 3 mds. at R3 a maund, and 4 mds. at R2 a maund.

The price of 2 mds. at R4 $=2\times 4$ or R8.

.... 3 .... R3 $=3\times 3$ or R9.

.... 4 .... R2 $=4\times 2$ or R8.

$\therefore$ .... 9 ....of mixture = R25,

and $\therefore$ .... 1 .... = R$\frac{25}{9}$

= R2. 12*a.* $5\frac{1}{3}$ pies.

253. The Rule for working out Examples of Alligation Alternate may be deduced thus :—

First let there be only 2 ingredients at prices R12 and R20 per maund, to be mixed so that the price of the mixture may be R18 per maund.

Let $x : y$ or $\frac{x}{y}$ be the ratio of the quantities reqd.

Then by mixing x parts at R12, and y parts at R20, we shall have a mixture of $x+y$ parts at R18 ;

$$\therefore \frac{12\times x+20\times y}{x+y}=18 \text{ (by the preceding Rule) ;}$$

or multiplying both sides by $x+y$,

$$12\times x+20\times y=18\times x+18\times y\ ;$$

or subtracting $12 \times x$ from both sides,

$$20 \times y = 18 \times x - 12 \times x + 18 \times y\ ;$$

or subtracting $18 \times y$ from both sides,

$$20 \times y - 18 \times y = 18 \times x - 12 \times x,$$

$$\text{or } y \times (20 - 18) = x \times (18 - 12)\ ;$$

or dividing both sides by $y \times (18 - 12)$,

$$\frac{x}{y} = \frac{20 - 18}{18 - 12}\ ;$$

i. e., the quantity of each kind is proportional to the difference between the price of the other and that of the mixture.

Next, let there be several ingredients at prices R12, R14, R20 and R22.

Arrange the prices in the manner shown in the annexed figure, and join the prices of the ingredients in pairs such as 12 and 22, and 14 and 20, so that each pair may consist of one price below and one above the price of the mixture. Then for each pair, find the quantities as in the first case. Thus corresponding to the prices 12 and 22, we have the quantities $22 - 18$ and $18 - 12$, *i. e*, 4 and 6 respectively ; and similarly, corresponding to the prices 14 and 20, the quantities 2 and 4 respectively. And mixing each pair of quantities, we have a mixture of 10 parts at R18, whereof 4 parts are at R12 and 6 at R22, and another mixture of 6 parts at R18, whereof 2 parts are at R14 and 4 at R20.

	12	4
	14	2
18	20	4
	22	6

Now since each of these two mixtures of two ingredients has the given price R18, a mixture of any quantities of these mixtures will have the same price R18. Therefore a mixture of the 10 parts of the first mixture and the 6 parts of the second will give a mixture of 16 parts at R18, whereof 4 parts are at R12, 2 at R14, 4 at R20, and 6 at R22.

Hence we deduce the following general Rule :—

Rule. On one side of a vertical line place the price of the mixture, and on the other, the prices of the ingredients in descending order. Alligate the prices in pairs so that each pair may consist of one price below and one above that of the mixture, and so that every price may be joined with one or more others. Opposite each price, place the difference or the differences between the price or prices with which it is linked and the price of the mixture. The number or the sum of the numbers opposite to each price will represent the corresponding quantity.

Ex. Find the quantities when their prices are R2, R4, R6, R8 and R10 a seer respectively, so that the price of the mixture may be R7 a seer.

Proceeding by the Rule, we take to modes of linking which are marked I and II.

$$\begin{array}{c|c|c} & \multicolumn{2}{c}{\text{I}} \\ & 2 & 3 \\ & 4 & 1 \\ 7 & 6 & 1 \\ & 8 & (3+1) \\ & 10 & 5 \end{array} \qquad\qquad \begin{array}{c|c|c} & \multicolumn{2}{c}{\text{II}} \\ & 2 & 3 \\ & 4 & 1 \\ 7 & 6 & 3 \\ & 8 & 3 \\ & 10 & (5+1) \end{array}$$

In I, we link 2 and 10, 4 and 8, 6 and 8; and opposite to 8 we have two numbers 3 and 1 which are respectively the differences between 7 and the numbers 4 and 6 with which it is linked: and opposite to each of the others, we have one number.

In II, we link 2 and 10, 4 and 8, 6 and 10, and get two numbers opposite to 10.

We can verify each of the results thus :—

$$\frac{2\times3+4\times1+6\times1+8\times4+10\times5}{3+1+1+4+5}=\frac{98}{14}=7$$

$$\frac{2\times3+4\times1+6\times3+8\times3+10\times6}{3+1+3+3+6}=\frac{112}{16}=7.$$

Thus we see that we can have as many modes of mixing as there are modes of joining the prices two and two according to the Rule.

From the process of *alligating* or joining the prices, this method of operation is called Alligation.

Examples XLIX.

1. Find the price per maund of a mixture of 3 mds. of sugar at R15 a maund, 4 mds. at R13. 8*a.*, and 5 mds. at R11.

2. Find the price per dram of a mixture of 2 oz. of quinine at R12 an ounce, 3 oz. at R11. 8*a.* an ounce, and 4 oz. at R9 an ounce.

3. Find the price per maund of a mixture of 10 mds. of rice at R4 a maund, 12 mds. at R4. 8*a.*, and 8 mds. at R3. 4*a.*

4. Find the price per pound of a mixture of 20 lbs. at £1. 10*s.* a pound, 3qrs. at 18*s.* a pound, and 1 cwt. at £1. 7*s.* a pound.

5. Find the proportion of the ingredients when their prices are R2 and R4 per seer, so that the price of the mixture may be R3 per seer.

6. Find the proportion of the ingredients when their prices are ₨3, ₨5, and ₨6 per maund, so that the price of the mixture may be ₨4 per maund.

7. Find the proportion of the ingredients when their prices are ₨1. 8*a*. a seer, 12*a*. a seer, and ₨40 a maund, so that the price of the mixture may be ₨50 a maund.

8. The price of an alloy per seer is 14*a*., and the prices of its ingredients per seer are 12*a*., and ₨1. 4*a*. Find the proportion of the ingredients.

9. Find the proportion of the ingredients when their prices are ₨3, ₨4, ₨5, and ₨6 per seer, so that the price of the mixture may be ₨4 per seer.

CHAPTER XIV.

EXCHANGE.

254. Defs. The method of converting any sum of money of one country into an equivalent sum of money of another is called **Exchange.**

The **Par of Exchange** means the *intrinsic* value of a coin of one country as compared with that of another.

The **Rate of Exchange** means the *actual* value at any time of a coin of one country in terms of a coin of another.

Thus, a rupee weighs 180 grs. Troy, whereof $\frac{11}{12}$ths or 165 grs. are pure silver (Art. 159) ; and a shilling weighs $\frac{1}{66}$ of 1 lb. Troy or $\frac{12\times20\times24}{66}$ grs. whereof $\frac{37}{40}$ths that is $80\frac{8}{11}$ grs. are pure silver (Art. 142) : so that the quantity of silver contained in a rupee is a little more than what is contained in 2 shillings ; and assuming the expense of coinage to be the same in both cases, the intrinsic value of R1 : that of 1*s.* :: 165 : $80\frac{8}{11}$, *i. e.*, the intrinsic value of R1 is a little more than that of 2*s.* Hence at *par* R1 ought to exchange for a little more than 2*s.* But actually the expense of coinage and a variety of other causes produce a variable *rate of exchange*, a rupee exchanging sometimes for 2*s.*, sometimes for a little more and sometimes for a little less. At present a rupee is worth only a little more than one shilling.

Arbitration or Comparison of Exchanges is the method of finding the rate of exchange between the first and the last of a given number of places, when the rates between the first and the second, the second and the third, &c., of these places are known.

255. We have already in Art. 193 indicated the method of converting the money of one country to that of another. We shall here add a few more Examples.

Ex. 1. A person in Calcutta wishes to remit £6700 to his agent in London. What must he pay when the exchange is at 2*s.* $1\frac{1}{8}d.$ a rupee ?

Let x = sum reqd. in rupees.

Then $\because$ R1 is worth 2*s.* $1\frac{1}{8}d.$ or $\frac{67}{32}s.$,

and Rx are worth $(6700\times20)s.$,

$\therefore x : 1 :: 6700\times20 : \frac{67}{32}$;

whence $x=\frac{6700\times20\times32}{67}=64000$, the no. of rupees reqd.

Ex. 2. A person pays R64000 for a bill of exchange, at 2*s.* $1\frac{1}{8}d.$ per rupee. What is the amount of the bill ?

Let x = amount reqd. in pounds.

Then $\because$ R1 is worth $\frac{67}{32}s.$ or £$\frac{67}{640}$,

and R64000 are worth £x,

$\therefore x : \frac{67}{640} :: 64000 : 1$;

whence $x = \frac{67 \times 64000}{640} = 6700$, the no. of pounds reqd.

Ex. 3. A person has to pay R64000 for a bill of exchange for £6700. What is the rate of exchange?

Since R64000 = £6700

$$\text{R}1 = £\tfrac{6700}{64000}$$
$$= £\tfrac{67}{640}$$
$$= 2s.\ 1\tfrac{1}{8}d.$$

Ex. 4. If the exchange between London and Calcutta be at $2s.\ 1d.$ a rupee, and that between Calcutta and New York at R2·24 a dollar, what is the rate of exchange between London and New York?

$$\text{R}1 = 2s.\ 1d. = \tfrac{25}{12}s.;$$

and 1 dollar = R2·24

$$= \tfrac{224}{100} \times \tfrac{25}{12}s.$$
$$= \tfrac{56}{12}s. = 4s.\ 8d.$$

Ex. L.

1. A person in Calcutta wishes to remit £340 to his agent in London. What must he pay when the exchange is at $2s.\ 1\frac{1}{2}d.$ a rupee?

2. What is the value in Indian money of £500 when a rupee is worth $1s.\ 1d.$?

3. A person has to pay R4500. 8a. for a debt due to a merchant in London, when R1 = $1s.\ 9d.$ What is the amount of the debt in English money?

4. A person has to pay R6000 for a debt of £550 due to a creditor in London. What is the value of a rupee in English money?

5. When the exchange between England and India is $1s.\ 10d.$ per rupee, and between England and St. Petersburg, $2s.\ 9d.$ per ruble, what is the rate of exchange between India and St. Petersburg?

6. If the exchange between England and India, and between England and St. Petersburg be as in the preceding Example, and that between India and St. Petersburg be R$1\frac{1}{2}$ per ruble, is it more advantageous to a person in Calcutta to remit money directly to London, or circuitously through St. Petersburg?

CHAPTER XV.

SQUARE ROOT. CUBE ROOT.

SECTION I. SQUARE ROOT.

256. Defs. The **Square** of a given number is the product of that number multiplied by itself. It is also called the second power of that number (Art. 42), and is denoted by placing the figure 2 above the number a little to its right. Thus $9^2 = 9 \times 9 = 81$.

The **Square Root** of a given number is the number whose square is equal to the given number. It is denoted by the sign $\sqrt{\ }$ placed before the given number.

Thus $\sqrt{16} = 4$.

257. Since

$$\sqrt{1} = 1$$
$$\sqrt{100} = 10$$
$$\sqrt{10000} = 100$$
$$\sqrt{1000000} = 1000$$
$$\&c. = \&c.,$$

∴ the square root of any number
between 1 and 100 must consist of 1 figure,
.... 100 and 10000 .... 2 figures,
...... 10000 and 1000000 .... 3 ....,
and so on.

Hence if we place a dot over the figure in the units' place of any given number, and thence over every second figure to the left, the number of dots will be equal to the number of figures in the integral part of the root. Thus the number 2436 with the dots will stand thus, $2\dot{4}3\dot{6}$, shewing that there are two figures in the integral part of the root.

258. *To find the square root of a given number.*

By pointing the given number as in Art. 257, we can ascertain the number of figures in the integral part of the root. Now let us see how the root itself is to be found out.

Take any number 48.
Then $48^2 = 2304$
and also $= (40+8)^2 = (40 + 8) \times (40 + 8)$
$$= 40 \times (40 + 8) + 8 \times (40+8)$$
$$= 40^2 + 8 \times 40 + 8 \times 40 + 8^2$$
$$= 40^2 + 2 \times 8 \times 40 + 8^2.$$

.. In this last form in which 48^2 can be written, we see how the parts of the root enter into the composition of the power. Now let us see how we can obtain 48 or $40+8$ from the last expression. The first part of the root, *i. e.*, 40 is the square root of 40^2. Subtracting 40^2 from the given number, we have $2\times8\times40+8^2$ left. Now if we divide $2\times8\times40$ by 2×40, we get 8, the second figure in the root. And multiplying $2\times40+8$ by 8, we get $2\times40\times8+8^2$; and subtracting this product from $2\times8\times40+8^2$, the portion of the given number left after deducting 40^2, we have nothing more left.

If the root be a number consisting of more figures than two, for instance, if it be 483, it may be written as $480+3$, and its square as $480^2+2\times3\times480+3^2$. Here having found 4 and 8 as before, we may find the third figure 3 in the same way as above, by supposing 480 to stand in the place of the number 40 in the first case.

The process may be stated thus :—

$$
\begin{array}{r|l}
 & 40^2+2\times8\times40+8^2\ (40+8 \\
 & 40^2 \\ \hline
2\times40+8 & 2\times8\times40+8^2 \\
 & 2\times8\times40+8^2 \\ \hline
\end{array}
$$

or thus :—

$$
\begin{array}{r|l}
 & 1600+640+64\ (40+8 \\
 & 1600 \\ \hline
80+8 & 640+64 \\
 & 640+64 \\ \hline
\end{array}
$$

or thus (omitting cyphers, and representing the given number as one number without breaking it into parts) :—

$$
\begin{array}{r|l}
 & 2304\ (48 \\
 & 16 \\ \hline
88 & 704 \\
 & 704 \\ \hline
\end{array}
$$

Hence we deduce the following Rule :—

Rule. Place a dot over the figure in the units' place of the given number, and thence over every second figure to the left, thus dividing the number into several periods.

Find the greatest number whose square is not greater than the first period on the left : this will be the figure in the highest place in the root required. Place it in the form of a quotient, subtract its square from the first period, and to the remainder, if any, annex the figures in the next period in the given number. Divide the number thus obtained, omitting its last figure, by twice the part

of the root already obtained, annex the quotient to the part of the root already obtained and to the divisor, and multiply the resulting divisor by the quotient, and subtract the product from the number formed by the first remainder and the second period. To the remainder annex the next period, and proceed as before. Repeat this process for each successive period.

Ex. Extract the square root of 12769.

$$\begin{array}{r|l}
 & \dot{1}2\dot{7}6\dot{9}\ (113 \\
 & 1 \\
\hline
21 & 27 \\
 & 21 \\
\hline
223 & 669 \\
 & 669 \\
\hline
\end{array}$$

259. Since $\sqrt{24\cdot66}=\sqrt{\frac{2466}{100}}=\frac{\sqrt{2466}}{10}$,

$$\sqrt{1\cdot634}=\sqrt{\frac{1634}{1000}}=\sqrt{\frac{16340}{10000}}= \qquad 100$$

$$\sqrt{\cdot0066}=\sqrt{\frac{66}{10000}}=\frac{\sqrt{66}}{100},$$

&c. = &c. = &c. ;

$\therefore$ it follows that in extracting the square root of a decimal with or without an integral part, if we make the number of decimal places even by annexing a cypher when necessary, and extract the square root of the resulting number regarded as an integer, and in the root thus obtained, point off a number of decimal places equal to half the number of those in the square, we shall obtain the true root.

Again $\because$ $\sqrt{12}=\sqrt{\frac{1200}{100}}=\frac{\sqrt{1200}}{10}$,

$$\sqrt{133}=\sqrt{\frac{1330000}{10000}}=\frac{\sqrt{1330000}}{100},$$

$$\sqrt{1\cdot25}=\sqrt{\frac{12500}{10000}}=\frac{\sqrt{12500}}{100},$$

&c. = &c. = &c. ;

$\therefore$ where the operation does not terminate, we can carry it on to any length by annexing periods of two cyphers to the given number and obtaining one decimal place in the root corresponding to every such period.

Ex. 1. Extract the square root of 6·25.

$$\begin{array}{r|l}
 & \dot{6}\cdot2\dot{5}\ (2\cdot5 \\
 & 4 \\ \hline
45 & 225 \\
 & 225 \\ \hline
\end{array}$$

Ex. 2. Extract the square root of 2 to 3 places of decimals.

$$\begin{array}{r|l}
 & 2\cdot000000\ (1\cdot414 \\
 & 1 \\ \hline
24 & 100 \\
 & 96 \\ \hline
281 & 400 \\
 & 281 \\ \hline
2824 & 11900 \\
 & 11296 \\ \hline
 & 604
\end{array}$$

260. The square root of a fraction may be found thus :—

Let it be required to find the square root of $\frac{3}{5}$.

Now $$\sqrt{\tfrac{3}{5}} = \sqrt{\frac{3\times5}{5\times5}} = \frac{\sqrt{(3\times5)}}{5} = \frac{\sqrt{15}}{5};$$

so that the square root of $\frac{3}{5}$ is found by finding that of 15 and dividing the result by 5.

Or $\sqrt{\frac{3}{5}}$ may be found by reducing $\frac{3}{5}$ to a decimal, and then extracting the square root of that decimal by the method indicated in Art. 259.

261. Since the number of square units in a square = the square of the number of linear units in a side,

∴ the number of linear units in a side of a square = the square root of the number of square units in the area.

Ex. A square plot of land measures 1 acre. Find the length of its side in yards.

$$1 \text{ ac.} = 4\times40\times\tfrac{121}{4} \text{ sq. yds.} = 4840 \text{ sq. yds.};$$

∴ the length reqd. = $\sqrt{4840}$ yds.

Now $\sqrt{4840}=69\cdot57$...as shewn below :—

```
          4840.0000 (69·57
          36
 129     |1240
         |1161
 1385     |7900
          |6925
 13907      |97500
            |97349
               151
```

∴ the length reqd. $=69\cdot57$......yds. approximately.

The result can be verified thus :—

The area of the square whose side is $69\cdot57$ yds.

$=(69\cdot57\times69\cdot57)$ sq. yds.

$=4839\cdot9849$ sq. yds.

which differs from 1 ac. by $(4840-4839\cdot9849)$ sq. yds., *i. e.*, by $\cdot0151$ sq. yd., *i. e.*, by less than $\frac{1}{60}$ sq. yd.

If we take a few more decimal places in the root, the difference will become smaller still. In this way, we can find the side to any degree of approximation.

Examples LI.

1. Find the square roots of

(1) 441 ; 961 ; 9801 ; 7921 ; 12321 ; 49284.

(2) 1681 ; 2601 ; 6241 ; 4761 ; 110889.

(3) 625 ; 1225 ; 2025 ; 3025 ; 4225 ; 5625.

(4) 7225 ; 9025 ; 15129 ; 54756 ; 18225.

(5) 1522756 ; 1234321 ; 4937284 ; 1002001.

(6) 11108889 ; 4080400 ; 25010001.

2. Extract (to 4 places of decimals where the operation does not terminate) the square roots of

(1) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

(2) 11, 12, 13, 14, 99, 77, 166.

(3) $1\cdot2$, $2\cdot3$, $3\cdot4$, $2\cdot34$, $1\cdot44$.

(4) $\cdot1$, $\cdot002$, $\cdot0004$, $100\cdot001$, $\cdot9$.

(5) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$.

(6) $\frac{2}{3}$, $\frac{3}{11}$, $\frac{25}{144}$, $\frac{26}{361}$, $\frac{25}{225}$.

3. A square plot of land contains 50 bghs. Find the length of its side.

4. A rectangular plot of land whose length is equal to its breadth, contains 5 acres. What is its length ?

5. A rectangular piece of land whose length is equal to its breadth, contains as much land as another piece, which is 5 bghs. long and 2 bghs. broad. Find the length of the former.

6. A rectangular piece of land whose length is double of its breadth, measures 800 ft. along its length. What must be the length of another field whose breadth is equal to its length, in order that it may contain the same area?

7. The perimeter of a rectangular field whose breadth is half of its length, is 6 bghs. Find the perimeter of a square plot of land containing the same area.

8. Two square plots of land contain respectively 2 square miles and 10 acres. Find the difference between their sides.

SECTION II. CUBE ROOT.

262. Defs. The **Cube** of a number is the continued product of that number repeated as a factor thrice. This product is also called the third power of that number (Art. 42), and is denoted by placing the figure 3 above the number a little to its right.

Thus $8^3 = 8 \times 8 \times 8 = 512$.

The **Cube Root** of a given number is the number whose cube is equal to the given number. It is denoted by the sign $\sqrt[3]{\ }$ placed before the given number.

Thus $\sqrt[3]{125} = 5$.

263. Since

$$\begin{aligned} \sqrt[3]{1} &= 1 \\ \sqrt[3]{1000} &= 10 \\ \sqrt[3]{1000000} &= 100 \\ \sqrt[3]{1000000000} &= 1000 \\ \&c. &= \&c., \end{aligned}$$

∴ the cube root of any number
between 1 and 1000 must consist of 1 figure
........1000 and 1000000..................2 figures
...1000000 and 1000000000.............3..........
and so on.

Hence if we place a dot over the figure in the units' place of any given number, and thence over every third figure to the left, the number of dots will be equal to the number of figures in the integral part of the root. Thus the number 2346785 with the dots will stand thus, $\dot{2}34\dot{6}78\dot{5}$, shewing that there are three figures in the integral part of the root.

264. *To find the cube root of a given number.*

By pointing the given number as in Art. 263, we can ascertain the number of figures in the integral part of the root. Now let us see how the root itself is to be found out.

Take any number 18.

Then $18^3 = 5832$

and also $= (10+8)^3 = (10+8)\times(10+8)\times(10+8)$

$$
\begin{aligned}
&= (10^2+2\times10\times8+8^2)\times(10+8)\\
&= 10\times(10^2+2\times10\times8+8^2)\\
&\quad +8\times(10^2+2\times10\times8+8^2)\\
&= 10^3+2\times10^2\times8+8^2\times10\\
&\quad +8\times10^2+2\times8^2\times10+8^3\\
&= 10^3+3\times10^2\times8+3\times10\times8^2+8^3.
\end{aligned}
$$

In this last form in which 18^3 can be written, we see how the parts of the root enter into the composition of the power. Now let us see how we can obtain 18 or $10+8$ from the last expression. The first part of the root, *i. e.*, 10 is the cube root of 10^3. Subtracting 10^3 from the given number, we have $3\times10^2\times8+3\times10\times8^2+8^3$ left. Now if we divide $3\times10^2\times8$ by 3×10^2, we get 8, the second figure in the root. And multiplying $3\times10^2+3\times10\times8+8^2$ by 8, we get $3\times10^2\times8+3\times10\times8^2+8^3$, and subtracting this product from $3\times10^2\times8+3\times10\times8^2+8^3$, the portion of the given number left after deducting 10^3, we have nothing more left.

If the root be a number consisting of more figures than two, for instance, if it be 185, it may be written as $180+5$, and its cube as $180^3+3\times180^2\times5+3\times180\times5^2+5^3$. Here having found 1 and 8 as before, we can find the third figure 5 in the same way as above, by supposing 180 to stand in the place of the number 10 in the first case.

The process may be stated thus :—

$$
\begin{array}{ll}
 & 10^3+3\times10^2\times8+3\times10\times8^2+8^3\,(10+8 \\
 & 10^3 \\
3\times10^2 & \\
\quad +3\times10\times8 & 3\times10^2\times8+3\times10\times8^2+8^3 \\
\qquad +8^2 & \\
\hline
3\times10^2+3\times10\times8+8^2 & 3\times10^2\times8+3\times10\times8^2+8^3
\end{array}
$$

or thus :—

$$
\begin{array}{rl}
 & 1000+2400+1920+512\ (10+8 \\
 & 1000 \\
3\times10^2=300 & 2400+1920+512 \\
3\times10\times8=240 & \\
8^2=\ \ 64 & \\
\hline
300+240+64 & \underline{2400+1920+512}
\end{array}
$$

or thus (omitting cyphers and representing the given number as one number without breaking it into parts) :—

$$
\begin{array}{rl}
 & 5832\ (18 \\
 & 1 \\
3\times 10^2 = 300 & 4832 \\
3\times 10\times 8 = 240 & \\
8^2 = \underline{\ 64} & \\
604 & \underline{4832}
\end{array}
$$

Hence we deduce the following Rule :—

Rule. Place a dot over the figure in the units' place of the given number, and thence over every third figure to the left, thus dividing the number into several periods.

Find the greatest number whose cube is not greater than the first period on the left : this will be the figure in the highest place in the root required. Place it in the form of a quotient, subtract its cube from the first period, and to the remainder annex the figures in the next period in the given number. Divide the number thus obtained by thrice the square of the part of the root already obtained (regarded as so many tens), in order to find by trial the greatest number such that the product of that number multiplied by the value of $3\times$the square of the part of the root already obtained (regarded as so many tens)$+3\times$the part of the root already obtained (regarded as so many tens)$\times$that number$+$the square of that number, is not greater than the dividend under consideration. That number will be the next figure in the root. Subtract the product above mentioned from the number composed of the first remainder and the second period. To the remainder annex the third period if any, and proceed as before. Repeat this process for each successive period.

Ex. Find the cube root of 6859.

$$
\begin{array}{rl}
 & \dot{6}85\dot{9}\ (19 \\
 & 1 \\
3\times 10^2 = 300 & 5859 \\
3\times 10\times 9 = 270 & \\
9^2 = \underline{\ 81} & \\
651 & \underline{5859}
\end{array}
$$

265. Since $\sqrt[3]{18\cdot775} = \sqrt[3]{\dfrac{18775}{1000}} = \dfrac{\sqrt[3]{18775}}{10}$

$$\sqrt[3]{2\cdot5786} = \sqrt[3]{\frac{2578600}{1000000}} = \frac{\sqrt[3]{2578600}}{100}$$

$$\sqrt[3]{\cdot006781} = \sqrt[3]{\frac{6781}{1000000}} = \frac{\sqrt[3]{6781}}{}$$

&c. &c. =

∴ it follows that in extracting the cube root of a decimal with or without an integral part, if we make the number of decimal places equal to 3 or a multiple of 3 by affixing cyphers when necessary, and extract the cube root of the resulting number regarded as an integer, and in the root thus obtained, point off a number of decimal places equal to one-third of the number of those in the cube, we shall obtain the true root.

$$\text{Again}\quad \sqrt[3]{13} = \sqrt[3]{\frac{13000}{1000}} = \frac{\sqrt[3]{13000}}{10}$$

$$\sqrt[3]{14\cdot4} = \sqrt[3]{\frac{14400000000}{1000000000}} = \frac{\sqrt[3]{14400000000}}{1000}$$

$$\sqrt[3]{1\cdot26} = \sqrt[3]{\frac{1260000}{1000000}} = \frac{\sqrt[3]{1260000}}{100}$$

&c. = &c. = &c.;

when the operation does not terminate, we can carry it on to any length by annexing periods of three cyphers to the given number, and obtaining one decimal place in the root corresponding to every such period.

Ex. 1. Extract the cube root of ·12500.

$$\sqrt[3]{\cdot12500} = \sqrt[3]{\frac{125000}{1000000}} = \frac{\sqrt[3]{125000}}{100}$$

$$\begin{array}{l} 12\dot{5}00\dot{0}\ (50 \\ \underline{125} \end{array}$$

∴ ·50 is the root reqd.

Ex. 2. Find the cube root of 4 to 2 places of decimals.

$$\begin{array}{rr} & 4\cdot000000\ (\ 1\cdot58 \\ & \underline{1} \\ 3\times10^2=300 & 3000 \\ 3\times10\times5=150 & \\ 5^2=\ \ 25 & \\ \hline 475 & \underline{2375} \\ 3\times150^2=67500 & 625000 \\ 3\times150\times8=\ \ 3600 & \\ =\ \ \ 64 & \underline{569312} \\ \hline 71164 & 55688 \end{array}$$

266. The cube root of a fraction may be found thus :—

Take as an example, $\sqrt[3]{\frac{2}{3}}$.

$$\text{Now, } \sqrt[3]{\tfrac{2}{3}} = \sqrt[3]{\frac{2\times3\times3}{3\times3\times3}} = \frac{\sqrt[3]{18}}{3};$$

so that the cube root of $\frac{2}{3}$ is found by finding that of 18, and dividing the result by 3.

Or $\sqrt[3]{\frac{2}{3}}$ may be found by reducing $\frac{2}{3}$ to a decimal, and extracting the cube root of that decimal by the method indicated in Art. 265.

267. Since the number of solid units in a cube = the cube of the number of linear units in an edge of it,

∴ the number of linear units in an edge of a cube = the cube root of the number of solid units in the cube.

Ex. A cubical room contains 1728 cub. feet. Find its height.

Height reqd. in feet $= \sqrt[3]{1728}$

$= 12.$

Examples LII.

1. Find the cube roots of

(1) 1331 ; 1728 ; 2197 ; 2744.

(2) 3375 ; 4096 ; 4913 ; 5832 ; 6859.

(3) 15625 ; 1367631 ; 10941048.

2. Extract (to 2 places of decimals where the operation does not terminate) the cube roots of

(1) 1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9.

(2) $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$.

(3) ·1 ; ·2 ; ·3 ; ·001 ; ·01.

(4) 123·456 ; 166·375 ; 287·496.

3. Find the length of a cubical room which contains 2744 cub. ft.

4. A cube contains 6859 cub. in. Find its edge.

ANSWERS TO THE EXAMPLES.

Examples I. Pages 12, 13.

1. (1) 10; 12; 15; 19; 28; 44; 56; 61; 84; 92.

(2) 101; 110; 154; 300; 405; 560; 774.

(3) 1001; 2051; 3263; 4000; 5500; 6780.

(4) 100001; 200300; 306709; 456004; 567432.

(5) 2000001; 3000029; 4000560; 5600074; 6754321.

(6) 3000000000000; 4000000000005; 5000000000708; 7000913579135.

(7) 19000000000000000000; 20000000000000000024; 31000000556709827520. (8) 100001; 203003; 561720; 1530612. (9) 20000002; 30507009; 56432178.

(10) 2165016718.

2. (1) Eighteen; twenty; thirty-seven; fifty-eight; sixty-nine; eighty-five; ninety-seven.

(2) Two thousand and three; three hundred and forty; four hundred and fifty-six; six hundred and ninety; seven hundred and eight; nine hundred and ninety-one.

(3) One hundred and nine; two thousand and twenty-nine; three thousand six hundred and ninety; four thousand eight hundred and sixty-two.

(4) One hundred and two thousand and thirty; two hundred and thirty thousand four hundred and fifty; three hundred thousand and four; seven hundred and forty-five thousand six hundred and twenty-one.

(5) One hundred and twenty-three millions, four hundred and fifty-six thousand seven hundred and eighty-nine; nine hundred and eighty-seven millions, six hundred and fifty-four thousand three hundred and twenty-one; one hundred and two millions, thirty thousand four hundred and five.

(6) Two thousand four hundred and sixty-eight millions, one hundred and one thousand two hundred and fourteen; two hundred

and forty-eight thousand one hundred and sixty-three millions, two hundred and sixty-four thousand one hundred and twenty-eight.

(7) Fifty billions, one hundred thousand two hundred millions, three hundred thousand four hundred; thirty-six thousand nine hundred and twelve billions, one hundred and fifty-one thousand eight hundred and twenty-one millions, two hundred and forty-two thousand seven hundred and thirty.

(8) Two trillions, three hundred and five thousand eight hundred and forty-three billions, eight thousand one hundred and thirty-nine millions, nine hundred and fifty-two thousand one hundred and twenty-eight; one hundred and thirty-seven thousand four hundred and thirty-eight millions, six hundred and ninety-one thousand three hundred and twenty-eight.

3. One lac two thousand and thirty; two lacs thirty thousand four hundred and fifty; three lacs and four; seven lacs forty-five thousand six hundred and twenty-one.

Twelve crores thirty-four lacs fifty-six thousand seven hundred and eighty-nine; ninety-eight crores seventy-six lacs fifty-four thousand three hundred and twenty-one; ten crores twenty lacs thirty thousand four hundred and five.

4. $10+8$; 20; $30+7$; $50+8$; $60+9$; $80+5$; $90+7$.
$200+3$; $300+40$; $400+50+6$; $600+90$; $700+8$; $900+90+1$.
$1000+9$; $2000+20+9$; $3000+600+90$; $4000+800+60+2$.

5. (1) XXV; XXXIII; XLVI; LXXXVII; XCIX.
(2) CI; CCXX; CCCXIV; DXVI; CMXCIX.
(3) MI; MDCCCLVI; MDCCCLXIV.

6. (1) 27; 34; 45; 46. (2) 99; 301; 1040; 650. (3) 1856; 1582; 1009.

Examples II. Pages 16, 17.

(1) 45. (2) 135. (3) 225. (4) 459.
(5) 2919. (6) 3330. (7) 2643. (8) 3108.
(9) 2997. (10) 46998. (11) 25908. (12) 3053.
(13) 37170. (14) 34655782. (15) 4327333. (16) 98549.
(17) 2420346420. (18) 1607804.

2. 6221449083221. **3.** 536468087. **4.** 14805; 6840.

5. 1269184; 295424. **6.** 546. **7.** 280; 92.

8. 171; 231; 256; 512; 360; 1580.

Examples III. Pages 20, 21.

1. (1) 6. (2) 9. (3) 10. (4) 11.
(5) 18. (6) 78. (7) 669. (8) 10.
(9) 63. (10) 1. (11) 178882.
(12) 433430. (13) 90909. (14) 164421.
(15) 90909. (16) 801244. (17) 71533517.
(18) 864197532. (19) 9999. (20) 10000998.

2. (1) 4500. (2) 495000. (3) 59700000.

3. 22 ; 45 ; 64 ; 120 ; 129 ; 110 ; 1284 ; 11112 ; 133334 ; 49995.

4. 2000000 ; 200000.

5. (1) 10. (2) 10. (3) 10. (4) 4. (5) 3.
(6) 8. (7) 6. (8) 7. (9) 7. (10) 5.

6. 0 ; 9 ; 4 ; 332 ; 1018 ; 4 ; 544 ; 261 ; 233 ; 140.

7. (1) 167. (2) 12.

8. 33 ; 132 ; 231 ; 139 ; 422 ; 2355.

Examples IV. Pages 29, 30.

1. (1) 492 ; 615 ; 738 ; 984 ; 1107. (2) 3192 ; 3648 ; 4104 ; 4560 ; 5016. (3) 2367 ; 4734 7101 ; 9468 ; 11835.
(4) 617283945 ; 1234567890 ; 1851851835 ; 2469135780 ; 3086419725. (5) 7901234568 ; 11851851852 ; 15802469136; 19753086420 ; 23703703704. (6) 56447784 ; 80958006 ; 57561885. (7) 97406784 ; 121851072 ; 98517888.
(8) 405811215 ; 505009512 ; 450901350.
(9) 3276941063 ; 3677951156 ; 3605040230.
(10) 82519021020 ; 245939043040 ; 409359065060.
(11) 14446089217728 ; 75640328065008.
(12) 121932631112635269 ; 134113580248594.
(13) 109820000000 ; 82678000000 ; 55530000000.
(14) 516760458000000 ; 688741400000.
(15) 450851566000 ; 539061655000.

2. (1) 1975296. (2) 18172480. (3) 36344960.
(4) 126419200. (5) 5598842958. (6) 3258960.

3. (1) 362880. (2) 46080. (3) 29160. (4) 19019.

4. (1) 285. (2) 2025. (3) 2109375.

5. 1000000000000; 24000000000000.

6. 9999000000000; 19998000000000.

7. 891000000000.

Examples V. Pages 37, 38.

1. (1) 617; 411, rem. 1; 308, rem. 2; 246, rem. 4.
   (2) 1152; 864; 691, rem. 1; 576.
   (3) 1135, rem. 3; 946, rem. 2; 811, rem. 1; 709, rem. 6.
   (4) 11272, rem. 6; 9863, rem. 6; 8767, rem. 7; 7891.
   (5) 24691357, rem. 4; 12345678, rem. 9; 8230452, rem. 9; 6172839, rem. 9; 4938271, rem. 14.
   (6) 123456790, rem. 1; 82304526, rem. 9; 61728395 rem. 1; 49382716, rem. 1; 41152263; 1130, rem. 9.
   (7) 271, rem. 213; 189, rem. 183; 266, rem. 99.
   (8) 156, rem. 372; 125, rem. 81; 154, rem. 564.
   (9) 2474, rem. 33; 1988, rem. 51; 2226, rem. 303.
   (10) 5006, rem. 753; 4461, rem. 19; 4551, rem. 217.
   (11) 79314, rem. 721; 26612, rem. 521; 15988, rem. 1721.
   (12) 99720, rem. 10128; 19045, rem. 5103.
   (13) 8000051200, rem. 65145; 8000000, rem. 9012345.
   (14) 1386, rem. 4600; 1841, rem. 5300; 2742, rem. 1000.
   (15) 900090009, rem. 1; 90009000, rem. 10000; 9000900, rem. 10000.
   (16) 100010001; 10001000, rem. 1111; 11112222, rem. 3333; 1111222, rem. 23331.
2. (1) 1728; 1152; 864; 691, rem. 1; 576.
   (2) 3394, rem. 5; 1697, rem. 3; 1131, rem. 7; 848, rem. 11.
   (3) 500000; 333333, rem. 1; 250000; 200000; 166666, rem. 4; 142857, rem. 1.
   (4) 13888888, rem. 7; 12345679; 11111111, rem. 1; 10101010, rem. 1; 9259259, rem. 3.
   (5) 170940, rem. 2; 158730, rem. 2; 148148, rem. 2; 138888, rem. 14; 130718, rem. 16; 123456, rem. 14.

(6) 1754385, rem. 18; 1666666, rem. 13; 1587301, rem. 12; 151515, rem. 11; 1388888, rem. 21.

(7) 2739197559, rem. 22; 2656191573, rem. 1; 2578068291, rem. 16; 2504409197, rem. 15.

(8) 587643, rem. 6; 560932, rem. 4; 514187, rem. 36; 503694, rem. 6; 483941, rem. 2n

3. 99, rem. 99902; 9890, rem. 209.

4. 1, rem. 2000000.

5. 111111, rem. 100000; .10000.

Examples VI. Pages 49, 50.

1. (1) 6; 16; 12; 16; 14; 19; 19; 23.

(2) 12; 19; 25; 16; 19; 17; 19; 19.

(3) 15; 8; 4; 11; 17; 289; 121; 361.

(4) 38; 34; 28; 43. (5) 291; 237; 438; 213. (6) 202; 321; 453; 724. (7) 12; 18; 15; 863. (8) 4; 4; 6. (9) 9; 33.

2. (1) 6. (2) 12. (3) 5. (4) 36. (5) 19. (6) 66.

3. (1) 108; 42; 126; 126; 234; 2185; 595; 931.

(2) 234; 858; 225; 2023; 10830; 1104.

(3) 9372; 6636; 15554. (4) 1999998; 1294125.

(5) 135480701236261 41.

4. (1) 2520. (2) 45045. (3) 1680. (4) 6350400. (5) 4200.

(6) 360. (7) 1085040. (8) 803440. (9) 114578640.

(10) 139230. (11) 457200. (12) 118800.

5. (1) $2\times2\times23$; $2\times2\times3\times3\times3$; 3×37; 7×17; $3\times3\times19$; $3\times3\times3\times7$.

(2) $2\times2\times3\times17$; $2\times2\times2\times3\times3\times3$; $2\times2\times5\times13$; 17×17; $2\times2\times2\times2\times19$.

(3) $2\times2\times3\times3\times3\times3$; 19×19; $2\times3\times67$; 13×31; $2\times2\times101$.

(4) $2\times7\times29$; 11×37; 5×83; $2\times2\times2\times2\times2\times2\times2\times2\times2$; $2\times3\times103$.

(5) $3\times11\times23$; 11×71; $2\times5\times79$; $2\times2\times3\times3\times23$; $3\times3\times3\times5\times7$.

(17) $5\frac{9511}{32032}$. (18) $\frac{871}{1170}$.

2. (1) $1\frac{17}{60}$. (2) $\frac{1237}{1260}$. (3) $\frac{241}{420}$. (4) $\frac{3589}{3960}$.
(5) $1\frac{1}{4}$. (6) 1. (7) $1\frac{121}{280}$. (8) $52\frac{179}{2520}$.
(9) $\frac{73}{126}$. (10) $5\frac{11}{120}$. (11) $31\frac{23}{70}$. (12) $108\frac{47}{264}$.
(13) $29\frac{229}{440}$. (14) $29\frac{1297}{1400}$. (15) $84\frac{683}{1224}$.

Examples XI. Page 78.

1. (1) $\frac{1}{6}$. (2) $\frac{1}{15}$. (3) $\frac{1}{10}$. (4) $\frac{1}{12}$.
(5) $\frac{1}{20}$. (6) $\frac{1}{30}$. (7) $\frac{3}{20}$. (8) $\frac{1}{30}$.
(9) $\frac{1}{48}$. (10) $4\frac{1}{2}$. (11) $3\frac{1}{12}$. (12) $4\frac{53}{60}$.
(13) $20\frac{19}{45}$. (14) $24\frac{2}{3}$. (15) $17\frac{61}{88}$. (16) $\frac{23}{116}$.
(17) $81\frac{2}{11}$. (18) $1\frac{1}{30}$. (19) $1\frac{1}{3}$. (20) $11\frac{2}{3}$.

2. (1) $\frac{31}{60}$. (2) $\frac{104}{315}$. (3) $1\frac{13}{150}$. (4) $\frac{49}{120}$.
(5) $\frac{79}{462}$. (6) $\frac{1337}{8300}$.

3. $\frac{1}{2}$. 4. $\frac{8}{15}$. 5. $\frac{1}{30}$. 6. $\frac{5}{8}$.

Examples XII. Page 80.

1. (1) $\frac{1}{3}$. (2) $\frac{1}{2}$. (3) 1. (4) $\frac{1}{4}$.
(5) $\frac{5}{14}$. (6) $\frac{5}{6}$. (7) $\frac{3}{14}$. (8) $\frac{5}{16}$.
(9) $\frac{9}{35}$. (10) $1\frac{1}{3}$. (11) $\frac{9}{28}$. (12) $\frac{1}{16}$.

2. (1) $\frac{1}{30}$. (2) $\frac{1}{9}$. (3) $\frac{4}{539}$. (4) $\frac{3}{14}$.
(5) $\frac{49}{64}$. (6) $\frac{1}{50}$.

3. (1) $1\frac{7}{15}$. (2) $1\frac{113}{210}$. (3) $3\frac{91}{132}$.

Examples XIII. Pages 81, 82.

1. (1) 2. (2) 3. (3) $\frac{9}{16}$. (4) $\frac{3}{8}$.
(5) 9. (6) $\frac{2}{3}$. (7) 9. (8) $3\frac{3}{7}$.
(9) $\frac{15}{64}$. (10) $1\frac{3}{7}$. (11) 72. (12) $1\frac{82}{243}$.

2. $2\frac{2}{7}$. 3. 20. 4. 9.

Examples XIV. Pages 85—88.

I. 3. $\frac{82}{91}$. 4. $\frac{9}{250}$. 5. 1. 6. The former.

II. 1. $\frac{27}{35}$, $\frac{8}{35}$. 2. 30 feet. 3. 45 years, 20 years, and 15 years.
4. 30, 6, 5 5. $\frac{113}{120}$. 6. $\frac{7}{12}$.

III. 1. $\frac{1}{12}$. 2. $\frac{5}{96}$. 3. $\frac{17}{30}$. 4. 1800, 400. 5. $\frac{3}{20}$. 6. 36, 8.

IV. 2. $\frac{16}{81}$. 3. $2\frac{1}{2}$. 4. 14 years and 10 years. 5. $\frac{7}{20}$. 6. 240.

V. **3.** 20. **4.** $\frac{4}{9}$. **5.** $\frac{35}{44}$. **6.** $\frac{3}{8}$.

VI. **1.** (1) $\frac{10}{81}$. (2) $1\frac{1}{8}$. **2.** $\frac{1}{10}$, $\frac{3}{10}$. **3.** 36, 18, 12. **4.** $2 \times 3\frac{1}{2}$.
5. $\frac{8}{9}$, $\frac{4}{9}$. **6.** 80 parts.

Examples XV. Pages 92, 93.

(1) ·3 ; ·7 ; ·05 ; ·66 ; 5·05 ; 660·69 ; 100·01.

(2) ·001 ; 99·009 ; 123·045.

(3) 1000000·000001 ; 5000050·002053.

2. (1) Two hundredths ; one and three hundredths ; twenty-one and twelve hundredths ; one and one ten-thousandth ; twenty and two ten-thousandths.

(2) One hundred and twenty-three and four hundred and fifty-six thousandths ; seven thousand eight hundred and ninety-one and one thousand one hundred and twelve hundred-thousandths ; thirteen and one thousand five hundred and seventeen ten-thousandths.

(3) One millionth ; fifty hundred-millionths ; five hundred hundred-thousandths.

3. (1) $\frac{1}{10}$; $\frac{12}{100}$; $12\frac{34}{100}$; $567\frac{8}{10}$; $\frac{1}{100}$.

(2) $\frac{1}{10000}$; $\frac{2}{1000}$; $\frac{3}{100}$; $100\frac{2}{1000}$.

(3) $351\frac{970}{1000}$; $\frac{200}{100000}$; $12\frac{321}{1000}$.

4. (1) $\frac{1}{4}$; $2\frac{1}{2}$; $\frac{1}{400}$; $\frac{1}{400}$. (2) $56\frac{16}{25}$; $72\frac{7}{2500}$; $1\frac{1}{500}$.

(3) $\frac{416}{125}$, $17\frac{7}{25}$, $\frac{2}{625}$, $61\frac{16}{25}$.

5. (1) ·1, ·5, 2·3, 5·3, 9·2. (2) ·0011, 32·109, 68·14, 57·2.

(3) 24·69, 1234·56789, 1·00200.

6. (1) 3, 30, 300. (2) ·03, ·3, 3. (3) 2000·020, 200002.

(4) 1560, 15600. (5) 202, 2020.

7. (1) ·003, ·000003. (2) ·03156, ·003156.

(3) ·35671, 356·71. (4) 987·65005, 9·8765005.

(5) ·031415, ·00031415.

Examples XVI. Pages 94, 95.

1. (1) 1371·65295. (2) 1234·62345. (3) 12·061623.
(4) 13703·70569. (5) 1583·00955. (6) 80641·003579.

2. (1) 200·387053. (2) ·616077. (3) 1050000·170001.

3. (1) 446·806. (2) 755·5863. (3) 1017·191.
(4) ·00535823. (5) 2596·217. (6) 388·85.

Examples XVII. Pages 95, 96.

1. (1) 44·37. (2) 44·44. (3) 1.97. (4) 1·28.
(5) 18·8944. (6) 643·9904.

2. (1) ·9. (2) 2·97. (3) ·999999. (4) 999999·999999.
(5) 6·3. (6) ·81.

3. (1) ·014704380. (2) ·29001. (3) 1·3653. (4) 46·559.
(5) 96·656. (6) 5594·5056.

Examples XVIII. Page 97.

1. (1) 108·78. (2) 330·5565. (3) 5·29.
(4) ·3136. (5) 1074·176103. (6) 35·940476.

2. (1) 1. (2) ·056. (3) ·1. (4) 44·89. (5) 5. (6) ·18.

3. (1) ·004096. (2) ·196928646. (3) 45722·668222.
(4) ·00000000003752. (5) ·66. (6) 5·14411722.

Examples XIX. Page 99.

1. (1) ·24. (2) 170. (3) 2·69. (4) 6·712.
(5) 30300. (6) ·003. (7) 1133·67. (8) 13490700.
(9) 1179000. (10) 2532. (11) 12132. (12) 420.

2. (1) 18·6423. (2) 659204·3478. (3) 5778·2962.
(4) ·0003. (5) 3·6366. (6) ·0002. (7) ·9348.
(8) ·2777. (9) 1115·7190. (10) 9·5890.

3. (1) 1000. (2) 10 (3) 100. (4) ·011. (5) 100000. (6) 19.

4. 252. **5.** 560. **6.** ·08125.

Examples XX. Page 104.

1. (1) ·5, ·25, ·125, ·0625, ·03125. (2) ·5, ·2, ·75, ·2 ·3.
(3) ·2, ·04, ·008, ·0016. (4) ·75, ·8, ·625, ·8, ·04.
(5) ·44, ·9765625, ·8, ·4. (6) ·59375, ·32, ·25, ·375.

2. (1) ·24324, ·30769, ·46666, ·53333, ·90909.
(2) ·33333, ·14285, ·11111, ·09090, ·07692.
(3) ·13333, ·17647, ·21052, ·23809, ·26086.
(4) ·91666, ·92307, ·92857, ·93333.
(5) ·95454, ·95652, ·95833, ·92307.
(6) 7·42857, 8·44444, ·05785, ·04733.

3. (1) $\cdot\dot{4}2857\dot{1}$, $\cdot\dot{5}$, $\cdot\dot{8}\dot{1}$, $\cdot\dot{8}4615\dot{3}$, $\cdot9\dot{2}8571\dot{4}$, $\cdot9\dot{3}$.

(2) $\cdot\dot{3}8461\dot{5}$, $\cdot\dot{9}5238\dot{0}$, $\cdot8\dot{3}$, $\cdot\dot{5}7142\dot{8}$.

(3) $\cdot\dot{4}2857\dot{1}$, $\cdot\dot{4}2857\dot{1}$, $\cdot\dot{6}$, $\cdot\dot{4}\dot{5}$. (4) $\cdot\dot{7}$, $\cdot2\dot{1}4285\dot{7}$, $\cdot02\dot{2}8571\dot{4}$, $\cdot\dot{2}$.

(5) $\cdot\dot{3}$, $\cdot\dot{4}2857\dot{1}$, $\cdot\dot{5}3846\dot{1}$, $\cdot\dot{8}\dot{1}$. (6) $\cdot\dot{7}$, $\cdot\dot{6}\dot{9}$, $\cdot08\dot{3}$, $\cdot01\dot{6}$.

4. (1) $\frac{7}{9}$, $\frac{89}{99}$, $\frac{9}{111}$, $\frac{31}{33}$, $\frac{41}{333}$. (2) $5\frac{71}{90}$, $4\frac{62}{99}$, $2\frac{17}{45}$, $9\frac{8}{45}$.

(3) $1\frac{11}{30}$, $5\frac{11}{15}$, $6\frac{28}{99}$, $\frac{34}{45}$. (4) $\frac{13}{18}$, $\frac{5}{6}$, $\frac{73}{90}$, $8\frac{8}{9}$, $\frac{61}{90}$.

(5) $\frac{1}{45}$, $\frac{29}{9000}$, $\frac{17}{25}$, $\frac{41}{111}$. (6) $\frac{1}{333}$, $\frac{1}{300}$, $\frac{7}{90}$, $\frac{65}{999}$.

Examples XXI. Pages 110, 111.

1. (1) ·06255, ·12346, ·98765. (2) ·13579, ·24681, ·51153.

(3) ·86429, ·13935, ·72337. (4) ·27333, ·02737, ·09239.

2. (1) 70·37035. (2) ·40972. (3) 7·45606.

(4) $63\cdot67\dot{5}\dot{9}$. (5) $84\cdot19\dot{4}6\dot{5}$. (6) $3\cdot5\dot{1}$.

(7) 2·46913. (8) 335·14620. (9) 204·31234.

(10) $1\cdot\dot{3}$; $\cdot3\dot{0}3\dot{7}$; ·03198. (11) $\cdot12\dot{3}\dot{7}$; ·41975 ; $12\cdot\dot{2}$.

(12) $\cdot\dot{8}$; 3·69135 ; 67·02469. (13) 1·24999.

(14) 1·45009. (15) $\cdot0\dot{5}$, $\cdot\dot{2}$, $\cdot018\dot{5}$, 45.

Examples XXII. Pages 112—115.

I. **2.** $\frac{13}{400}$. **3.** 269500·34995. **4.** ·20425. **5.** 5. **6.** 2.

II. **1.** 8·3751. **2.** 4798·9026. **3.** 1000·8999.

4. ·3375, 0. **5.** ·615, ·115. **6.** $2\times2\cdot357$.

III. **1.** ·18305, 18304·81695. **2.** 10 years, 5 years. **3.** 14, 8.

4. R100, R150 and R540. **5.** ·2. **6.** R1000, R350, R150.

IV. **1.** ·25, 2·5, 2500. **2.** 70000. **3.** ·0464.

4. The 3rd is the greatest, and the 1st the least.

5. 1·7598. **6.** ·8.

V. **2.** $\cdot\dot{1}17647058823529\dot{4}$. **3.** $\frac{15}{768}=\frac{5}{256}=\frac{5}{2^8}$.

4. $\cdot12\dot{5}6\dot{7}$, $\cdot\dot{4}0\dot{5}$. **6.** $\frac{14}{37}$, $\frac{20}{55}$, $\cdot\dot{2}8571\dot{4}$, $\cdot5\dot{0}26$.

VI. **1.** ·7853. **2.** $6\cdot08\dot{3}$. **3.** 1·05067. **4.** 4, 12, 27.

5. ·7853. **6.** 1·82896.

Examples XXIII. Pages 134, 135.

1. (1) 2464*p.* ; R5. 3*a.* 4*p.* (2) 4589*p.* ; R15. 10*a.*

(3) 30535*p.* ; R31. 4*a.* (4) 3759*a.* ; R12. 4*a.*

(5) 5067*d.*; £2. 1*s.* 8*d.* (6) 30882*q.*; £0. 8*s.* 4*d.*

(7) 27000*d.*; 44 half-guineas, 7*s.* 5*d.* (8) £66. 2*s.* 6*d.*, 209 crowns.

(9) 6792 grs.; 2 oz. 11 dwts. 10 grs.

(10) 3154 dwts.; 21 lbs. 5 oz. 2 dwts. 6 grs.

(11) 8372 lbs.; 62 lbs. 8 oz.

(12) 90272000 grs.; 4 tons 9 cwt. 1 qr. 4 lbs.

(13) 2412 rattis; 10 tolas 40 rattis.

(14) 81420 kanchas; 12½ seers.

(15) 320000 tolas; 3 mds. 5 seers.

(16) 3371½ yds.; 1 mile 240 yds.

(17) 859 in.; 14 yds. 1 ft. 9 in.

(18) 7350¾ sq. yds.; 0 ac. 2 ro. 19 po. 5¼ sq. yds.

(19) 25947 cub. in.; 18 cub. yds. 14 cub. ft.

(20) 8200 cubits; 1 kros 95 rasis 78 cubits.

(21) 41600 cubits; 10 kroses.

(22) 575 kths.; 26 bghs. 5 kths.

(23) 796 kths.; 5 bghs. 3 kths. 2 chts.

(24) 1230 grams; 987·654 grams.

(25) 456000 centimetres; 78·91 dekam.

2. 24000*d.*; £6. 13*s.* 4*d.* **3.** 409600 acres; 7 bghs. 4 kths.

4. 525960 min.; 420 dandas.

Examples XXIV. Pages 136, 137.

(1) £23. 7*s.* 7½*d.* (2) £89. 13*s.* 3¼*d.* (3) £152. 15*s.* 11*d.*

(4) R136. 6*a.* 6*p.* (5) R179. 6*a.* 10*p.* (6) R289. 2*a.* 8*p.*

(7) 149 tons 8 cwt. 14 lbs. (8) 48 tons 11 cwt. 3 qrs. 26 lbs.

(8) 137 mds. 20 seers 11 chts. (10) 210 mds. 25 seers 8 chts.

(11) 32 yds. 1 ft. 7 in. (12) 207 miles 6 fur. 33 po.

(13) £69. 16*s.* 7½*d.* (14) £103. 12*s.* 4½*d.*

(15) £209. 7*s.* ¾*d.* (16) R260. 7*a.* 2*p.*

(17) R109. 14*a.* 9*p.* (18) R141. 14*a.* 2*p.*

2. 50 bghs. 13 kths. 7 chts. **3.** 77 sq. yds. 6 sq. ft. 30 sq. in.

4. 27 lbs. 5 oz. 4 dwts. **5.** R208. 12*a.* 8*p.*

6. 108 miles 2 fur. 31 po.

Examples XXV. Pages 138, 139.

1. (1) £10. 18s. 9¾d. (2) £46. 13s. 10d. (3) £48. 12s. 3½d.
(4) £265. 11s. 1d. (5) R3. 0a. 11p. (4) R17. 14a. 9p.
(7) R39. 9a. 9p. (8) R22. 0a. 6p. (9) 24 lbs. 8 oz. 18 dwts.
(10) 17 cwt. 2 qrs. 22 lbs. 10 oz. 1 dr. (11) 13 mds. 30 seers 12 chts.
(12) 11 mds. 19 seers 3 chts. (13) 68 miles 3 fur. 34 po.
(14) 2 kroses 95 rasis. (15) 16 sq. yds. 7 sq. ft. 108 sq. in.
(16) 8 cub. ft. 1721 cub. in. (17) 8 bghs. 17 kths. 14 chts.
(18) 2 kilog. 7 hectog. 8 dekag. 1 gram.

2. (1) 7 weeks 4 days 19 hrs. (2) 5 dandas 44 pals.
(3) 15 hrs. 54′ 57″. (4) 12° 55′ 55″.
(5) 5a. 7 gan. 2 cowr. 2 kr. (6) 2a. 13 gan. 1 cowr. 1 kr.

Examples XXVI. Pages 140, 141.

1. (1) £31. 1s. ; £46. 11s. 6d. ; £77. 12s. 6d.
(2) £66. 6s. 6d. ; £82. 18s. 1½d. ; £99. 9s. 9d.
(3) £125. 4s. 8¼d. ; £143. 2s. 6d. ; £161. 0s. 3¾d.
(4) £286. 6s. 8d. ; £343. 12s. ; £429. 10s.
(5) R166. 6a. ; R249. 9a. ; R332. 12a.
(6) R129. 12a. 3p. ; R155. 11a. 6p. ; R207. 10a.
(7) R130. 2a. ; R195. 3a ; R390. 6a.
(8) R170. 10a. 9p. ; R341. 5a. 6p. ; R512. 0a. 3p.
(9) 40 lbs. 3 oz. 3 drs. 2 scr. ; 50 lbs. 4 oz. 2 drs. 1 scr. 15 grs. ; 100 lbs. 8 oz. 5 drs. 10 grs.
(10) 78 cwt. 2 qrs. ; 157 cwt. ; 235 cwt. 2 qrs.
(11) 31 mds. 24 seers 2 powas ; 42 mds. 6 seers ; 47 mds. 16 seers 3 powas.
(12) 260 mds. 39 seers 1 cht. ; 347 mds. 38 seers 12 chts. ; 556 mds. 30 seers.
(13) 42 wks. 11 hrs. 30′ ; 56 wks. 15 hrs. 20′ ; 82 wks. 5 days 5 hrs. 20′.
(14) 182 days 10 hrs. 45′ ; 188 days 8 hrs. ; 235 days 10 hrs.
(15) 32 kilog. 5 hectog. 3 dekag. 3 decig. 5 centig. ; 65 kilog. 6 dekag. 7 decig. ; 97 kilog. 5 hectog. 9 dekag. 1 gram. 5 centig.

(16) 1 hectom. 7 dekam. 5 metres 2 decim.; 3 hectom. 5 dekam. 4 decim. ; 7 hectom. 8 decim.

2. (1) £46. 12*s*. 6*d*. ; £466. 5*s*.

(2) ₨186. 9*a*. ; ₨1865. 10*a*.

(3) 785 cwt. 2 qrs. 24 lbs. ; 1571 cwt. 1 qr. 20 lbs.

(4) 321 lbs. 3 oz. 11 dwts 16 grs.; 3212 lbs. 11 oz. 16 dwts. 16 grs.

(5) 154 mds. 33 seers 2 chts. ; 1548 mds. 11 seers 4 chts.

(6) 305 mds. 5 seers ; 457 mds. 27 seers 8 chts.

Examples XXVII. Pages 142, 143.

1. (1) £4. 15*s*. $2\frac{1}{2}$*d*. ; £3. 3*s*. $5\frac{2}{3}$*d*. ; £2. 7*s*. $7\frac{1}{4}$*d*.

(2) £11. 3*s*. $1\frac{1}{5}$*d*. ; £9. 5*s*. 11*d*. ; £7. 19*s*. $4\frac{2}{7}$*d*.

(3) £8. 9*s*. $8\frac{3}{8}$*d*. ; £7. 10*s*. $10\frac{1}{9}$*d*. ; £6. 15*s*. $9\frac{1}{10}$*d*.

(4) £3. 2*s*. $11\frac{5}{9}$*d*. ; £3. 2*s*. $1\frac{15}{43}$*d*. ; £3. 0*s*. $5\frac{1}{3}$*d*.

(5) ₨5. 11*a*. $\frac{3}{10}$*p*. ; ₨4. 11*a*. $10\frac{1}{4}$*p*. ; ₨4. 1*a* $\frac{3}{14}$*p*.

(6) ₨10. 0*a*. $10\frac{2}{15}$*p*. ; ₨9. 6*a*. $9\frac{1}{2}$*p*. ; ₨8. 6*a*. $\frac{4}{9}$*p*.

(7) ₨11. 14*a*. $\frac{10}{19}$*p*. ; ₨7. 4*a*. $5\frac{23}{31}$*p*.

(8) ₨80. 1*a*. $\frac{3}{4}$*p*. ; ₨35. 9*a*. $4\frac{1}{3}$*p*.

(9) 14 gals. $\frac{3}{4}$ qts. ; 4 gals. $\frac{3}{14}$ qts.

(10) 9 qrs. $1\frac{1}{4}$ pks. ; 4 qrs. 4 bus. $\frac{5}{8}$ pks.

(11) 1 cwt. 3 qrs. $23\frac{1}{3}$ lbs. ; 3 qrs. $19\frac{17}{19}$ lbs.

(12) 2 cwt. 2 qrs. $4\frac{4}{5}$ lbs. ; 1 cwt. 3 qrs. $17\frac{3}{5}$ lbs.

2. (1) $12\frac{19}{102}$. (2) $8\frac{252}{253}$. (3) $15\frac{111}{667}$. (4) 4_{82}.

(5) 19166400. (6) $4\frac{18}{47}$. (7) $4\frac{26}{51}$. (8) 3.

(9) $7\frac{9}{38}$. (10) $5\frac{5}{39}$. (11) $4\frac{5}{42}$. (12) $198\frac{1}{1}$

Examples XXVIII. Pages 146—149.

I. **2.** $87\frac{3}{11}$ grs. ; $2\frac{1}{16}$*s*. **3.** 400 drs. ; $877\frac{5}{7}$ drs. ; $32666\frac{2}{3}$ dwts.

4. 3960*q*. ; ₨$17\frac{3}{16}$. **5.** 4752000000. **6.** 126.

II. **1.** 187500 mds. ; 18750000 lbs. Troy ; 11719 carts, the last cart carrying only 12 mds.

2. ₨39062. 8*a*. ; ₨1002604. 2*a*. 8*p*.

3. 30000000 mds. ; ₨150000000. **5.** 113565760 bgh .

6. 8766 hrs.

III. **1.** £4. 6*s*. 4*d*. **2.** 5600 pice.

3. 21 years 45 days ; 10th September, 1847. 4. ₨29. 10*a.*
5. 27 mds. 18 seers ; ₨137. 4*a.* 6. 8180 days.

IV. 2. ₨44. 14*a.* 6*p.* ; 479 miles.
3. 25 rupees, 50 half-rupees, 75 four-anna pieces, 100 two-anna pieces.
4. ₨14. 1*a.* 3*p.* 5. 5 mds. 33 seers. 6. 2516800000 bghs.

V. 1. ₨275. 8*a.* 2. ₨4. 13*a.* 3. ₨1950.
4. 56940. 5. 12. 6. 5.

VI. 1. 45. 2. 15. 3. 3 days. 4. 8′ 20″.
5. *A* gets ₨400., *B*, ₨600., and *C*, ₨800.
6. ₨196. 10*a.* ; ₨3. 1*a.* $1\frac{7}{8}$*p.*

Examples XXIX. Pages 151, 152.

1. (1) £1. 2*s.* 6*d.* ; £3. 3*s.* 8*d.*
(2) £12. 18*s.* 6*d.* ; £2. 10*s.* 2*d.*
(3) £3. 19*s.* ; £33. 15*s.*
(4) £7. 17*s.* 11*d.* ; £26. 12*s.*
(5) ₨2. 14*a.* $2\frac{2}{3}$*p.* ; ₨5. 0*a.* 8*p.*
(6) 13*a.* 1*p.* ; ₨1. 7*a.* 1*p.*
(7) 13 cwt. 2 qrs. 14 lbs. 14 oz. ; 1 lb 7 oz. 13 dwts.
(8) 7 mds. 13 seers 4 chts. ; 7 mds. 19 seers 4 chts.
(9) 6 fur. 12 po ; 1 yd. 2 ft. 9 in.
(10) 2 days 2 hrs. 21′ ; 2 hrs. 39′.

2. (1) $\frac{3}{4}$; ·8. (2) $\frac{1}{40}$; ·25. (3) ·5.
(4) $\frac{1}{16}$; ·1. (5) $\frac{1}{48}$; ·03125. (6) ·125.
(7) $\frac{5}{16}$; ·25. (8) $\frac{2}{5}$; ·00625.

3. $\frac{5}{8}$; $\cdot\dot{3}$. 4. $\frac{1}{10}$; ·125. 5. $\frac{1}{4}$; $\frac{1}{16}$.

6. In order of value, the quantities will stand thus :—
1) $\frac{1}{15}$ of ₨1. 8*a.*, $\frac{1}{12}$ of ₨1, $\frac{2}{3}$ of 2*a.*
(2) $\frac{1}{3}$ of £1, $\frac{3}{8}$ of 2 crowns, $\frac{5}{6}$ of 1 florin.
(3) $\frac{3}{8}$ of 15 seers, $\frac{7}{8}$ of 1 pusury, $\frac{1}{15}$ of 1 md.
(4) $\frac{5}{9}$ of 3 ft, $\frac{3}{8}$ of 4 ft., $\frac{1}{15}$ of 2 yds.

Examples XXX. Page 154.

(1) £1. 2*s.* 4*d.* (2) 16*s.* $6\frac{2}{3}$*d.* (3) ₨4. 14*a.* $10\frac{2}{3}$*p.*
(4) 18*s.* 6*d.* (5) 2*s.* $5\frac{1}{30}$*d.* (6) ₨3. 10*a.* $2\frac{2}{3}$*p.*

(7) 2 mds. 14 seers 5 chts. (8) 2 ft. $11\frac{1}{4}$ in.
(9) 3 days 23 hrs. (10) 1 qr. 11 lbs. 10 oz.

Examples XXXI. Pages 154, 155.

(1) 0. (2) 3*s*. 9*d*. (3) £3. 13*s*. (4) R3. 1*a*. 8*p*.
(5) 2*a*. 4*p*. (6) R1. (7) 2qrs. 11 lbs. (8) 2 in.
(9) 16 seers. (10) 2 hrs. 28′ 40″.

Examples XXXII. Page 156.

(1) 17*s*. $\frac{2}{5}d$. (2) £6. 19*s*. 4*d*. (3) £7. 1*s*. $10\frac{3}{4}d$.
(4) R1. 4*a*. $\frac{3}{4}p$. (5) R17. 6*a*. $\frac{3}{7}p$. (6) R19. 7*a*. $9\frac{3}{4}p$.
(7) 1 qr. 9 lbs. 8 oz. (8) 15 seers $2\frac{1}{3}$chts. (9) $4\frac{1}{2}d$.
(10) 6*a*. $2\frac{1}{4}p$. (11) 1ft. $2\frac{4}{9}$ in. (12) 16*s*. $10\frac{29}{40}d$.

Examples XXXIII. Page 157.

(1) £13. 14*s*. $10\frac{3}{4}d$. (2) 75. (3) 12*a*. (4) 4.
(5) R16. 9*a*. (6) 5 ft. (7) 6ft. 8 in. (8) $\frac{22}{35}$.
(9) 10. (10) $\frac{1}{3}$. (11) 1 bus. $3\frac{1}{3}$pks. (12) 2.

Examples XXXIV. Pages 159, 160.

1. R560. **2.** R672. **3.** R1066. 10*a*. 8*p*.
4. S. R$94\frac{29}{64}$. **5.** £25. 1*s*. 3*d*. **6.** R162. 12*a*.
7. $8\frac{19}{35}$ Factory mds.; 5 cwt. 3 qrs. 4 lbs.
8. 105 lbs. Troy; $86\frac{4}{7}$ lbs. Avoir. **9.** 12 lbs. Avoir.
10. $20\frac{5}{96}$ lbs. Troy. **11.** 40656 bghs. **12.** $145\frac{75}{121}$ acres.
13. $38\frac{91}{160}$ bghs. **14.** $13\frac{3}{8}$ dandas; $6\frac{1}{10}$ hrs. **15.** $\frac{64}{75}$hr.; $22\frac{11}{12}$ dandas.

Examples XXXV. Pages 164—167.

I. **1.** $2\frac{13}{800}s$. **2.** R9600. **3.** $11\frac{11}{24}$srs.; $28\frac{1}{33}$lbs. Troy.
4. R560; £51. 6*s*. 8*d*. **5.** R2100; R1470.
6. $\frac{1}{2}$; R3262. 8*a*.

II. **1.** 4840 sq. yds.; 1600 sq. yds.; 1936 bghs.; $198\frac{48}{121}$ ac.
2. $2\frac{1}{4}$ miles an hour. **3.** $\frac{3}{40}$; $616\frac{7}{8}$ bghs.
4. $\frac{1}{640}$; $\frac{40}{121}$. **5.** ·0125; $\frac{1}{1760}$.
6. 330 ft.; $\frac{4}{11}$.

III. 1. R225. *7a.* *6p.* 2. R65625.

3. *A* gets R800, *B*, R1200, and *C*, R1500.

4. *A* gets R400, *B*, R300, and *C*, R90.

5. $\frac{1}{100}$; $\frac{36}{49}$.

6. $33\frac{1}{3}$ mds. of the first kind, and $66\frac{2}{3}$ mds. of the second kind.

IV. 1. R64. *0a.* $6\frac{1}{2}$*p.* 2. 6 days. 3. $395\frac{915}{2023}$.

4. 7 bghs. $17\frac{1}{2}$kths. 5. $266\frac{2}{3}$ yds. 6. $1\frac{107}{432}$.

V. 1. At 5 o'clock. 2. $3\frac{3}{5}$ hrs. 3. 365·242219 days, 365·25 days ; ·007781 of a day. $128\frac{4032}{7781}$ years.

4. 52 Sundays ; 5 Saturdays. 5. $\frac{11}{25}$; 26400 jojans.

6. $11\frac{21498029}{43882800}$ seconds.

VI. 1. 14400 grs.; 15432·34765625 grs. 2. $\cdot 621382\dot{5}\dot{7}$.

3. 30 miles from *A* ; 12 hrs.; 2 hrs. 12.′

4. 2 days. 5. 15 gals. 6. In 1883.

Examples XXXVI. Pages 172, 173.

1. (1) 3 sq. ft. 54 sq. in. (2) 9 sq. ft. 90 sq. in.
   (3) 14 sq. ft. 126 sq. in. (4) 26 sq. ft. 120 sq. in.
   (5) 34 sq. ft. 120 sq. in. (6) 10 sq. ft. 32 sq. in.
2. (1) 2 cub. ft. 918 cub. in. (2) 9 cub. ft. 1458 cub. in.
   (3) 11 cub. ft. 576 cub. in. (4) 33 cub. ft. 432 cub. in.
3. 277 sq. ft. 72 sq. in.; 74 ft.
4. 265 sq. ft. 90 sq. in.; 26 ft. $6\frac{2}{3}$ in.
5. 125 cub. in.; 420 cub. ft.
6. (1) 3 bghs. 12 kths. (2) 7 bghs. 17 kths. 10 dhools.
   (3) 13 bghs. 12 kths. (4) 25 bghs. 3 kths. 10 dhools.
   (5) 17 bghs. 17 kths. (6) 42 bghs. 10 kths.

Examples XXXVII. Page 175.

1. 3 sq. ft. 5′. 3″. 2. 15 sq. ft. 6′. 8″. 3. 21 sq. ft. 1′.
4. 34 sq. ft. 6′. 5. 50 sq. ft. 5′. 8″. 6. 50 sq. ft. 6′. 8″.
7. 60 sq. ft. 6′. 3″. 8. 58 sq. ft. 1′. 6″. 9. 33 sq. ft. 3′, 9″.
10. 58 sq. ft. 8′. 2″.

Examples XXXVIII. Pages 179, 180.

1. R33. **2.** 23 bghs. 17 kths. **3.** £3. 1*s*. $1\frac{1}{3}$*d*.
4. R27. 9*a*. **5.** 4 bghs. **6.** R28200.
7. R240. **8.** 22 yds. **9.** R51840.
10. 27 ft. **11.** R2479. 8*a*. **12.** 45″.
13. $149\frac{79087}{138637}$. **14.** 12 ft. 6 in. **15.** 96.
16. 1152 bricks ; R10. 15*a*. $1\frac{31}{125}$*p*. **17.** 63.
18. 42 cub. ft. 1512 cub. in.
19. 32 in the 1st class, 64 in the 2nd, and 192 in the 3rd.
20. R3536.

Examples XXXIX. Pages 184, 185.

1. R106. 4*a*.; R212. **2.** £108 ; £42. 12*s*. 6*d*.
3. £84 ; £225. **4.** R71. 4*a*.; R6. 12*a*. 6*p*.
5. R92. 8*a*.; R41. 4*a*. **6.** R275 ; R230. 10*a*.
7. £2376 ; £420. 7*s*. 6*d*. **8.** £2858. 8*s*.; £2108. 8*s*.
9. £15845. 16*s*. 8*d*.; £5087. 10*s*.
10. R2312. 8*a*.; R5156. 4*a*.
11. R6746. 4*a*.; R5361. 15*a*. 8*p*.
12. R6114. 7*a*.; R23437. 8*a*. **13.** £218. 5*s*.
14. £43. 15*s*. $10\frac{1}{2}$*d*. **15.** £184. 9*s*. $3\frac{3}{7}$*d*.
16. R103. 5*a*. 9*p*. **17.** R1107. 15*a*. $3\frac{3}{20}$*p*.
18. R408. 8*a*. 3*p*. **19.** £18. 8*s*. **20.** £21. 5*s*.
21. R11200. 8*a*. **22.** R13216. 8*a*. **23.** R4200.
24. R180. 0*a*. 3*p*.

Examples XL. Pages 199—205.

1. (1) 6. (2) $18\frac{2}{3}$. (3) £18. (4) 6*s*. (5) R$1\frac{1}{4}$.
(6) ·1. (7) $6\frac{1}{4}$ bghs. (8) 4 yds. 6 in.
2. (1) 45. (2) 14·4. (3) 3. (4) 12. (5) $1\frac{1}{4}$*s*.
(6) $\frac{1}{16}$*a*.
3. 30 ft. and 15 ft. **4.** 20 times. **5.** R18. 12*a*.
6. 16 mds. **7.** R278. 2*a*. **8.** 20 cwt.
9. The prices are as 5 : 3 ; 6*s*. per yard. **10.** R1. 14*a*.
11. R1396. 5*a*. $9\frac{9}{11}$*p*. **12.** £600. **13.** R38400.

14. 7*a*. **15.** R5500. **16.** R110000. **17.** R6.
18. R6. **19.** R19. 11*a*. **20.** 9 : 25.
21. $50{\cdot}25\dot{7}$ sq. ft. **22.** 18·8496 ft.
23. $840\frac{40}{119}$ yds. **24.** 24 5 sq. ft.
25. R225000. **26.** £128000.
27. R1 for paddy land, and R5 for mulberry land.
28. R1 for arable land, and R2. 8*a*. for homestead land.
29. R773. 8*a*. **30.** R$79\frac{5}{7}$.
31. *A* gets R600 and *B* gets R1000.
32. The shares of *A*, *B* and *C* are R300, R600 and R1500.
33. $\frac{4}{11}$ of an inch ; 110 yds.
34. 1 ft. 6·75 in. **35.** $\cdot 015\dot{9}\dot{0}$. **36.** 6 bghs.
37. 150 poles. **38.** 4 bghs. $16\frac{1}{3}$ kths.
39. 3 bghs. $7\frac{1}{2}$ kths. **40.** 60 ft. **41.** 564 miles.
42. 15 miles an hour. **43.** 15 miles an hour.
44. Velocity of the Earth : velocity of light : : 1309 : 13696875.
45. 80 ft. **46.** 45 men. **47.** 5 days.
48. 60 men. **49.** 14 days. **50.** 16 men.
51. $49\frac{81}{321}$ minutes past 12 o'clock P. M.; $10\frac{1}{41}$ minutes past 1 o'clock P. M.
52. 14′ 35″ ; exactly after 12 days **53.** $40\frac{1}{3}$yds.
54. 6 days. **55.** 54 men. **56.** 10 mds.
57. 456 mds. 10 seers ; 25 mds. 20 seers.
58. R866. 14*a*. **59.** 383 mds. 10 seers. **60.** 100.
61. R450. **62.** 12 oz. **63.** 20.
64. $1'\,\frac{5}{22}''$; 490 yds. **65.** 5 months. **66.** £10000.
67. $38\frac{2}{11}$ ′ past 1 o'clock. **68.** $5\frac{5}{11}$′ past 1 o'clock.
69. $18{\cdot}1\dot{3}$*s*. **70.** 39 days.

Examples XLI. Page 207.

1. (1) 3, 6, 9. (2) 6, 9, 12. (3) 9, 12, 15.
(4) 8, 24, 40, 56. (5) 100, 200, 300, 400.
(6) 165, 195, 225.
2. *A* gets R300, *B*, R900, and *C*, R1500.
3. *A* gets £1000, *B*, £800, and *C*, £600.

Examples XLII. Page 210.

1. $7\frac{1}{2}$. 2. $6\frac{2}{3}$; R2. 4*a.* 3. R3. 2*a.* 4. R8000.
5. £4000. 6. $1\frac{9}{16}$. 7. 26 8. R2050.

Examples XLIII. Pages 211, 212.

1. R600, R750. 2. *A* gets £720, *B*, £1080, and *C*, £1200.
3. *A* should have R600, and *B*, R1800.
4. *A* should have R288, *B*, R288, and *C*, R324.
5. *A* should have R315, *B*, R280, and *C*, R315.
6. *A* should have R9500, and *B*, R2500.

Examples XLIV. Pages 218—220.

1. (1) R$4\frac{1}{2}$, R$79\frac{1}{2}$. (2) R$14\frac{2}{5}$, R$94\frac{2}{5}$. (3) R$23\frac{7}{16}$, R$148\frac{7}{16}$.
   (4) R$1228\frac{4}{5}$, R$3788\frac{4}{5}$. (5) £210. 2*s.*, £1260. 12*s.*
   (6) £$866\frac{1}{4}$, £$6344\frac{1}{4}$. (7) R$425\frac{1}{4}$, R$1475\frac{1}{4}$. (8) R225, R975.
2. (1) R$4\frac{4}{5}$. (2) R11. 15*a.* 3*p.* (3) R$18\frac{3}{4}$. (4) R$31\frac{1}{5}$.
   (5) R13. 2*a.* (6) R$10\frac{1}{2}$. (7) R$30\frac{138}{365}$. (8) R$128\frac{808}{1825}$. (9) £$11\frac{17}{73}$.
3. (1) 10 years. (2) 7 years. (3) $6\frac{2}{3}$ years. (4) $6\frac{1}{4}$ years.
   (5) 16 months. (6) 4 years.
4. (1) $16\frac{2}{3}$ per cent. per annum. (2) $13\frac{1}{3}$ per cent. per annum.
   (3) $3\frac{1}{8}$ per cent. per annum. (4) $1\frac{7}{8}$ per cent. per annum.
   (5) $6\frac{1}{4}$ per cent. per annum. (6) 10 per cent. per annum.
5. (1) R500. (2) £800. (3) £500. (4) R7500.
   (5) R$33333\frac{1}{3}$. (6) R1600.
6. (1) £$89\frac{2}{7}$. (2) £500. (3) R$675\frac{25}{37}$. (4) R15625.
7. 25 years. 8. 20 per cent. per annum.

Examples XLV. Page 221.

1. R16·8, R96·8. 2. R15·9792, R90·9792.
3. £10. 4*s.*, £135. 4*s.* 4. £165. 10*s.*, £665. 10*s.*
5. R163·2, R2163·2. 6. R8275, R33275.

Examples XLVI. Pages 225, 226.

1. (1) R$89\frac{2}{7}$. (2) £$181\frac{9}{11}$. (3) £700.
   (4) R750. (5) R500. (6) R1525.

2. (1) £10. (2) £95. (3) ₨189.
(4) ₨360. (5) ₨$333\frac{1}{3}$. (6) ₨$535\frac{5}{7}$.

3. $8\frac{466058}{671819}d$. **4.** ₨$2203\frac{187}{371}$. **5.** ₨$8951\frac{31887}{71283}$.

6. The rate of interest is $17\frac{11}{17}$ per cent. per annum, and the rate of discount 15 per cent.

Examples XLVII. Pages 227, 228.

1. $3\frac{4}{5}$ months. **2.** After $20\frac{2}{3}$ months. **3.** At the end of 7 months.
4. $4\frac{1}{6}$ months. **5.** $15\frac{1}{8}$ months. **6.** After 13 months.

Examples XLVIII. Pages 231, 232.

(1) ₨5000. (2) ₨5000. (3) ₨60000.
(4) £4000. (5) £9500. (6) ₨25000.

2. (1) ₨9700. (2) ₨12600. (3) ₨202.
(4) £528. (5) £1728. (6) ₨26500.

3. (1) ₨400. (2) £450. (3) £560.
(4) ₨1800. (5) ₨1900. (6) ₨750.

4. (1) The latter. (2) The latter. (3) The latter.
(4) The former. (5) The latter. (6) The former.

5. (1) Increase of ₨100. (2) No change. (3) Increase of £63.

6. (1) $4\frac{2}{47}$. (2) $3\frac{4}{7}$. (3) $4\frac{7}{17}$. (4) $5\frac{5}{14}$.

7. ₨108. $11\frac{1}{4}a$. **8.** ₨56. $4a$.

Examples XLIX. Pages 235, 236.

1. ₨12. $13a$. $4p$. **2.** ₨1. $5a$. **3.** ₨4. **4.** £1. $3s$. $9\frac{1}{3}d$.
5. The ingredients are mixed in equal parts. **6.** 3, 1, 1.
7. 12, 4, 4. **8.** 6, 2. **9.** 3, 1, 1, 1.

Examples L. Page 238.

1. ₨3200. **2.** ₨6000. **3.** £393 $15s$. $10\frac{1}{2}d$. **4.** $1s$. $10d$.
5. ₨1 = $\frac{2}{3}$ of a ruble. **6.** Circuitously through St. Petersburg.

Examples LI. Pages 243, 244.

1. (1) 21 ; 31 ; 99 ; 89 ; 111 ; 222. (2) 41 ; 51 ; 79 ; 69 ; 333.
(3) 25 ; 35 ; 45 ; 55 ; 65 ; 75. (4) 85 ; 95 ; 123 ; 234 ; 135.
(5) 1234 ; 1111 ; 2222 ; 1001. (6) 3333 ; 2020 ; 5001.

2. (1) 1, 1·4142..., 1·7320..., 2, 2·2360..., 2·4494..., 2·6457..., 2·8284..., 3, 3·1622...

(2) 3·3166..., 2·4641..., 3·6055..., 3·7416..., 9·9498..., 8·7749..., 8·1240...

(3) 1·0954..., 1·5165..., 1·8439..., 1·5297..., 1·2.

(4) ·3162..., ·0447..., ·02, 10·0049..., ·9486...

(5) ·7071..., ·5773..., $\frac{1}{2}$, ·4472..., ·4082..., ·3779..., ·3535..., $\frac{1}{3}$.

(6) ·8164..., ·5222..., $\frac{5}{12}$, ·2683..., $\frac{1}{5}$.

3. 7·0710...bghs. **4.** 155·562...yds. **5.** 3·1622...bghs.

6. 565·68...ft. **7.** 5·6568...bghs. **8.** 1·2892...miles.

Examples LII. Page 248.

1. (1) 11 ; 12 ; 13 ; 14. (2) 15 ; 16 ; 17 ; 18 ; 19.

(3) 25 ; 111 ; 222.

2. (1) 1 ; 1·25... ; 1·44... ; 1·58... ; 1·70... ; 1·81... ; 1·91... ; 2 ; 2·08...

(2) ·79... ; ·69... ; ·62... ; ·58... ; ·55... ; ·52... ; $\frac{1}{2}$; ·48..

(3) ·46... ; ·58... ; ·66... ; ·1 ; ·21...

(4) 4·97... ; 5·5 ; 6·6.

3. 14 ft. **4.** 19 in.

THE END.

APPENDIXES.

APPENDIX A.

Additional Examples I.—Notation and Numeration.

1. Express in figures the following :

(1). Twelve ; nineteen ; seventeen ; twenty-seven.

(2). Fifty-two ; thirty-nine ; eighty ; ninety-four ; seventy-nine.

(3). Five hundred and twelve ; eight hundred and eighty-five; eight hundred and five ; nine hundred and twenty-one ; nine hundred and one.

(4). Three thousand ; twenty-seven thousand five hundred and four ; six hundred and seventy-six thousand and fifty ; two hundred and one thousand three hundred and twenty-four.

(5). Two hundred thousand; five hundred and eighty-four thousand nine hundred and eleven; eight hundred and thirty-four thousand five hundred and eight ; six hundred and two thousand and seventy-nine.

(6). Eighty-four millions, five hundred and twenty-five thousand eight hundred and four ; ninety-eight millions, four hundred and six thousand, six hundred and six ; seventy-four millions, three hundred and thirty-five thousand, nine hundred and nine.

(7). Seven billions, four hundred and sixty-two thousand, four hundred and seven.

(8). Four hundred and five billions, eighty-two millions, three hundred and fifty-one thousand, seven hundred and five.

2. Write in figures the greatest number of eight digits and the least number of ten digits.

3. Express five hundred crores in English and a trillion in Indian Numeration.

4. A boy when told to write in figures eighty millions, five hundred and two thousand and four, wrote 850020004. What was his mistake?

5. How many crores are there in three trillions ?

6. A boy was told to write out eight crores, eight lacs, eight hundred and eighty-eight ; he wrote 80088088. What mistakes did he commit ?

7. Point out the position of the zeros in the following :—
708001 ; 23400250 ; 80007002 ; 020807109.

8. Find the local value of each of the digits in the following :—
73 ; 173 ; 1234 ; 50378 ; 670000 ; 81240102.

9. Express in words the following :—
(1). 10203 ; 40025 ; 67809 ; 80007 ; 57000 ; 92004.
(2). 3687562 ; 2100007 ; 5960003 ; 8000345 ; 5500555.

10. Write down in words the greatest number of eight digits and the least number of seven digits.

11. Two boys were told to read 50070010 ; one of them read it as five millions, seven thousand and ten ; and the other as fifty millions, seven thousand and ten. Find out the mistakes committed by each.

Additional Examples II.—Addition.

1. Find the sum of 2 and 3 ; 3 and 4 ; 5 and 6 ; 7 and 8 ; 9 and 6 ; 6 and 4 ; 9 and 9.

2. Find the sum of 6 and 7 ; 8 and 9 ; 9 and 2 ; 9 and 3 ; 9 and 4 ; 9 and 5 ; 8 and 8.

3. Find the sum of 10 and 2 ; 20 and 7 ; 30 and 8 ; 40 and 9 ; 50 and 10 ; 60 and 8 ; 70 and 9.

4. Find the sum of 12 and 5 ; 13 and 6 ; 14 and 7 ; 15 and 8 ; 33 and 7 ; 61 and 9.

5. Find the sum of 22 and 12 ; 48 and 15 ; 37 and 10 ; 99 and 12 ; 10 and 25 ; 17 and 10.

6. Find the sum of 5 and 8 ; 5 and 17 ; 5 and 27 ; 5 and 30 ; 5 and 40 ; 5 and 50 ; 5 and 60.

7. Find the sum of 20 and 2 ; 21 and 3 ; 22 and 4 ; 23 and 5 ; 24 and 6 ; 28 and 8.

8. Find the sum of 38 and 6; 45 and 9; 84 and 7; 75 and 8; 98 and 3.

9. Add 2 to 3 ; 5 to 19 ; 8 to 17 : 9 to 20 ; 8 to 23.

10. Add 12 to 13, to 14, to 15, to 16, to 17, to 18.

11. Count aloud by increments of 8, starting at 7, at 9, at 11, at 13, at 15, at 17.

12. Add 5 apples to 7 apples; 6 mangoes to 8 mangoes; 5 cows to 9 cows.

13. Add 2 rupees to 12 rupees ; 8 annas to 7 annas ; 9 cowries to 6 cowries.

14. In a certain hall there are 10 pillars and in another 12 pillars. How many pillars are there in the two ?

15. I gave 14 rupees to Satis and 18 to Sarat. How many rupees did I spend ?

16. In a box there were 15 rupees at first and 5 rupees were kept afterwards in it. How many rupees are there in the box ?

17. Ram is 11 years old and his elder brother is 5 years older than he. What is his elder brother's age ?

18. In a rupee there are 16 annas. How many annas are there in 3 rupees ?

19. 20 Bombay mangoes can be had for one rupee. How many can be had for 2 rupees ?

20. Hari is 14 years old now. How old will he be 13 years hence ?

21. Ganesh purchased a piece of cloth for 15 rupees and at a gain of 25 rupees he sold it off. What was the selling price of the cloth ?

22. I have with Hari 6 rupees, with Syam 5 rupees more than what I have with Hari, and with Ram 7 rupees more than what I have with Syam. How many rupees have I in all ?

23. In a school there are 5 classes. In the first class, there are 10 boys; in the second, 2 boys more than in the first ; in the third, 3 boys more than in the second ; in the fourth, 4 boys more than in the third ; and in the fifth 5 boys more than in the fourth. How many boys are there in the school ?

24. From a piece of cloth, I gave away to each of my 2 servants 4 yards and to each of my 3 maid servants 5 yards, and found that there were 2 yards left. What was the length of the piece of cloth ?

25. Add together :—

(1).	17	(2).	53	(3).	98	(4).	78	(5).	100
	20		88		60		17		200
	34		55		82		71		300
	28		43		40		45		400

(6).	561	(7).	9002	(8).	12368	(9).	300	(10).	1
	320		1006		1807		49		10
	201		107		561		101		100
	119		29		69		26		1000
	100		9		73		10		10000

(11).	(12)	(13).	(14).
58083	627432	892764	1807353
57214	543201	936807	298743
7164	678641	9482	5987
725	548200	100	7600031
27	868759	152646	247
1	345678	11	50705

(15)	(16).	(17).	(18).
1047613	8000000	1	5678539
980476	700000	82	653215
33618	60000	543	20362
2579	5000	7654	4580
628	400	98765	753
18	00	109876	54
5	0	3210987	7

(19).	(20).	(21).	(22).
843771	121578	572337	6594372
457014	236943	232708	2453216
561246	467810	253301	363145
872454	956437	270087	47194
112349	615432	25612	42574
943576	804676	3251	8792
562790	718425	147	536
847321	263144	26	85
1431	283344	8	6

(23). 7384, 326, 6740 and 57.
(24). 201, 9073, 5612, 18 and 1001.
(25). 7512, 98721, 1002, 1000 and 2500.
(26). 98761, 685201, 5340 and 870062.
(27). 98934, 61425, 8000423, 46, 4060 and 7576732.

26. Find the value of :—

(1) 6740 + 9745 + 5649 + 8301 + 6543 + 2002.
(2) 83764 + 2474 + 423 + 2034 + 5481 + 3 + 7.
(3) 794316 + 4078 + 324010 + 9876001 + 49136.

27. Find the sum of 921 repeated 3 times, 5107 repeated 4 times and 90002 repeated twice.

28. Ram and Syam are two brothers. The income of Ram is Rs 7180 a year and that of Syam is Rs. 48113. What is their income together?

29. Hari purchased 3 baskets of mangoes in one of which he found 513 mangoes and in the other two 1006 and 4156 respectively. How many mangoes did he purchase?

30. What is the number from which if 6437 be taken away 476 remains?

31. A man had six sons of whom the eldest was 2 years older than the second; the second 3 years older than the third; the third 4 years older than the fourth; the fourth 5 years older than the fifth and the fifth 6 years older than the sixth. What was the eldest son's age when the youngest was 18 years old? The eldest son was born when the father was 25 years old. What was the father's age when the third son was 20 years old?

32. The income of Ram is three times that of Syam and the income of Jadu is four times that of Ram. If the income of Syam be 10,000 Rupees, what is the income of the three together?

33. What is the number from which if 5387 and 3464 be taken away, 786 remains?

Additional Examples III.—Subtraction.

1. Subtract 4 from 5, 5 from 6, 6 from 7, 7 from 8, 8 from 9.
2. Subtract 9 from 10, 10 from 11, 11 from 12, 12 from 13, and so on.
3. Find the difference between 14 and 9; 15 and 7; 19 and 10; 19 and 9.
4. Find the difference between 15 and 8; 17 and 13; 14 and 12; 15 and 7.
5. Find the value of :—
   (1) $18-7$; $16-9$; $11-3$; $16-8$; $19-10$.
   (2) $25-3$; $29-8$; $34-9$; $17-6$; $19-8$.
   (3) $42-8$; $64-7$; $66-11$; $81-9$; $71-11$.
   (4) $60-10$; $50-10$; $70-10$; $80-10$; $90-10$.
6. Subtract 4 from 18, from 19, from 16 and from 15.
7. Take 11 from each of the following :—82; 71; 63; 92.
8. Subtract 9 from the sum of 7 and 8; 7 from the sum of 6 and 5; 11 from the sum of 10 and 8.
9. Subtract 5 times 8 from 50; 6 times 7 from 52; 8 times 10 from 90.
10. Take 8 from $5+6$; 9 from $10+8$; 10 from $12+8$; 13 from $10+12$.

11. Syam received 8 pice from his father, with 3 pice of which he purchased a copy of note book. How many pice has he left ?

12. Jadu is 31 years old now. How old was he 12 years back ?

13. I had 49 rupees in my box, from which I gave away to Ram 14 rupees and to Syam 13 rupees. How many rupees were there left ?

14. A father has 5 sons of whom the second is 2 years younger than the first ; the third 3 years younger than the fourth ; and the fourth 4 years younger than the fifth. If the first son's age be 25 years, what is the youngest son's age ?

15. Find the difference of :—

(1).	98	(2).	516	(3).	826	(4).	879	(5).	721
	75		413		709		430		319
(6).	5123	(7).	7843	(8).	4030	(9).	6789	(10)	8004
	789		798		675		4567		7806

(11).	900000	(12).	5000701	(13).	9308762	(14).	1000000
	8887		4500001		61704		999999
(15).	99999990	(16).	78975687	(17).	817890043	(18).	1817143001
	8888888		56007630		763157304		459761732

16. Find the value of :—

(1). 268783 – 10009. (2). 10067 – 1067.
(3). 58637940 – 76846941. (4). 8010203 – 1710204.
(5). 1817519 – 817519. (6). 1111111 – 99999.
(7). 10000001 – 800008. (8). 975200781 – 70512008.

17. I took on various occasions 488, 456, 701 and 1000 rupees from a box containing 3000 rupees ; how much was there left in the box ?

18. By how much is the number eight thousand eight hundred and ninety-nine greater than five thousand and seven ?

19. Find the difference between the sum and difference of 1923250 and 170619.

20. What number must be subtracted from 80000 so as to leave 7509 as remainder ?

21. Ram was born in the year 1856. How old will he be in the year 1907?

22. By selling an article for 595856 rupees a merchant gained 95800 rupees. What was the cost price of the article?

23. Napoleon Bonaparte was born in the year 1769 and died in the year 1821. How old was he at his death?

24. Subtract the greatest number of six digits from the least number of seven digits.

25. A boy when told to write nine hundred and forty-eight thousand four hundred and six in figures, wrote 94846. How much less did he write?

27. A boy when told to write three millions, nine hundred and four, wrote 30000904. How much more did he write?

28. A civil surgeon whose fee is Rs. 4, had 650 calls in a certain month, his expenditure in that month was 1020 Rs. What sum did he save that month?

29. A pleader gets on an average 8000 rupees annually; he spends 450 rupees a month. What amount can he place in the Savings Bank in a year?

30. A man was 83 years old when he died in 1886; in what year was he born? and how old was he at the time when his first son was born in 1830?

31. A man at the time of his death gave away to each of his three sons 9528 rupees and the rest to his only daughter. If he had 30,000 rupees, what was left for the daughter?

Additional Examples IV.—Multiplication.

1. How much is

5 times 4; 7 times 3; 4 times 6; 5 times 7; 7 times 8; 8 times 9; 9 times 9; 12 times 13; 14 times 15; 15 times 16?

2. Multiply 9 by 7; 8 by 6; 15 by 8; 13 by 9; 18 by 5; 19 by 8.

3. How much is 9×9; 9×10; 11×11; 12×13; 14×15; 15×16; 16×17; 18×12; 19×7; 19×12?

4. What is the sum of 8 repeated 9 times, 15 repeated 7 times, and 13 repeated 13 times?

5. There are 20 oranges in a basket; how many are there in 8 such baskets?

6. A man can walk 5 miles in an hour; how many miles will he walk in 15 hours?

7. I can ride 12 miles an hour. How many miles shall I ride in 13 hours ?

8. A yard of velvet is worth 5 rupees. What will be the price of 19 yards of the same ?

9. The multiplicand is 15 and the multiplier 14. What is the product ?

10. 12 *lengra* mangoes can be had for a rupee ; how many can be had for 18 rupees ?

11. There are 16 annas in a rupee. How many annas are there in 15 rupees ?

12. There are 7 days in a week ; how many days are there in 13 weeks ?

13. There are 12 pence in a shilling ; how many pence are there in 16 shillings ?

14. By how much is 9 times 15 more than 105 ?

15. By how much is 15 times 13 less than 200 ?

16. What number exceeds 8 times 12 by 15 ?

17. Find the value of :—

(1). 173×4.	(2). 276×8.	(3). 891×9.
(4). 503×8.	(5). 4217×5.	(6). 8134×7.
(7). 5021×4.	(8). 7502×4.	(9). 73125×6.
(10). 85763×8.	(11). 854361×8.	(12). 273046×9.
(13). 257×100.	(14). 8976×1000.	(15). 2632×10000.
(16). 8543×200.	(17). 7617×3000.	(18). 9732×5000.
(19). 61234×700.	(20). 7145×9000.	(21). 85342×800000.

18. Subtract 3 times 8405 from 9 times 4374.

19. What is the price of 3523 maunds of rice at 5 rupees per maund ?

20. A train travels at the rate of 25 miles per hour. What distance will it pass over in 7 hours ?

21. 3520 cubits make one mile ; how many cubits will make 2385 miles ?

22. In a building there are 25 rooms; in each room there are 16 windows; and in each window there are 10 iron bars. How many iron bars are there in the whole building ?

23. A pound contains 20 shillings, each shilling 12 pence and each penny 4 farthings. How many farthings are there in 4318 pounds ?

24. A man purchased 321 yards of cloth at 5 rupees per yard and sold it at 9 rupees per yard. What was his gain ?

25. A man gave away to his first son 4 times 425 rupees and to his second son 7 times 125 rupees. How much had he ?

26. Multiply :—

(1). 95768023 by 871209. (2). 36453012 by 7634021.
(3). 789072005 by 81437. (4). 1200240048 by 12036.
(5). 12036530 by 63021. (6) 5678000 by 91011000.
(7). 7800643 by 8756008.

27. Find the squares of—
3, 6, 7, 8, 9, 5, 11, 12 and 13.

28. Find the values of—
64^2, 77^2, 182^2, 736^2, 9413^2, 67814^2.

29. Find the cubes of—
6, 3, 2, 7, 8 and 9.

30. Find the values of—
63^3, 76^3, 87^3, 126^3, 718^3, 976^3, 1234^3.

31. Find the values of— 129^4, 72^6.

Additional Examples V.—Division.

1. How many times is 6 contained in 18? 8 in 32? 9 in 72? 13 in 65? 15 in 105?

2. Divide 91 into 13 equal parts; 144 into 8 equal parts.

3. How many times can 6 be taken from 48; 7 from 49: and 9 from 81?

4. What is the fifth part of 115; the eighth part of 128; and the seventh part of 119?

5. What is the remainder when 6 is subtracted as often as possible from 65? 7 from 72? 9 from 83?

6. Find the value of :—

(1). $48 \div 12$; $64 \div 16$; $72 \div 18$; $57 \div 19$.
(2). $225 \div 15$; $196 \div 14$; $323 \div 17$; $288 \div 16$.
(3). $285 \div 19$; $289 \div 17$; $342 \div 18$; $361 \div 19$.

7. Find the quotient and the remainder when 50 is divided by 7; 60 by 8; 70 by 9; 120 by 11; and 160 by 15.

8. How many times does the fifth part of 90 contain 6 and the eighth part of 120 contain 5?

9. 135 rupees were divided equally among 15 boys. How many did each get?

10. There are 12 pence in a shilling. How many shillings are there in 192 pence ?

11. A man purchased 16 seers of *sandes.* for 208 annas. What is the price of a seer ?

12. Ram purchased 240 mangoes at 15 per rupee. How much did he pay ?

13. Divide:—

(1).	1324 by 25.	(2).	4104 by 37.	(3).	3216 by 41.
(4).	2467 by 82.	(5).	5674 by 97.	(6).	8763 by 99.
(7).	37213 by 134.	(8).	47632 by 713.	(9).	56783 by 893.
(10).	4136 by 213.	(11).	28563 by 517.	(12)	94307 by 854.

14. Find the value of :—

(1).	$800067 \div 50001.$	(2).	$957818 \div 53212.$
(3).	$63908920 \div 53213.$	(4).	$10962198 \div 69381.$
(5).	$1220024732 \div 200563.$	(6).	$1165574308 \div 17074.$
(7).	$35088008759188 \div 74921.$	(8).	$47827453540789 \div 8396317.$

15. Distribute 6345 oranges among 235 boys.

16. 12 months make one year. Find the monthly income of one whose yearly income is 6348 rupees.

17. 52 weeks make one year. Find the weekly income of one whose yearly income is 2080 rupees.

18. Find the number which when multiplied by 5678, will give 85170.

19. A man had 8 sons and 4 daughters. He at his death left 60,000 rupees to be divided among his children in such a manner that each daughter should receive half of what each son received. If the shares of the sons be all equal, find what each received.

20. Find the number which being multiplied by 532 will produce a number less than 20254 by 38.

21. The yearly expenditure of a family is 133225 rupees ; if a year consists of 365 days, find its daily expenditure.

22. A man purchased 273 sheep for 1042 rupees. At what price must he sell each to gain 50 rupees on the whole ?

23. A man purchased 15 cows and twice as many sheep for 360 rupees ; the price of a cow being 4 times that of a sheep, what is the price of each ?

24. Find by the method of short division the value of :—

(1).	789 − 7.	(2).	857 ÷ 6.	(3).	6132 ÷ 8.	(4).	2713 ÷ 11.
(5).	7531 ÷ 13.	(6).	9763 ÷ 8.	(7).	1857 ÷ 12.	(8).	5852 ÷ 14.
(9).	3442 ÷ 15.	(10).	9474 ÷ 17.	(11).	8157 ÷ 18.	(12).	2478 ÷ 19.

25. A man travels 18 miles a day ; in how many days will he travel 7596 miles?

26. If 7855 rupees be divided between Syam and 15 boys so that Syam shall receive what remains after divding the sum equally among the 15 boys, find the share of Syam and that of each of the 15 boys.

27. Divide by Factors :—

(1). 9463 by 16, 20, 32, 65 and 78.

(2). 2671851 by 128, 144, 135 and 168.

(3). 6284573 by 450, 840 and 252.

(4). 9784532 by 540, 384 and 243.

28. Divide :—

(1). 5321 by 30, 40, 50, 60 and 70.

(2). 67891 by 100, 200, 300, 500 and 700.

(3). 94601725 by 1200, 1300, 1400 and 1800.

(4). 8460172 by 1500, 15000 and 13000.

(5). 6324070 by 80, 900 and 9000.

(6). 78006700 by 780, 8900 and 566000.

Additional Examples VI.—Factors.

1. Resolve the following into their elementary factors :—

(1).	4 ;	8 ;	12 ;	15 ;	18 ;	20.
(2).	24 ;	25 ;	28 ;	30 ;	35 ;	39.
(3).	42 ;	49 ;	52 ;	56 ;	60 ;	64.
(4).	72 ;	75 ;	84 ;	88 ;	95 ;	96.
(5).	105 ;	112 ;	132 ;	188 ;	196 ;	200.

2. Resolve the following into their elementary factors :—

(1).	207 ;	252 ;	215 ;	288 ;	432 ;	495.
(2).	525 ;	625 ;	729 ;	999 ;	972 ;	1296.
(3).	1872 ;	1331 ;	1456 ;	1980 ;	2000 ;	3675.
(4).	4815 ;	7425 ;	5250 ;	7623 ;	9748.	
(5).	9009 ;	15015 ;	11025 ;	18413 ;	48510.	
(6).	118125 ;	74088 ;	405769 ;	750684 ;	750750.	

3. Find which of the following numbers are prime :—

199 ; 197 ; 936 ; 991 ; 2197 ; 2809.

Additional Examples VII.—Greatest Common Measure.

1. Find the G. C. M. of :—

(1).	6 and 8.	(2).	10 and 15.	(3).	12 and 18.
(4).	9 and 15.	(5).	10 and 16.	(6).	15 and 25.
(7).	20 and 35.	(8).	21 and 28.	(9).	25 and 80.
(10).	39 and 52.	(11).	46 and 69.	(12).	56 and 80.

(2). Find the G. C. M. of :—

(1).	324 and 328.	(2).	391 and 629.	(3).	2542 and 5487.
(4).	2809 and 6731.	(5).	2720 and 5152.	(6).	4559 and 7003.
(7).	4067 and 2573.	(8).	10395 and 16819.	(9).	80934 and 110331.

(10).	110319 and 127308.	(11).	66429 and 169037.
(12).	218707 and 526769.	(13).	2698703 and 54987261.
(14).	232, 290 and 493.	(15).	805, 1311 and 1978.
(16).	837, 1134 and 1347.	(17).	504, 5292 and 1520.
(18).	3328, 11008 and 1380.	(19).	16650, 10730 and 1961.
(20).	30, 110, 140 and 680.	(21).	12558, 20769, 47403 and 12581.
(22).	102, 612, 476, 816 and 428.	(23).	30, 1270, 255, 435 and 495.

4. What highest number will divide 91, 104 and 117, and leave the remainders 7, 8 and 9 respectively ?

5. Find the highest number which will divide 563, 1201 and 1566 and leave the remainders 17, 18 and 19 respectively.

6. The sum of two numbers is 119 and their G. C. M. is 17. Find the numbers.

7. The sum of two numbers is 400 and their G. C. M. is 50. Find the numbers.

8. What highest number will divide 739 and 916, leaving the remainders 4 and 6 respectively ?

9. Find the greatest number which will divide 1284, 1908 and 2532 leaving the remainders 34, 33 and 32 respectively.

10. Find the highest number which will divide 805, 1311 and 1978 and leave no remainder.

11. Find the highest number of beggars among whom 1485 rupees and 1809 pieces of cloth can be equally divided.

12. In a school 40 boys were Hindus, and 24 boys Musulmans. They were separately divided into equal groups. Find the highest number of groups that can be thus formed.

13. A labourer was engaged for Rs. 5. 5 as. for a certain number of days. He was absent for some days and got Rs. 2-13 as. only. Show that his daily wages were not more than 5 annas.

Additional Examples VIII.—Least Common Multiple.

1. Find the L. C. M. of :—

(1). 4 and 8. (2). 6 and 9. (3). 8 and 12. (4). 9 and 15.
(5). 12 and 16. (6). 20 and 30. (7). 2, 3 and 6. (8). 4, 8 and 12.
(9). 8, 12 and 16. (10). 10, 15 and 20.

2. Find by resolving into factors the L. C. M. of :—

(1). 28 and 63. (2). 72 and 80. (3). 108 and 144.
(4). 6, 15 and 20. (5). 4, 14 and 21. (6). 18, 20 and 30.
(7). 18, 45, 54 and 81. (8). 16, 24, 28 and 42.
(9). 556 and 580. (10). 790 and 852.

3. Find the L. C. M. of :—

(1). 12, 18, 24. (2). 9, 6, 24. (3). 8, 16, 20.
(4). 8, 12, 6, 10. (5) 6, 15, 24, 25, 30.
(6). 12, 18, 30, 48, 60. (7). 8, 16, 64, 40, 120.
(8). 8, 16, 32, 64, 12, 108. (9). 27, 33, 121, 484, 264.
(10). 5, 7, 16, 28, 48, 766. (11). 12, 16, 18, 28, 32, 40, 42.
(12). 10, 15, 21, 24, 35, 45, 63, 70.
(13). 24, 35, 52, 60, 91, 108, 126, 156, 315.
(14). 8, 9, 7, 17, 23. (15). 7, 11, 13, 17, 19.
(16). 32, 37, 41, 71, 83. (17). 103, 137, 163, 181.

4. Find the least number which when divided by 7, 8, 15 and 20 respectively will leave no remainder.

5. Find the least number which when divided by 28 and 35 will leave the remainder 9 in each case.

6. Find the least number which when added to 8 will be exactly divisible by 20, 25, and 30 respectively.

7. Find the least number from which 8 being taken away the remainder will be exactly divisible by 20, 25 and 30 respectively.

8. A, B and C walk round a circular field. They start at the same time, from the same place and at the same direction. A walks at the rate of 4 miles an hour, B 12 miles and C 16 miles. Find the time when they will meet one another for the first time.

9. Four bells begin to toll simultaneously, and they toll at intervals of 5, 7, 8 and 10 minutes respectively. When will they again toll together for the second time?

10. Three men can travel at the rate of 8, 12 and 16 miles per day. Find the least distance travelled by each in a complete number of days.

11. A debt can be cleared only by rupees or half-rupees or quarter-rupees or two-anna pieces. Find the least amount of any such debt.

12. The fore wheel of a carriage is 10 feet 4 inches and the hind wheel 15 feet 6 inches. Find the least distance in which both the wheels will make a complete number of revolutions.

13. The G. C. M. of two numbers is 5 and their L. C. M. is 975. Find the numbers.

14. The length of a road is less than 300 yards. It can be measured by four rods 8 cubits, 9 cubits, 10 cubits and 12 cubits long. Find the exact length of the road.

Additional Examples IX.—Miscellaneous.

1. Subtract 9009 from the sum of 6533, 412, 3017 and 90138.

2. The difference of two numbers is 30451 and one of them is 573210. Find the other.

3. The product of two numbers is 118400 and the half of one is 160. Find the other.

4. Find the number which when multiplied by 275 will produce the same result as 325×715.

5. The product of two numbers in 35103159 and the double of one is 9126. Find the other.

6. Find the number which when divided by 528 will give 36 as quotient and 44 as remainder.

7. If the price of 907 maunds of sugar is 10884 rupees, what is the price of a maund?

8. The sum of two numbers is 100043; and one of them when divided by 4 gives 1433 as quotient. Find the numbers.

9. The worth of a building is 3 times that of the land on which it stands. If the price of the land is 7231 rupees, find the price of both.

10. Find the number of which 2125 is both quotient and divisor.

11. Ram and Syam together earned 750083 rupees. Ram earned 999 rupees more than Syam. Find how much each earned.

12. Simplify :—$\{31+2\times(16\div4)\}\div13-3$.

13. Show that $\{17\times3+13\times6\}\div3=(8+1)(8-1)-4\times45\div9$.

14. Find the numbers which will divide 107 and leave a remainder 2.

15. What is the difference of two numbers the sum of which is 62900, and the larger of which is 3 times as great as the smaller?

16. How many times is 13 to be added to 201 so that the sum may be the same as 74×5?

17. B was born when A was 30 years old. C was 9 years old when B was married. A died at the age of 70 when C was 25 years old. What was the age of B when he was married?

18. Simplify :—$52\div13\times15\div3\times50\div10\times3$

19. Find the sum of all the numbers made up of the three digits 7, 0 and 9.

20. Find the number which being added to 5 and the sum being multiplied by 13, the result is the same as seven times 390.

21. The dividend is 111111, the quotient 100 and the remainder 11. Find the divisor.

22. The price of a horse and a carriage is 1200 Rs. and the price of the horse is 2 times that of the carriage; what is the price of each?

23. Simplify:—$33\times(80\div16+90\div18)\div10+43\times(210\div7-220\div55)\div13$.

24. In a cistern there are 3 pipes attached to it; by two of the pipes 825 and 713 maunds of water enter into the cistern and by the third 1728 maunds of water go out per hour. Find how much the cistern holds if the cistern when full and when all the three pipes work be emptied in 325 hours.

25. Find the least number which when added to 66665, will make the sum exactly divisible by 39.

26. A man rides for 6 hours at the rate of 10 miles an hour and comes back at the rate of 6 miles an hour. How long will he take in coming back?

27. Find the number which being added to 315 and the sum being divided by 11 and the quotient being multiplied by 15 the product will be 450.

28. If in dividing a number by 189 we divide it by the factors 3, 7, 9 in succession and the successive remainders be 1, 0 and 1, what is the complete remainder ?

29. Show that $(837921-77709)-(837923-89754) = (89752-999)-(77709-999)$.

30. Two ships sail at the rate of 8 and 10 miles per hour respectively. If they sail in the same direction, when will there be a distance of 18 miles between them ?

31. A's age is 3 years less than B's, B's age 5 years less than C's, and C's age 10 years less than 50 years. Find the respective ages of A and B.

32. Ram had with him 39 rupees more than what Hari had and Hari had 39 rupees more than 81 rupees. Gopal had 3 times as much as Ram and Hari together. Find what Gopal had.

33. A book contains 231 pages, each page contains 28 lines, and each line contains 15 words. Find how many words there are in 578 copies of such a book.

34. A gentleman in distributing a certain sum among 87 men at 8 rupees per head found that he had 5 rupees too short. How much had he with him ?

35. Find the least number which being added to 91134, the sum will be exactly divisible by 437.

36. Subtract $(1+2+3)\times(17+19-32)$ from $8\times99\div11\times108\div36\times55\div5$.

37. Divide 515 rupees among A, B and C so that A will have 15 rupees less than B and B 10 rupees less than C.

38. A man earns 5 Rs. a day and spends 19 Rs. in 4 days. If a year consists of 365 days, find what he will save in a year.

39. What a man saves by spending 400 Rs. a month in 8 years, he spends by spending 600 Rs. a month in 8 years. What is his monthly income ?

40. A man travelled from Calcutta to his house in the mufussil in 8 days. Another man travelled twice the distance in 20 days walking at the rate of 12 miles a day. Find the rate of the first per day.

41. In a farm there are employed 15 men, 30 women and 120 children. The work of a man is twice that of a woman and of a woman 3 times that of a boy. Find how many women alone must be employed to do the work of the farm.

Additional Examples X.—(Transformation of Fractions).

1. Convert each of the whole numbers 5, 7, 8, 9, and 12 into a fraction with the denominator 13.

2. Convert 12, 14, 19 and 22 into fractions with the denominators 15, 16, 18 and 19 respectively.

3. Reduce 100, 200, aod 300 to fractions with denominators 200, 300, and 400 fespectively.

4. Reduce the following to their lowest terms

(1). $\frac{3}{6}$, $\frac{4}{8}$, $\frac{6}{8}$, $\frac{6}{10}$, $\frac{8}{12}$, $\frac{6}{16}$, $\frac{3}{9}$, $\frac{6}{15}$.

(2). $\frac{7}{14}$, $\frac{8}{16}$, $\frac{9}{15}$, $\frac{12}{18}$, $\frac{10}{20}$, $\frac{21}{28}$, $\frac{24}{36}$, $\frac{18}{27}$.

(3). $\frac{25}{30}$, $\frac{32}{40}$, $\frac{33}{44}$, $\frac{39}{52}$, $\frac{36}{63}$, $\frac{48}{72}$, $\frac{27}{63}$, $\frac{22}{88}$.

(4). $\frac{66}{121}$, $\frac{144}{156}$, $\frac{115}{230}$, $\frac{187}{836}$, $\frac{280}{945}$, $\frac{975}{1000}$, $\frac{256}{1024}$.

(5). $\frac{3100}{10925}$, $\frac{10265}{14371}$, $\frac{1632}{2976}$, $\frac{6816}{10656}$, $\frac{114135}{220661}$, $\frac{135795}{222210}$.

(6). $\frac{6\times8\times15}{9\times12\times10}$, $\frac{16\times22\times39}{52\times28\times11}$, $\frac{2\times3\times8\times12}{12\times6\times4\times16}$, $\frac{11\times21\times4\times9}{51\times35\times2\times64}$.

5. Reduce the following to improper fractions:—

(1). $2\frac{2}{3}$, $2\frac{3}{4}$, $3\frac{2}{3}$, $6\frac{2}{3}$, $5\ 6$, $9\frac{7}{8}$, $9\frac{8}{9}$, $7\frac{7}{12}$.

(2). $14\frac{2}{5}$, $13\frac{1}{3}$, $11\frac{2}{5}$, $10\frac{9}{10}$, $16\frac{2}{3}$, $17\frac{3}{4}$, $18\frac{2}{3}$, $19\frac{3}{4}$.

(3). $27\frac{3}{4}$, $36\frac{9}{11}$, $47\frac{2}{5}$, $62\frac{13}{18}$, $75\frac{7}{11}$, $95\frac{13}{36}$.

(4). $317\frac{11}{15}$, $620\frac{13}{14}$, $626\frac{57}{307}$, $427\frac{5}{1107}$, $8888\frac{71}{90}$, $9999\frac{91}{99}$.

6. Convert the following to mixed or whole numbers :—

(1). $\frac{3}{2}$, $\frac{4}{3}$, 8, 12 16 21 $\frac{27}{6}$, $\frac{28}{9}$ 63

(2). $\frac{53}{12}$, $\frac{49}{14}$, $\frac{112}{16}$, $\frac{29}{12}$, $\frac{41}{18}$, $\frac{113}{10}$, $\frac{256}{15}$, $\frac{302}{14}$.

(3). $\frac{509}{16}$, $\frac{819}{13}$, $\frac{974}{19}$, $\frac{521}{25}$, $\frac{963}{21}$, $\frac{889}{55}$, $\frac{775}{65}$, $\frac{9726}{70}$.

(4). $\frac{1942}{105}$, $\frac{4723}{480}$, $\frac{9756}{129}$, $\frac{5627}{370}$, $\frac{23082}{721}$, $\frac{10027}{500}$, $\frac{123456}{1201}$.

7. Simplify :—

(1). 3 of $\frac{2}{7}$, 7 of $\frac{2}{3}$, 5 of $\frac{3}{5}$, $2\frac{2}{3}$ of 4, $9\frac{3}{4}$ of $1\frac{3}{13}$, $8\frac{7}{8}$ of $\frac{24}{213}$.

(2). $\frac{5}{6}$ of $1\frac{1}{4}$ of $4\frac{4}{5}$, $7\frac{2}{9}$ of $\frac{1}{2}$ of $10\frac{1}{8}$, 12 of $\frac{3}{4}$ of $\frac{1}{18}$ of 18,

$\frac{5}{8}$ of $\frac{3}{7}$ of $\frac{2}{3}$ of $\frac{1}{15}$ of 28.

(3). $2\frac{1}{3}$ of $3\frac{1}{7}$ of $\frac{3}{11}$ of $\frac{25}{26}$, $4\frac{3}{5}$ of $8\frac{5}{4}$ of $2\frac{3}{11}$ of $\frac{8}{15}$,

$\frac{25}{36}$ of $\frac{81}{125}$ of $\frac{100}{333}$ of $\frac{111}{625}$ of $\frac{512}{576}$.

8 Arrange in order of their magnitude :—

(1). $\frac{4}{7}$, $\frac{5}{9}$, $\frac{11}{21}$, $\frac{13}{42}$. (2). $\frac{5}{12}$, $\frac{7}{18}$, $\frac{11}{27}$, $\frac{13}{30}$, $\frac{25}{60}$.

(3). $\frac{48}{25}$, $1\frac{29}{30}$, $1\frac{49}{50}$. (4). $\frac{15}{4}$, $3\frac{1}{3}$, $\frac{2}{7}$ of $9\frac{3}{5}$.

(5). $\frac{5}{7}$ of $\frac{7}{25}$, $\frac{25}{26}$ of $\frac{39}{50}$, $\frac{7}{9}$ of $\frac{15}{28}$.

6). $\frac{35}{33}$ of $\frac{44}{63}$, $\frac{9}{13}$ of $\frac{2}{15}$, $\frac{17}{39}$ of $\frac{26}{51}$.

Additional Examples XI.—(Addition of Fractions).

Add together :—

1. $\frac{1}{3}+\frac{1}{4}$. 2. $\frac{1}{7}+\frac{2}{7}$. 3. $\frac{3}{4}+\frac{1}{3}+\frac{5}{6}$.

4. $\frac{1}{4}+\frac{1}{2}+\frac{5}{12}$. 5. $1\frac{1}{2}+2\frac{1}{2}$. 6. $2\frac{1}{3}+3\frac{2}{3}$.

7. $6\frac{2}{5}+7\frac{2}{5}$. 8. $1\frac{2}{3}+2\frac{1}{3}+3\frac{2}{5}$.

9. $\frac{3}{16}+\frac{2}{9}+\frac{5}{12}+\frac{1}{2}$.

10. $\frac{69}{81}+\frac{18}{30}+\frac{1}{3}+\frac{16}{60}+\frac{7}{63}$.

11. $\frac{3}{10}+\frac{7}{20}+\frac{9}{40}+\frac{11}{50}$.

12. $\frac{10}{11}+\frac{6}{7}+\frac{5}{14}+\frac{1}{2}+\frac{5}{22}$.

13. $1+7\frac{3}{23}+5\frac{5}{46}+\frac{1701}{69}$.

14. $3\frac{2}{15}+1\frac{9}{20}+1\frac{1}{12}+2\frac{5}{18}$.

15. $\frac{2}{3}$ of $\frac{9}{10}$ of $25+\frac{3}{4}$ of $\frac{7}{8}$ of 8.

16. $\frac{7}{11}+\frac{5}{19}$ of $2\frac{3}{8}+\frac{8}{15}+8\frac{9}{14}$ of $4\frac{17}{22}+30\frac{19}{30}$.

Additional Examples XII.—(Subtraction of Fractions).

1. Find the value of :—

(1). $\frac{2}{3}-\frac{1}{3}$. (2). $\frac{3}{4}-\frac{1}{4}$. (3). $\frac{2}{3}-\frac{1}{6}$.

(4). $\frac{7}{8}-\frac{5}{8}$. (5). $\frac{7}{12}-\frac{1}{2}$. (6). $\frac{3}{7}-\frac{2}{7}$.

(7). $1-\frac{6}{7}$. (8). $3-\frac{3}{4}$. (9). $2-\frac{5}{6}$.

(10). $10-8\frac{2}{9}$. (11). $\frac{49}{50}-\frac{9}{10}$. (12). $\frac{21}{64}-\frac{7}{48}$.

(13). $\frac{19}{30}-\frac{19}{40}$. (14). $\frac{17}{18}-\frac{11}{12}$. (15). $200-188\frac{3}{16}$

(16). $10\frac{26}{35}-8\frac{40}{49}$. (17). $19\frac{5}{432}-12\frac{11}{54}$. (18). $88-77\frac{1}{9}$.

2. Simplify :—

(1). $7\frac{7}{9}+8\frac{1}{3}-5\frac{2}{3}-4\frac{1}{9}$.

(2). $27\frac{5}{12}-12\frac{7}{15}-21\frac{39}{60}+10\frac{1}{3}$.

(3). $7\frac{3}{8}-2\frac{3}{5}+10-1\frac{5}{32}$.

(4). $47\frac{5}{27}-3\frac{2}{3}-31\frac{3}{54}-2\frac{3}{8}$.

(5). $10+\left\{2\frac{1}{2}+\left(4\frac{1}{2}-1\frac{1}{4}\right)\right\}$.

(6). $5\frac{3}{4}-\left[5\frac{3}{4}-\left\{5\frac{3}{4}-\left(5\frac{3}{4}-5\frac{3}{4}\right)\right\}\right.$.

Additional Examples XIII.—(Multiplication of Fractions)

Find the value of :—

1. $\frac{1}{4}\times 3$, $\frac{3}{5}\times 4$, $\frac{1}{9}\times 4$, $\frac{2}{11}\times 5$, $\frac{2}{9}\times 5$, $\frac{3}{17}\times 4$.

2. $\frac{7}{10}\times 7$, $\frac{3}{4}\times 5$, $\frac{6}{7}\times 7$, $\frac{13}{23}\times 3$, $\frac{14}{65}\times 13$, $\frac{24}{35}\times 7$.

3. $\frac{1}{2}\times\frac{1}{3}$, $\frac{1}{3}\times\frac{1}{4}$, $\frac{1}{4}\times\frac{1}{5}$, $\frac{1}{5}\times\frac{1}{6}$, $\frac{1}{8}\times\frac{1}{3}$, $\frac{1}{8}\times\frac{1}{8}$.

4. $\frac{1}{7}\times\frac{1}{8}$, $\frac{1}{9}\times\frac{1}{7}$, $\frac{1}{4}\times\frac{1}{7}$, $\frac{1}{6}\times\frac{1}{10}$, $\frac{1}{12}\times\frac{1}{9}$, $\frac{1}{15}\times\frac{1}{8}$.

5. $\frac{13}{24}\times 36$, $\frac{70}{121}\times 11$, $\frac{14}{33}\times 88$, $\frac{13}{34}\times 51$, $\frac{41}{112}\times 128$, $\frac{37}{47}\times 94$.

6. $\frac{41}{71}\times 639$, $\frac{4}{99}\times 88$, $\frac{91}{625}\times 125$, $\frac{77}{80}\times 10$, $\frac{43}{112}\times 252$, $\frac{100}{308}\times 880$.

7. $8\frac{5}{14}\times 35$, $3\frac{17}{22}\times 11$, $10\frac{13}{34}\times 17$, $4\frac{98}{115}\times 23$, $14\frac{5}{63}\times 147$.

8. $4\frac{3}{8}\times 2\frac{13}{28}\times\frac{4}{23}$, $19\frac{5}{7}\times\frac{3}{5}\times\frac{10}{23}$, $4\frac{5}{8}\times\frac{27}{35}\times 7\times 1\frac{3}{37}$, $\frac{1}{7}\times 14\times\frac{3}{2}\times 3\frac{5}{8}$.

9. $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}$, $\frac{7}{8}\times\frac{8}{9}\times\frac{11}{14}\times\frac{18}{33}\times\frac{12}{25}$, $2\frac{2}{3}\times\frac{9}{52}\times 4\frac{1}{3}\times\frac{4}{33}$.

10. $5\frac{1}{3}\times 3\frac{3}{4} - 4\frac{4}{5}\times 1\frac{7}{8} + 17\frac{1}{2}\times\frac{6}{7}$.

11. $\left\{\frac{8}{17}+\frac{3}{5}\times 7\frac{1}{2}\right\}\times\frac{17}{13}\times\left(1\frac{3}{4}\times\frac{2}{7}+5\frac{1}{2}\right)$.

12. $18+\frac{1}{9}\times\left\{\left(\frac{3}{4}-\frac{1}{2}\right)-\frac{1}{4}\right\}+1+\left\{\frac{3}{4}\times\frac{2}{3}+11\times\frac{1}{22}\right\}$.

Additional Examples XIV.—(Division of Fractions).

1. Find the quotient of :—

(1). $\frac{1}{2} \div 2$; $\frac{1}{3} \div 3$; $\frac{1}{4} \div 4$; $\frac{1}{5} \div 5$; $\frac{1}{6} \div 6$; $\frac{1}{7} \div 7$.

(2). $\frac{3}{7} \div 5$; $\frac{3}{8} \div 2$; $\frac{2}{9} \div 7$; $\frac{12}{13} \div 6$; $\frac{7}{9} \div 7$; $\frac{10}{11} \div 5$.

(3). $\frac{4}{19} \div 4$; $\frac{5}{19} \div 15$; $\frac{7}{17} \div 14$; $\frac{11}{21} \div 11$; $\frac{18}{23} \div 9$.

(4). $\frac{1}{5} \div \frac{1}{4}$; $\frac{1}{3} \div \frac{1}{2}$; $\frac{1}{9} \div \frac{1}{7}$; $\frac{1}{10} \div \frac{1}{7}$; $\frac{1}{11} \div \frac{1}{5}$; $\frac{2}{3} \div \frac{3}{4}$.

(5). $\frac{3}{4} \div \frac{4}{5}$; $\frac{1}{6} \div \frac{6}{1}$; $\frac{2}{3} \div \frac{5}{7}$; $\frac{2}{7} \div \frac{5}{4}$; $\frac{3}{5} \div \frac{2}{3}$; $\frac{1}{9} \div \frac{7}{9}$.

(6). $\frac{84}{112} \div 7$; $\frac{108}{123} \div 36$; $\frac{125}{259} \div 100$; $2\frac{3}{7} \div 34$; $\frac{125}{126} \div 75$.

(7). $3\frac{5}{8} \div 116$; $4\frac{1}{4} \div 85$; $5\frac{1}{7} \div 12$; $9\frac{1}{10} \div 455$; $17\frac{2}{7} \div 33$.

(8). $\frac{28}{57} \div \frac{7}{19}$; $\frac{29}{43} \div \frac{87}{215}$; $\frac{31}{60} \div \frac{3}{20}$; $\frac{7}{22} \div \frac{112}{375}$; $\frac{23}{60} \div \frac{46}{120}$.

(9). $143 \div \frac{11}{12}$; $\frac{208}{135} \div \frac{130}{441}$; $\frac{11}{12} \div \frac{121}{144}$; $152 \div \frac{19}{21}$.

(10). $3\frac{23}{35} \div 2\frac{2}{3}$ of $3\frac{1}{5}$. (11). $3\frac{3}{4}$ of $2\frac{3}{16} \div 5\frac{5}{8}$ of $8\frac{8}{9}$.

(12). $3\frac{3}{4}$ of $6\frac{2}{3}$ of $\frac{1}{4} \div 18\frac{3}{4}$. (13). $5\frac{1}{3}$ of $3\frac{3}{4} \div \left(7\frac{2}{3} - 3\frac{2}{3}\right)$.

2. Find the number of which $\frac{3}{4} + 1\frac{2}{3}$ of $1\frac{4}{5} = \frac{1}{8}$.

3. Divide the sum of $2\frac{1}{3}$, $3\frac{1}{4}$ and $5\frac{1}{6}$ by the sum of $4\frac{1}{2}$ and $6\frac{1}{4}$, and to the quotient add the difference of $10\frac{1}{10}$ and $5\frac{1}{5}$.

Additional Examples XV.—(Fractions—Miscellaneous).

[*Note.* When two or more fractions are reduced to their equivalent ones with the least common denominator, they may each be regarded as

consisting of an integral number of secondary units or parts of the primary unit, the number of secondary units in each being equal to its new numerator, and the magnitude of each secondary unit being a part of the primary unit resulting from its division by the least common denominator. Viewed in this light, fractions may have their Greatest Common Measure and their Least Common Multiple, which are respectively the Greatest Common Measure and the Least Common Multiple of the integral numbers representing their values in terms of the secondary unit, that is, the Greatest Common Measure and the Least Common Multiple respectively of their new numerators, divided by their least common denominator.

Thus taking any two fractions, $\frac{2}{3}$ and $\frac{8}{9}$, their equivalents with the least common denominator being $\frac{6}{9}$ and $\frac{8}{9}$, their G. C. M. and L. C. M. respectively are $\frac{2}{9}$ and $\frac{24}{9}$.]

1. Simplify :—

(1). $\frac{3}{7} \times 1\frac{2}{5} \times 12\frac{1}{2} \div 6\frac{2}{3}$. (2). $3\frac{3}{4} \div 4\frac{2}{7} \div 1\frac{3}{4} \times \frac{2}{3}$.

(3). $2\frac{1}{4}$ of $1\frac{1}{2} \div 1\frac{1}{8} \times 2\frac{2}{5}$. (4). $3\frac{2}{5} \times \frac{10}{11} \div 3\frac{3}{5}$ of $4\frac{7}{11}$ of $\frac{5}{54}$.

(5). $\frac{3}{4}$ of $\frac{7}{11} \times 8\frac{1}{5} \div \frac{1}{55}$ of $71\frac{3}{4}$. (6). $3\frac{1}{250}$ of $5\frac{5}{14} \div \frac{84}{490}$ of $563\frac{1}{4}$.

(7). $3\frac{3}{4}$ of $\frac{75}{18} \div 1\frac{4}{12}$ of $\frac{50}{33} \times \frac{16}{165}$. (8). $8\frac{2}{3} \times \frac{2}{13} \div 1\frac{1}{3} \times \frac{1}{2}$.

(9). $2\frac{1}{3}$ of $2\frac{4}{21} \div 3\frac{2}{7} \times 2\frac{1}{3}$.

(10). $9\frac{1}{3} \div 4\frac{1}{3} \times \frac{2}{9}$ of $\frac{4}{5} \div \frac{4}{9}$ of $\frac{3}{7} \div \frac{7}{40} \times 1\frac{17}{28}$.

(11). $12 - 2\frac{1}{8} \div 3\frac{2}{5} + 4\frac{1}{9} \div 6\frac{1}{6}$. (12). $13 - 2\frac{3}{4} \times \frac{2}{11} - 10\frac{1}{2} + \frac{9}{17} \div \frac{4}{17}$.

(13). $6 + \left(\frac{2}{5} - \frac{1}{18}\right) \times 2\frac{9}{11}$.

(14). $\left(\frac{2}{19} + \frac{1}{3}\right) \div \left(3 - \frac{1}{3}\right) \times \left(\frac{1}{3} + \frac{1}{5}\right)$.

(15). $1 \div \left[5 - 1 \div \left\{2 - 1 \div \left(1 - \frac{3}{4}\right)\right\}\right]$.

(16). $\left\{1 + \frac{5}{8} + \frac{3}{8} + \frac{11}{12}\right\} \div \left\{\frac{3}{8} - \frac{5}{6}\right\}$.

(17). $14-2\frac{1}{2}\div\left(\frac{3}{2}\text{ of }6\frac{1}{7}+\frac{3}{7}\text{ of }9\right)\div2\frac{39}{61}$.

(18). $2\frac{1}{3}-\left\{\frac{1}{3}-\left(1+\frac{1}{12}\right)\right\}\div3\left(1-\frac{3}{8}\text{ of }2\frac{1}{2}\right)$. (19). $\frac{\frac{2}{17}}{\frac{10}{51}}$.

(20). $\frac{5\frac{3}{8}}{\frac{4}{43}}$. (21). $\frac{13\frac{9}{10}}{1\frac{1}{14}}$. (22). $\frac{2\frac{1}{6}-1\frac{1}{8}}{3\frac{5}{8}-2\frac{4}{5}}$. (23). $\frac{2\frac{1}{2}+3\frac{1}{4}}{1\frac{1}{3}-\frac{3}{8}}$.

(24). $2\frac{1}{2}\times\frac{1}{3\frac{1}{3}+\frac{4}{17}}$. (25). $\frac{3\frac{5}{8}+2\frac{5}{12}-1\frac{5}{24}}{5\frac{4}{5}+2\frac{9}{10}-1\frac{9}{20}}$. (26). $\frac{3\frac{1}{2}\times3\frac{1}{2}\times3\frac{1}{2}-1}{3\frac{1}{2}\times3\frac{1}{2}-1}$.

(27). $\frac{3\frac{4}{5}+12\frac{5}{6}-15\frac{1}{8}}{4\frac{2}{5}-\frac{3}{4}\text{ of }\frac{5}{3\frac{3}{4}}}$. (28). $\frac{3+\frac{1}{4}\text{ of }\frac{21}{8\frac{1}{6}}-\frac{1}{4}-\frac{1\frac{1}{3}}{2\frac{1}{2}}}{10\times\frac{1\frac{2}{3}}{108}\text{ of }5}$.

(29). $\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{2+\frac{1}{2}}}}$. (30). $3+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{8+\frac{1}{5}}}}$.

(31). $9\div\cfrac{1}{8+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{1+\frac{2}{7}}}}}$. (32). $\cfrac{8}{3+\cfrac{1}{5+\cfrac{1}{7+\cfrac{1}{3+\frac{1}{2}}}}}$

(33). $1\frac{9}{67}\text{ of }\frac{\frac{4}{7}}{\frac{3}{7}}\times\frac{5\frac{1}{4}+3\frac{1}{3}}{5\frac{1}{4}-3\frac{1}{3}}-\frac{3\frac{1}{4}+\cfrac{1}{3+\cfrac{1}{3+\frac{3}{4}}}}{329}$

(34). $\cfrac{2}{4+\cfrac{2}{3+\frac{1}{4}}}+\cfrac{3}{3-\cfrac{2}{3-\frac{1}{4}}}+\cfrac{4}{4\div\cfrac{2}{4\div\frac{1}{4}}}+\cfrac{5}{3+\cfrac{\frac{1}{7}}{2+\frac{1}{5}}}$.

(35). $\frac{1+\frac{3}{4}+2\frac{1}{4}}{9\frac{2}{5}+3\frac{1}{2}}\text{ of }9\frac{3}{8}\text{ of }\frac{30}{56}$. (36). $\frac{3\frac{1}{2}-2\frac{1}{6}}{\frac{1}{2}\text{ of }\left(\frac{1}{6}+\frac{1}{7}\right)}\div15\frac{3}{4}$.

(37). $\frac{3\frac{1}{3}\text{ of }\frac{1}{5}}{6\frac{1}{3}-\frac{1}{3}}\div3\frac{\frac{1}{4}}{1\frac{1}{5}}+1-\frac{3}{4}\text{ of }1\frac{1}{3}$. (38). $\frac{\frac{\frac{3}{8}}{1-\frac{1}{25}}+\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{7}\left(\frac{\frac{3}{8}}{1-\frac{1}{25}}+\frac{1}{8}\right)}$

(39). $\frac{\frac{1}{3}+\frac{2}{11}+\frac{7}{24}-\frac{1}{3}\text{ of }\frac{2}{11}\text{ of }\frac{7}{24}}{1-\frac{1}{3}\text{ of }\frac{2}{11}-\frac{2}{11}\text{ of }\frac{7}{24}-\frac{1}{3}\text{ of }\frac{7}{24}}$. (40). $\frac{\frac{816}{166463}-\frac{1}{1393}}{1+\frac{1}{1393}\times\frac{816}{166463}}$.

(41). $\frac{9}{200}$ of $1\frac{2}{5}$ of $\frac{1\frac{1}{5}}{5\frac{1}{6}}$ of $\frac{\frac{3}{8}+\frac{1}{2}}{\frac{3}{2}-\frac{1}{3}}$ of

$$71\frac{3}{7} \text{ of } \frac{\frac{\frac{2}{5}}{1-\frac{1}{25}}+\frac{1}{2}+\frac{1}{7}}{1-\frac{1}{7} \text{ of } \left\{\frac{\frac{2}{5}}{1-\frac{2}{5}}+\frac{1}{3}\right\}}$$

(42). $\frac{1-1\frac{1}{5} \text{ of } \frac{3}{4}+9\frac{1}{3}\div\frac{7}{5}}{3\frac{1}{5}+\frac{1}{4} \text{ of } \frac{1}{6}\times\frac{1}{3}\div 1\frac{1}{2}} - \frac{4-\frac{1}{8}+\frac{1}{4} \text{ of } 1\frac{1}{4}\div\frac{5}{16}}{3\frac{1}{8}+\frac{8}{25}-\frac{1}{3} \text{ of } 2\frac{1}{4}}$.

(43). $\frac{7}{5-\frac{8}{3}} \div \frac{3-\frac{2}{3-\frac{3}{2}}}{4-\frac{3}{2}} - \frac{5}{7}$ of $\left\{\frac{1}{1\frac{3}{7}}+\frac{6}{5} \text{ of } \frac{3\frac{1}{3}-2\frac{1}{2}}{\frac{47}{21}-2}\right\}$.

(44). $\left\{\frac{3\frac{1}{15}+4\frac{1}{5}-3\frac{1}{6}+4\frac{1}{4}-7\frac{1}{2}}{3\frac{3}{4}+4\frac{3}{4}+5\frac{1}{2}+6\frac{1}{6}-7\frac{3}{4}} \div \frac{\frac{3}{4}-\frac{1}{2}}{4\frac{3}{4}}\right\} \div \frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}-\frac{1}{2}}$.

(45). $\left\{\frac{1+\frac{\frac{1}{2}+\frac{2}{3}}{1\frac{1}{6}}}{3\frac{1}{2} \text{ of } \frac{10}{49} \text{ of } 1}\right\} \times \left\{13+\frac{\frac{3\frac{1}{5}+4\frac{2}{3}+5}{3\frac{1}{2}+2\frac{1}{4}}}{1\frac{1}{3}\times 5}\right\} \div \frac{1}{3}$ of 2.

(46). $\frac{(\frac{4}{7}+1\frac{3}{4})\times\frac{4}{13}}{(\frac{3}{4}-\frac{4}{7})\times\frac{4}{15}}$ of $\left\{\frac{\frac{7}{15}-\frac{3}{20}}{\frac{5}{12}-\frac{4}{15}} - \frac{4\frac{2}{5}+2\frac{4}{15}}{5\frac{1}{7} \text{ of } 8\frac{3}{4}} + \frac{7\frac{2}{3}-6\frac{1}{3}}{\frac{1}{15}+\frac{1}{12}}\right\}$.

2. Find the G. C. M. and the L. C. M. of :—

(1). $\frac{2}{3}$ and $\frac{4}{5}$. (2). $\frac{8}{15}$ and $\frac{16}{25}$. (3). $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{8}{9}$.

(4). $\frac{15}{28}$, $1\frac{5}{18}$ and $1\frac{1}{20}$. (5). $\frac{7}{8}$, $\frac{3}{16}$ and $\frac{2}{5}$ of $\frac{5}{8}$.

(6). $9\frac{1}{10}$, $2\frac{1}{8}$ and $\frac{117}{200}$. (7). $1\frac{37}{38}$, $3\frac{5}{62}$ and $7\frac{25}{26}$.

3. Find the greatest number by which $8\frac{1}{2}$ and $5\frac{1}{10}$ being divided respectively the quotients will be integers.

4. Find the least number which when divided by $3\frac{1}{7}$, $5\frac{1}{2}$ and $10\frac{1}{3}$ respectively, the quotients will be integers.

5. The circumferences of three wheels are $5\frac{1}{3}$ ft., $3\frac{1}{7}$ ft., and $8\frac{2}{3}$ ft. respectively ; find the least distance in which the wheels will make complete revolutions.

6. Three bells commence tolling together ; they toll at intervals of 2, $2\frac{1}{2}$ and $3\frac{1}{3}$ minutes respectively. After what interval of time will they toll together again ?

7. Find a number to which $\frac{3}{4}$ of $\frac{2}{3}+\frac{5}{8}$ being added the sum will be 15.

8. Divide the sum of $5\frac{1}{3}$ and $4\frac{1}{2}$ by their difference.

9. Find the number from which $7\frac{1}{5}$ of $1\frac{2}{3}$ being subtracted the remainder will be equal to the sum of $\frac{2}{3}$ and $\frac{4}{5}$.

10. Find the two numbers of which the sum is $7\frac{3}{5}$ and the difference $6\frac{1}{5}$.

11. Find the least number to which $9\frac{7}{13}$ being added, the sum will be an integer.

12. What fraction is Rs. 50 of Rs. 500 ?

13. Find the number of which $\frac{3}{4}$ is greater than its $\frac{2}{3}$ by 625.

14. Find the number which being multiplied by $\frac{118}{315}$, the product will be the least possible integer.

15. Find the number of which $\frac{1}{4}$ and $\frac{1}{3}$ being added the sum will be less than its $\frac{3}{4}$ by 50.

16. Find two integers so that $\frac{1}{5}$ of the first is equal to $\frac{3}{7}$ of the second.

17. A pole has $\frac{1}{4}$ of its length in the mud, $\frac{1}{3}$ in water and 12 ft. above the water. Find the length of the pole.

18. A man gives away $\frac{3}{5}$ of his property to his eldest son and the remainder to his second son. The eldest receives 4000 Rs. more a year than the second. Find the annual income of the property, and also the income of each of the brothers.

19. A man bequeathed $\frac{2}{7}$ of his property to his eldest son, $\frac{8}{11}$ to the second and $\frac{1}{5}$ to the third and the rest to his fourth son. If the income of the first and the fourth together be 19000 Rs. a year, what is the yearly income of the whole property ?

20. Find the least fraction which being added to the sum of $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{11}{12}$, the result will be a whole number.

21. A boy was asked to divide one half of a certain number by 11, and the other half by 13, and then to add the quotients. To save trouble he divided the number by 12, and his result was wrong by unity. What was the number ?

22. Divide 240 into two such parts that $\frac{1}{4}$ of the first being added to $\frac{1}{10}$ of the second, the sum will be 36.

23. In a meeting there were 1200 men present of whom $\frac{1}{4}$ were Bengalis ; there were $1\frac{1}{2}$ times as many Bengalis as Hindustanis ; $1\frac{1}{3}$ times as many Hindustanis as Englishmen ; and $\frac{1}{5}$ as many Rajputs as there were Bangalis, Hindustanis and Englishmen together ; and the rest were Shikhs. Find the number of each nationality.

Additional Examples XVI.—(Decimals—Conversion.)

1. Express as vulgar fractions :—

(1). ·21 ; ·3 ; ·31 ; ·01 ; ·011 ; ·0019 ; 39·021.

(2). 87·5319 ; ·489 ; 33·0719 ; 888·4317 ; 5·00009.

2. Express as vulgar fractions in their lowest terms :—

(1). ·2 ; ·4 ; ·5 ; ·8 ; ·15 ; ·05 ; ·08.

(2). ·0005 ; ·00025 ; 1·05 ; 2·1875 ; 4·375 ; ·8125.

(3). ·37875 ; 6.5625 ; 6·0025 ; ·00006875 ; ·0000096875.

3. Express the following decimals as mixed numbers :—

5·5 ; 8·15 ; 12·625 ; 11·025 ; 140·0725 ; 13·675.

4. Express as decimals :—

(1). $\frac{7}{10}$; $\frac{9}{10}$; $\frac{121}{100}$; $\frac{57}{100}$; $\frac{91}{100}$; $\frac{82353}{10000}$; $\frac{5721}{10000}$.

$\frac{61387}{1000000}$; $\frac{2367}{10000000}$; $\frac{71}{100000}$; $\frac{5135}{100000}$; $\frac{1}{10000000}$.

Additional Examples XVII.—(Decimals—Addition.)

1. Add together

(1). 9·4, 8·07, 5·2, ·117.

(2). 12·1, 12, 1200, 12·01.

(3). ·675, ·4213, 4·7512, 2·401, ·61342.

(4). 7·3, 573·45, ·008742, 961, ·01, ·009.

(5). 2·3, 108·72, 97·000071, 337·000125.

2. Find the value of

(1). 503·219 + 8·4314 + 387·6 + ·0001 + ·0007.

(2). £713·1 + £91 + £71·02 + £512·73.

(3). Rs·000725 + Rs 725 + Rs ·725 + Rs 7·25.

(4). 531·1 miles + 51·01 miles + 5101 miles + .512 miles.

Additional Examples XVIII.—(Decimals—Subtraction).

1. Subtract

(1). 6·432 from 7·43 ; ·999 from 9 ; ·518 from 13 ; ·9999 from 1.

(2). 5·0002 from 7·51 ; ·00001 from ·1 ; 12·02 from 31·01.

2. Find the difference between

(1). ·0001 and ·1 ; 712·001 and 72·120001 ; 80·14 and 90.
(2). Seven hundred and seven hundredths.

3. Find the value of :—

(1). Rs10 − Rs9·336 ; £21 − £19·0021 ; £1 − £·9901752.
(2). 500 − 201·3 − 12·004 − 13·1 − 14·6 − 19.
(3). 27 − ·27 + ·027 − ·0027 − ·000027.
(4). 4·25 − ·32 − (7·13 − 1·03) + 12·001 − ·0001.

4. Of 1·1718 and 1·1719 which is nearer in value to 1·171819 ?

Additional Examples XIX.—(Decimals—Multiplication.)

Find the value of

(1). 2·3 × 3·2 ; 4·5 × 4·05 ; 71 × ·21 ; 41·71 × 3·4.
(2). 8751·2 × 73·1 ; 692 × ·035 ; 13.14 × 11·17 ; ·0703 × ·00055.
(3). 53·7021 × 3734 ; 74·192 × 2·008 ; 5·4925 × 1·498.
(4). ·0606 × ·0001 ; 8753 × ·8753 ; 4·7 × 4·7 × 4·7.
(5). ·4005 × 4·4 × 8·03 × 5 ; 2·1 × 21 × 210 × 2100.
(6). $(5·73 - 4·63)^2$; $(·6)^3 - (·06)^3$; 8·253 − 2·12 × ·201.
(7). $875 \times ·4 + (·003 \times 5)^2 - (·8 + ·7)^3 - (·711 - ·71)$.

Additional Examples XX.—(Decimals—Division.)

1. Find the value of

(1). 74·339 ÷ ·079 ; 10·543 ÷ 13 ; 615·6 ÷ 5·13.
(2). 140·53 ÷ ·00611 ; ·01001 ÷ ·001 ; 41·825 ÷ 1·28.
(3). 145·817 ÷ 563 ; 8886·66 ÷ ·0037 ; ·8748 ÷ 1·08.
(4). ·06227 ÷ 1300 ; ·0009197 ÷ 1360 ; 2078·61 ÷ 579.

2. Divide ·001596 by 42, ·42, ·042, ·0042, ·00042.

3. Find the value (to four places of decimals) of
156·25 ÷ 25 ; 456·8 ÷ 3060·125 ; ·000001 ÷ ·0000431.

4. Find by short division the value (to 4 places of decimals) of
229 ÷ ·007 ; ·0012 ÷ 13 ; 2·4567 ÷ ·04 ; ·214 ÷ ·17.

Additional Examples XXI.—(Decimals—Conversion.)

1. Reduce to decimals :—

(1). $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{16}$; $\frac{3}{8}$; $\frac{3}{4}$; $\frac{19}{25}$; $\frac{7}{8}$; $\frac{13}{16}$.

(2). $\frac{97}{128}$; $\frac{47}{125}$; $\frac{177}{250}$; $\frac{111}{125}$; $\frac{7}{6125}$; $\frac{71}{625}$.

2. Compare the values (by reducing to decimals) of :—

(1). $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$. (2). $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{8}$. (3). $\frac{3}{10}$, $\frac{7}{12}$, $\frac{4}{15}$. (4). $\frac{4}{5}$, $\frac{11}{20}$, $\frac{1}{24}$.

3. Find the value of

(1). ·272 of $\frac{3}{16}$. (2). ·00125 of $\frac{81}{625}$. (3). 5·12 of $\frac{1}{8}$ ÷ ·003 of $\frac{1}{4}$.

(4). $\frac{\cdot 0075 \times 2\cdot 1}{\cdot 0175}$. (5). $\frac{4\cdot 255 \times \cdot 0064}{\cdot 00032}$. (6). $\frac{5\cdot 4 \times 21\cdot 25}{7\cdot 5 \times \cdot 046875}$.

4. Reduce to recurring decimals :—

(1). $\frac{7}{13}$; $\frac{8}{15}$; $\frac{4}{15}$; $\frac{7}{18}$; $\frac{14}{33}$; $\frac{13}{22}$; $\frac{7}{75}$; $\frac{1}{41}$.

(2). $\frac{10}{13}$; $\frac{9}{17}$; $3\frac{6}{35}$; $1\frac{8}{27}$; $5\frac{1}{48}$; $\frac{1}{96}$; $\frac{243}{375}$.

(3). $7\frac{25}{132}$; $\frac{541}{1084}$; $\frac{1000}{407}$; $\frac{715}{52}$; $\frac{43}{444}$; $4\frac{19}{75}$; $\frac{1}{995}$.

5. Reduce to vulgar fractions in their lowest terms :—

(1). $\cdot\dot{2}\dot{7}$; $\cdot\dot{0}\dot{3}$; $\cdot 0\dot{0}\dot{2}$; $\cdot\dot{4}\dot{2}$; $\cdot\dot{3}\dot{6}$; $\cdot\dot{6}2\dot{1}$; $\cdot 1\dot{5}\dot{6}$.

(2). $3\cdot\dot{1}9047\dot{6}$; $10\cdot 02\dot{2}\dot{7}$; $60\cdot 9\dot{2}\dot{4}$; $\cdot 0012\dot{3}\dot{6}$.

Additional Examples XXII.—(Recurring Decimals.)

1. Find the sum of

(1). $\cdot 7\dot{3} + \cdot 1\dot{5}\dot{6}$. (2). $\cdot 00\dot{5} + \cdot\dot{1}\dot{8} + \cdot\dot{4}2$.

(3). $4\cdot 0\dot{3} + 1\cdot 0\dot{2} + 2\cdot 0\dot{1}3$. (4). $1\cdot\dot{1} + \cdot 06\dot{1}\dot{2} - \cdot 4\dot{1}2\dot{3}$.

(5). $3\cdot\dot{7}67\dot{1} + \cdot 0\dot{6}2\dot{1} + \cdot 0\dot{6}\dot{1} + \cdot 24$.

(6). $5\cdot 0\dot{6} + 6\cdot 1\dot{7}\dot{3} + 1\cdot 2\dot{4}3\dot{1} + 6\cdot 15\dot{7}\dot{2}$.

(7). $81 + 4\cdot 4\dot{2}\dot{3} + 2\cdot 00\dot{5}\dot{1} + \cdot\dot{0}1\dot{2}$.

(8). $23\cdot\dot{0}7\dot{6} + 19\cdot\dot{2}4\dot{5} + 31\cdot 2\dot{0}\dot{3} + 5000$.

2. Find (to five places of decimals) the sum of :—

(1). $3\cdot6\dot{5}2\dot{1}+21\cdot0\dot{0}3\dot{4}+6\cdot1\dot{4}5\dot{7}+9\cdot21\dot{4}\dot{1}$.

(2). $5\cdot213\dot{4}+6\cdot4\dot{1}\dot{3}+7\cdot\dot{1}3\dot{4}+48+5\cdot21\dot{3}4\dot{6}$.

3. Find the value of :—

(1). $20\cdot\dot{3}\dot{1}-17\cdot\dot{2}\dot{5}$. (2). $3\cdot2\dot{3}45\dot{1}-1\cdot5\dot{2}056\dot{1}$.

(3). $4\cdot1\dot{3}0\dot{2}-1\cdot0\dot{5}\dot{2}$. (4). $4\cdot0\dot{0}1\dot{2}-.\dot{9}12\dot{3}$.

(5). $17\cdot\dot{5}7\dot{3}-14\cdot5\dot{7}$. (6). $1\cdot49\dot{8}-\cdot630\dot{6}\dot{4}$.

4. Find the value (correct to five places of decimals) of :—

(1). $7\cdot123\dot{6}0\dot{1}-2\cdot\dot{3}\dot{4}$. (2). $9\cdot2\dot{0}5\dot{1}-8.002\dot{4}$.

5. Multiply

(1). $71\cdot\dot{3}4\dot{3}$ by $1\cdot012$. (2). $7.36\dot{7}\dot{4}$ by 560.

(3). $\cdot41\dot{3}$ by $\cdot\dot{2}\dot{7}$. (4). $21\cdot\dot{0}\dot{9}$ by $\cdot\dot{1}\dot{4}$.

6. Find the value (correct to 5 places of decimals) of :—

(1). $13.5791113\times24\cdot681012$.

(2). 3.14159×391393.

7. Divide

(1). $40\cdot010\dot{2}$ by $\cdot102$. (2). $3\cdot46$ by $2\cdot3$.

(3). $3\cdot6$ by $\cdot02\dot{7}$. (4). $\cdot\dot{3}\dot{7}$ by $\cdot\dot{1}4\dot{8}$.

8. Find the value (correct to 4 places of decimals) of :—

(1). $3\cdot648726\div7\cdot7$. (2). $\cdot5361085\div23\cdot450842$.

Additional Examples XXIII.—(Decimals—Miscellaneous.)

1. Simplify

(1). $\dfrac{\cdot375\times\cdot375-\cdot025\times\cdot025}{\cdot375-\cdot025}$.

(2). $\dfrac{2\cdot8+1\cdot1+9\frac{1}{11}}{4\cdot5-2\cdot5+9\frac{1}{11}}$. (3). $\dfrac{2\times\cdot2\dot{5}-\frac{3}{\cdot4}\text{ of }\cdot16}{1\cdot9}$.

(4). $\dfrac{1}{\cdot 4+\dfrac{1}{\cdot 5+\dfrac{1}{\cdot 4}}}$. (5). $\dfrac{3\frac{1}{2}}{\cdot 6-\dfrac{3}{\cdot 71-\cdot 21}}$

(6). $1\frac{1}{11}-\dfrac{1-\frac{7}{22}}{2-\frac{1}{3}}+\dfrac{1\cdot 4}{3\cdot 5}-\dfrac{6\frac{1}{4}}{6\cdot 25}$ of $\left\{\cdot 2-\dfrac{\cdot 5-\cdot\dot{3}}{4\cdot 75-3\cdot\dot{2}}\right\}$.

(7). $\dfrac{\cdot 2\times\cdot 2\times\cdot 2+\cdot 02\times\cdot 02\times\cdot 02}{\cdot 4\times\cdot 4\times\cdot 4+\cdot 04\times\cdot 04\times\cdot 04}$

(8). $1\frac{3}{5}$ of $\dfrac{\cdot 5+\cdot 25+\cdot 75}{1\frac{2}{3}-1\frac{7}{12}+\frac{3}{4}}\div\left\{\dfrac{1}{4\frac{2}{3}}\text{ of }\dfrac{4\frac{2}{3}}{2\cdot 25}+\dfrac{2}{4\cdot 5}\text{ of }\dfrac{\frac{3}{5}}{\frac{7}{5}}-\dfrac{2\frac{3}{7}}{18}\right\}$.

2. Find the value of :—

(1). $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+$&c. correct to 3 places of decimals.

(2). $1+\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+$&c. correct to 3 places of decimals.

(3). $1+\dfrac{1}{10}+\dfrac{1}{100}+\dfrac{1}{1000}+$&c. correct to 4 places of decimals.

(4). $1+\dfrac{1}{1\times 3}+\dfrac{1}{1\times 3\times 5}+\dfrac{1}{1\times 3\times 5\times 7}+$&c. correct to 5 places of decimals.

3. Find the number from which $\cdot 635$ being subtracted, the remainder will be $7\cdot 632$.

4. Find the number to which $\cdot 45$ of itself being added, the sum will be $36\cdot 25$.

5. Simplify :—

$$(3\cdot 05-1\cdot 995)\times(\cdot 0005\div\cdot 025)+2\cdot 109.$$

6. A person having $\cdot 9375$ of a mine, sells $\cdot 458\dot{3}$ of his share ; what fractional part of the mine has he still left ?

7. A riband is $6222\cdot 8$ inches long ; how many pieces each $2\cdot 304$ inches long can be cut off from it ? and how many inches will be left ?

8. A man gave away $\cdot\dot{3}1$ of his property to his son and the remainder to his wife. If the wife's property be valued at ₨124, what ought to be the value of the son's property ?

9. $\cdot 75$ of a property is worth £8000. What is the worth of the whole ?

10. Of the whole number of boys in a school ·75 are Hindus, ·075 are Christians and the rest are Mahomedans. If the number of Mahomedans be 70, what is the number of each of the other two sects ?

11. Ram can do ·03 of a piece of work in a day. Jadu and Hari together can do ·17 of the same work in one day. In what time can Ram, Jadu and Hari together finish the whole work ?

12. In a garden there are 10,000 trees of which ·5725 are mango-trees, ·4203 jack-trees, and the rest cocoanut-trees. Find the number of trees of each kind.

13. The product of two numbers is ·00264 and their L. C. M. is ·00132. Find their G. C. M.

14. A man walked 60 miles in 6 days. In each of the first five days he walked an equal distance, and in the sixth day he walked 8·75 miles. What distance did he walk in the first day ?

15. Find the least number of articles costing Rs. 3·71875 each, that can be purchased for an integral number of rupees.

16. How many times can a vessel whose capacity is 3·456 gallons, be filled from a cask containing 192 gallons, and how many gallons will be left ?

17. A book containing 301 leaves is 1·876 inches thick ; allowing ·07 of an inch for the cover, find the thickness of the paper.

18. What decimal added to $\cdot\dot{0}7692\dot{3} - \cdot\dot{2}3076\dot{9} + \cdot\dot{3}0769\dot{2}$ will make the result equal to $\cdot\dot{5}3846\dot{1}$?

19. How many cards each 3·75 inches long and 1·5 inches broad can be cut out of a piece of card-board 72·25 inches long and 19·75 inches broad ?

20. A boy gave ·16 of his pocket money to one companion, $\cdot08\dot{3}$ of what remained to another, and $\cdot0\dot{5}\dot{4}$ of what still remained to a third ; and he found that he had Rs26 still left. How many rupees had he at first ?

21. Find the G. C. M. of :—

(1). 1·44 and ·108. (2). ·002 and ·06.

(3). ·09, ·027 and 4·1. (4). 8·7, 1·45 and 23·2.

22. Find the L. C. M. of :—

(1). ·12, 5·4, and ·42. (2). ·14, ·021, ·0035, and 42.

(3). ·22, ·33, ·044, and ·297. (4). 15, ·35, 5·5, ·075, and 1·65.

Additional Examples XXIV.—Reduction.

1. Reduce to pice :—

(1). Rs21. (2). Rs32. 14 as. (3). Rs122. 8 as.
(4). Rs134. 13 as. 5 *gandas*. (5) Rs246. 5 as. 10 *gandas*.

2. Reduce to pies :—

(1). Rs24. 14 as. 3 pies. (2). Rs45. 10 as. 1 pie.
(3). Rs38. 10 as. 6 pies. (4). Rs63. 9 as. 7 pies.

3. Reduce to *gandas* :—

(1). Rs12. (2). Rs130. 4 as. 9 *gandas*.
(3). Rs121. 13 as. 13 *gandas*.

4. Reduce to *karas* :—

(1). Rs13. (2). Rs813. 3 as. 14 *gandas* 2 *karas*.
(3). Rs432. 8 as.

5. Reduce to farthings :—

(1). £20. (2). £21. 2s. (3). £123. 17s. 2d.
(4). £231. 15s. 4d. (5). 55 guineas. (6). 25 florins.
(7). 35 crowns. (8). 55 moidores.

6. Reduce to pence :—

(1). £506. (2). £324. (3). 428 half-guineas.
(4). 712 guineas.

7. If the price of a book be 1*s*., how many copies of the same can be had for £171. 15*s*. ?

8. If the price of an apple be 1*d*., how many apples can be had for £712. 17*s*. 9*d*. ?

9. Reduce to rupees :—

(1). 2034 pies. (2). 7832 pies. (3). 7132 pice. (4). 10202 pice.
(5). 7325 annas. (6). 6721 two-anna pieces.
(7). 2356 four-anna pieces. (8). 23621 *karas*.

10. Reduce to pounds :—

(1). 2346*s*. (2). 9743*d*. (3). 98749*q*.

11. Reduce to guineas :—

(1). 2136*s*. (2). 979*q*. (3). 28379*q*.

12. Reduce to rupees, annas and pies, when 1*d*. = 10 pies :—

(1). £120. 13*s*. 8*d*. (2). £425. 17*s*. (3). £1000. 17*s*. 7*d*.

13. Reduce to crowns :—

(1). £56. 12*s*. (2). £812. 5*s*. (3). £900. 10*s*.

14. Reduce to guineas :—

(1). £27. 6*s*. (2). £515. (3). £5040.

15. Reduce to pounds, shillings and pence, when 1*d*. = 10 pies :—

(1). Rs. 2. 6 as. 4 p. (2). Rs. 25. 10 as.

(3). Rs. 80. 5 as. 10 p.

16. Reduce to maunds :—

(1). 9118 chts. (2). 2567 kanchas. (3). 5213 chts.

(4). 97268l2 rattis.

17. Reduce :—

(1). 12713 oz. to cwt. (2). 73214 oz. to cwt.

(3). 5 tons to ounces. (4). 43705 pounds to tons.

18. Reduce to bighas :—

(1). 6215 sq. cubits. (2). 9621 chts. (3). 78632 chts.

19. Reduce 6 ac. 2 ro. 13 po. 6 sq. yds. to sq. inches.

20. Reduce 9625678 sq. inches to acres.

Additional Examples XXV.—(Compound Addition.)

Add together :—

1.

Rs.	as.	p.
611	12	10
512	11	9
410	10	8
309	9	7

2.

Rs.	as.	*g*.	*k*.
602	9	11	3
53	13	13	2
25	13	1	3
27	12	10	1

3.

£.	*s*.	*d*.	*q*.
10	12	5	2
9	7	8	3
12	14	11	1
17	16	15	2

4.

£.	*s*.	*d*.	*q*.
30	17	10	1
36	19	9	2
38	4	0	1
	5	1	0

5.

Mds.	seers.	chts.
175	39	2
207	31	5
900	2	3
504	35	4
200	29	10

6.

tons.	cwt.	qrs.	lbs.	oz.
25	13	3	7	6
29	4	1	9	11
23	6	2	7	5
26	0	0	2	1
91	2	3	0	5

7.

lbs.	oz.	dwts.	grs.
7	6	17	13
18	7	9	13
28	13	18	23
27	6	7	12
61	5	14	19

8.

Miles.	fur.	po.	yds.
125	6	24	2
903	7	28	2
423	6	23	1
931	5	0	3

9.

days.	hrs.	mins.	secs.
18	6	19	28
15	16	39	31
21	23	50	51
18	20	15	35

10.

Ac.	ro.	po.	sq. yds.	sq. ft.
125	2	29	25	7
197	1	28	17	6
21	3	25	27	8
617	2	30	28	5

Additional Examples XXVI.—(Compound Subtraction).

Subtract the smaller number from the greater in the following Examples :

1.

Rs.	as.	pies.
732	13	10
239	9	8

2.

Rs.	as.	pies.
5000		
325	10	6

3.

£.	*s.*	*d.*
139	8	6
87	13	2

4.

£.	*s.*	*d.*	*q.*
534	13	10	3
456	12	11	2

5.

guineas.	*s.*	*d.*
122	13	11
103	15	10

6.

Mds.	seers.	chts.
132	11	9
88	39	14

7.

Tons.	cwts.	qrs.	lbs.	oz.
124	19	2	22	11
88	17	3	20	10

8.

lbs.	oz.	dwts.	grs.
70	5	3	2
57	10	16	9

9.

Miles.	fur.	po.	yds.
931	3	2	2
817	7	25	4

10.

Ac.	ro.	po.	sq. yds.	sq. ft.
923	1	29	28	5
732	3	30	29	6

Additional Examples XXVII.—(Compound Multiplication).

1. Multiply :—

(1). Rs. 25. 9 as. 4 pies by 12, 14, 16 and 18.
(2). £12. 17*s.* 4*d.* by 6, 8, 5 and 7.
(3). Rs. 9. 10 as. 13 g. by 47, 53, 59 and 67.
(4). £5. 6*s.* 6*d.* by 40, 48, 56 and 60.
(5). 6 miles. 5 fur. 20 po. 2 yds. by 18, 27, 35, 39 and 648.
(6). 48 bghs. 16 kths. 14 chts. by 8, 9, 27 and 36.
(7). 127 maunds. 25 seers. 11 chts. by 21, 33, 34 and 96.
(8). 10 tons. 10 cwt. 8lbs. by 120, 934 and 625.
(9). 5 lbs. 14 dwts. 2 grs. by 726, 841 and 1036.
(10). 80 sq. yds. 6 sq. ft. 125 sq. in. by 8, 27, 45, 64 and 182.

2. If the price of one maund of sugar be Rs. 8. 10 as. 6 pies, what will be the price of 50 maunds ?

3. The price of two horses and a cart is Rs. 287. 2 as. 3 pies, and the price of the cart is Rs. 182. 6 as. 3 pies. What is the price of a horse ?

Additional Examples XXVIII.—(Compound Division).

1. Divide :—

(1). Rs. 2482. 9 as. 6 pies by 6, 9 and 13.
(2). £530866. 17 *s.* 6*d.* by 3, 7, 8, 5 and 11.
(3). 14436 mds. 14 srs. 1cht. by 5, 7 and 9.
(4). 938 tons. 1 cwt. 3qrs. by 3479.
(5). 2827 lbs. 9 oz. 18 dwts. by 1008 and 1584.
(6). 984 miles 7 fur. 8 po. by 6, 7, 11, 21 and 33.

(7). 796 bghs. 2 kths. 8 chts. by 22, 33, 60 and 132.

(8). 15 years 7 mo. 13 d. 4 h. 34 min. by 33, 34 and 78.

(9). 2307 ac. 1 ro. 20 po. by 3, 5, 7, 15, 21 and 35.

2. The price of a house with its furniture is Rs 1202. 9 as. If the price of the furniture be 3 times that of the house, what is the price of each ?

3. If the value of a shilling be 10 as. 3 pies, how many shillings can be had for Rs 19. 3 as. 6 pies ?

Additional Examples XXIX.—(Concrete Fractions).

1. Find the value of :—

(1). $\frac{3}{8}$ of Rs 3. 5 as. (2). $\frac{1}{6}$ of Rs 25. 6 as.

(3). $1\frac{1}{6}$ of Rs 28. 13 as. 15 g. (4). $\frac{1}{6}$ of £2. 2*s.*

(5). $3\frac{1}{4}$ of £3. 11*s.* 6*d.* (6). $\frac{7}{8}$ of 3 yds. 2 ft. 3 in.

(7). $\frac{3}{5}$ of $\frac{1}{4}$ of $\frac{2}{3}$ of Rs 25. 6 as. (8). $\dfrac{1\frac{4}{9}}{2\frac{1}{6}}$ of $1\frac{1}{6}$ of 5 mds. 35 srs. 7 chts.

(9). $\dfrac{\frac{1}{3}\text{ of }\frac{3}{7}}{\frac{2}{5}\text{ of }\frac{2}{3}}$ of 5 ac. 2 ro. 30 po. (10). $\dfrac{10\frac{19}{16}}{\frac{89\times 15}{10}}$ of 5 weeks.

(11). ·375 of Rs 50. 14 as. 8 pies. (12). ·725 of Rs 28. 2 as.

(13). ·875 of £ 3. 5*s.* 6*d.* (14). 5·75 of £13. 12*s.* 8*d.*

(15). $4\cdot\dot{3}0\dot{9}$ of £2. 15*s.* (16). $\cdot\dot{2}\dot{5}$ of £2. 7*s.* 8*d.*

(17). 3·0396 of 1 mile. 530 yds. (18). 3·3275 of 3 qrs. 6 lbs.

2. Add together :—

(1). Rs.	as.	pies.	(2). £.	s.	d	(3). cwt.	qrs.	lbs.
10	5	$3\frac{1}{3}$	9	8	$3\frac{1}{2}$	3	2	$18\frac{1}{4}$
9	7	$2\frac{1}{2}$	7	6	$1\frac{1}{3}$	4	1	$13\frac{2}{5}$
8	6	$1\frac{1}{2}$	6	12	$5\frac{3}{4}$	6	1	$3\frac{1}{4}$
12	13	$4\frac{1}{2}$	10	10	$10\frac{1}{6}$	4	2	$\frac{1}{4}$

3. Subtract :—

(1). Rs.	as.	pies.	£.	s.	d.	(3). cwt.	qrs.	lbs.
10	12	$7\frac{3}{5}$	25	18	$10\frac{1}{4}$	27	2	$13\frac{1}{3}$
8	10	$6\frac{1}{4}$	10	9	$7\frac{1}{2}$	17	1	$25\frac{2}{5}$

4. Multiply :—

(1). Rs17. 9 as. $10\frac{1}{3}$ pies by 3, 7, 9, 8 and 12.

(2). 18 mds. 32 srs. $5\frac{1}{4}$ chts. by 7, 8, 12 and 11.

(3). £18. 17s. $9\frac{1}{4}d.$ by 6, 5, 9 and 22.

(4). 3 tons 13 cwt. 2 qrs. $13\frac{1}{8}$ lbs. by 9, 13, 18 and 32.

5. Divide :—

(1). Rs125. 1 a. $10\frac{3}{4}$ pies by 9, 12, 28 and 48.

(2). 90 tons 18 cwt. 3 qrs. $12\frac{3}{8}$ lbs. by 7, 9, 17, and 30.

(3). 100 yds. 2 ft. $7\frac{1}{3}$ in. by 6, 8, 24 and 50.

(4). £80. 18s. $3\frac{7}{8}d.$ by £2. 9s. $3\frac{1}{4}d.$

6. Simplify :—

(1). Rs$4\frac{3}{8}$ + $2\frac{1}{2}$ annas + $3\frac{3}{4}$ pies.

(2). £$7\frac{1}{4}$ + $3\frac{3}{8}s.$ + $2\frac{1}{3}d.$

(3). $3\frac{7}{11}$ miles − $7\frac{7}{18}$ fur. + $35\frac{32}{33}$ po.

(4). $31\frac{1}{4}$ cwt. + $2\frac{3}{4}$ qrs − $8\frac{3}{8}$ lbs.

(5). $25\frac{1}{12}$ yds. − $13\frac{1}{2}$ yds. + $2\frac{1}{3}$ ft. + $10\frac{1}{2}$ in.

(6). $\frac{5}{8}$ of Rs7. 14 as. + $\frac{7}{8}$ of Rs48. 3 as. 6 p. − $\frac{3}{8}$ of Rs10. 6 as. 2 p.

(7). ($\frac{3}{8}$ + $\frac{1}{14}$) of 1 guinea − $\frac{1}{12}$ of £1 − $\frac{3}{4}$ of 1 crown.

(8). ·265 of Rs25 + ·32 of Rs48.

(9). ·375 of £1. 10s. + $\frac{3}{5}$ of $1\frac{1}{3}$ of $\frac{4}{5}$ of £1. 5s. − ·5625 of £5.

(10). ·625 of £1. 1s. + $\cdot\dot{5}\dot{4}$ of 8s. 3d. + $\cdot0\dot{2}\dot{7}$ of £2. 15s.

7. Reduce :—

(1). Rs25 to the fraction of Rs28. 2 as.

(2). Rs8. 3 as. 4 p. to the fraction of Rs10. 4 as. 6 p.

(3). £4. 17s. to the fraction of £5.

(4). £18. 7s. 6d. to the fraction of £3.

(5). 2 cwt. 3 qrs. 14 lbs. to the fraction of 5 tons.

(6). 3 cwt. 2 qrs. 14 lbs. to the fraction of 1 cwt.

(7). $4\frac{3}{8}$ of £1 to the fraction of $\frac{3}{4}$ of 1 guinea.

(8). 2 days 3 hours 5 minutes to the fraction of 6·13 hours.

(9). 2 ft. $4\frac{1}{2}$ in. to the fraction of 1 yard.

(10). 2 acres 31 po. to the fraction of 5 acres 2 ro.

(11). $\frac{2}{3}$ of 2 bghs. 7 kths. to the fraction of 4 bghs. 5 kths.

(12). 1 mile 220 yds. 2 ft. to the fraction of 4 miles.

(13). $2\frac{2}{3}$ of $3\frac{3}{4}$ of $5\frac{1}{2}$ of 12 cwt. to the fraction of 1 ton.

(14). Rs15. 10 as. 10 pies to the decimal of Rs25.

(15). Rs10. 6 as. 6 pies to the decimal of Rs20.

(16). £8. 10*d.* to the decimal of $5\frac{1}{4}$*d.*

(17). £4. 15*s.* $7\frac{1}{2}$*d.* to the decimal of £1.

(18). 3 acres 2 ro. 10 po. to the decimal of 4 acres 3 ro.

(19). £1, 4*s.* $10\frac{3}{4}$*d.* to the decimal of £3. 19*s.* 8*d.*

(20). 1 cwt. 2 qrs. $3\frac{1}{4}$ lbs. to the decimal of 1 ton 4 cwt. 1 qr. 24 lbs.

(21). 1 md. 12 seers 8 chts. to the decimal of $\frac{3}{4}$ of 4 mds. 24 seers.

(22). 20 hours 20 mi. 20 sec. to the decimal of 4 days 2 h. 30 m.

8. Express the value of ·002 of £ 50 + ·02 of £ 100 − ·21 of £ 55 as the decimal of $\cdot 0\dot{3}$ of £ 80.

9. What decimal of 3 tons. 4 cwt. 2 qrs. together with 32 tons 12 cwt. is equivalent to ·375 of 5 tons. 4 cwt. 2 qrs. ?

10. From what decimal of 500 yards, must 25 yds. 2 ft. 3 in. be subtracted, that the remainder may be 8 yds. 2 ft. 9 inches ?

Additional Examples XXX.—(Concrete numbers—Miscellaneous).

1. A man having purchased 80 maunds of sugar for Rs 650 sells them for Rs 666. 10 as. 8 pies. What will be his gain per seer ?

2. A man sold 25 horses for Rs 1750 and thereby incurred a loss of of Rs 125 on the whole. What was the cost price of a horse ?

3. A man sold a cow for £ 1. 2*s.* 3*d.* at a loss of 4*s.* 3*d.* What would have been his gain had it been sold for £ 2 ?

4. A man buys tea at 5*s.* 6*d.* a pound and sells it at 6*s.* a pound. If his gain be £ 1. 10*s.*, how much tea did he purchase ?

5. If the value of a guinea be Rs 16. 2 as. 3 pies, how many guineas can be had for Rs 1033 ?

6. If the value of a shilling be 10 as., how many rupees can be given for £ 500 ?

7. 16 maunds of flour can be had for 20 maunds of rice at Rs. 3. 8 as. per maund. What is the price of flour per maund ?

8. A shop-keeper mixes 10 mds. of rice at Rs. 3. 12 as. per maund with 8 mds. of rice at Rs. 4 per maund. At what price per maund must he sell the mixture so as neither to gain nor to lose ?

9. How much water must a vintner add to wine worth £73. 6*s*. 8*d*. at 7*s*. 4*d*. a gallon, in order that by selling it at 4*s*. 2*d*. a gallon, he may just recover the amount of his outlay?

10. A fruiterer mixes 1250 mangoes at Rs. 3 per hundred with 1325 mangoes at Rs. 2 per hundred. At what price per hundred must he sell the mixed mangoes so as to gain Rs. 39 on the whole?

11. A wine merchant buys 250 gallons of spirits at 12*s*. a gallon. What quantity of water must he add to it, so that by selling the mixture at 8*s*. a gallon, he may gain £15 on his outlay?

12. Divide Rs. 512. 6 as. 9 p. between *A* and *B*, so that *A* may receive Rs. 50. 3 as. 3 p. more than *B*.

13. Divide Rs. 24. 10 as. 7 p. between *A*, *B* and *C* in such a way that *B* may receive Rs. 2. 6 as. 3 p. more than *A*, and *C* Rs. 3. 7 as. 1 p. more than *B*.

14. Divide Rs. 35. 3 as. 6 p. between Ram and 9 other boys so that Ram may receive Rs. 4. 6 as. 2 p. more than each of the others.

15. Divide Rs. 46. 4 as. between *A*, *B* and *C* in such a way that *B* may receive Rs. 4. 6 as. more, and *C*, Rs. 4. 6 as. less than *A*.

16. Divide £9. 9*s*. between two persons in such a way that one may receive thrice as much as the other.

17. Divide Rs. 75 among three men in such a way that the second may receive three times as much as the first, and the third four times as much as the second.

18. Divide £49. 9*s*. among 2 men, 3 women and 5 boys in such a way that each woman may receive twice as much as each boy, and each man thrice as much as each woman.

19. Divide Rs. 109. 2 as. between 2 persons in such a way that one may receive Rs. 5. 6 as. less than three times what the other receives.

20. In a heap there are altogether Rs. 15. 10 as. worth of fruits, of which there are three times as many pears and four times as many plums as oranges. The price of oranges is 6 as., of pears 5 as., and of plums 1 anna per 20. Find the number of each kind of fruit.

21. An equal number of rupees, half-rupees, four-anna pieces, two-anna pieces, double pice and pice amount to Rs. 615. Find the number of each coin.

22. A person lays out Rs. 1175 in the purchase of an equal number of buffaloes, cows and goats. The price of a buffalo is Rs. 40, of a cow Rs. 16. 8 as., and of a goat Rs. 2. 4 as. Find the number of each.

23. Divide £118 into an equal number of guineas, pounds, crowns, and half-crowns.

24. In a box there are 3 times as many half-rupees, 4 times as many four-anna pieces and 5 times as many two-anna pieces as there are rupees; and the whole sum in the box is Rs 99. Find the number of each coin.

25. The price of a horse and a cow is Rs 121; of a horse and an ass, Rs 116; and of a cow and an ass, Rs 36. Find the price of each.

26. The sum of the ages of Jadu and Gopal is 24 years 1 month, of Gopal and Haradhan 21 years 11 months, and of Jadu and Haradhan 23 years. Find the age of each.

27. How many revolutions will a wheel make in going over 112 yds., whose circumference is 10 ft. 6 inches?

28. A wheel whose circumference is 9 yds. 6 in. makes 912 revolutions in going from one place to another. Find the distance between the two places.

29. The circumferences of the fore-wheel and the hind-wheel of a carriage are 8 ft. and 10 ft. respectively. How many revolutions will the one make more than the other in going over 546 yds. 2 ft.?

30. In passing a distance of 150 miles, the fore-wheel and the hind-wheel of a carriage make 17600 and 12000 revolutions respectively. By how much is the circumference of the hind-wheel greater than that of the fore-wheel?

31. In passing a distance of 7 miles the fore-wheel of a carriage whose circumference is 9 yds. 1 ft. makes 528 revolutions more than the hind-wheel. Find the circumference of the hind-wheel.

32. *A*, *B* and *C* together earned Rs10. 10 as. 10 *gandas*. *A* and *C* each earned 5 *gandas* more than twice what *B* did. Find the earning of each.

33. A man consumes Rs31. 5 as. 17½ *gandas* worth of rice a year If his daily consumption be 1 sr. 6 chts., and if the price of rice be Rs 2. 8 as. a maund, of how many days does the year consist?

34. The daily wages of a labourer is 5 as. 6 p. But he has to pay a fine of 2 as. 9 p. every day he is absent from work. He receives Rs7. 11 as. 9 p. for the month of June. For how many days was he absent?

35. Divide Rs31. 12 as. into two parts so that one part may be 5 times as great as the other.

36. 24 men can do a piece of work in 8 days. How many men will do the same piece of work in 12 days ?

37. A person started with a certain sum of money in his pocket. To the first beggar whom he met on his way, he gave $\frac{1}{2}$ of what he had and 3 pice more ; to the second beggar whom he met, he gave $\frac{2}{3}$ of what remained and 3 pice more ; and to the next beggar he gave $\frac{3}{4}$ of what still remained and 3 pice more, after which he had nothing more left. With what sum did he start ?

38. If 1 rupee be equal to 1*s*. 9*d*., what is the value in Indian money of £ 98. 9*s*. 4*d*. ?

39. Jadu earns Rs 3. 9 as. a month more than Keshab. At the end of a year their earnings amounted to Rs 113. 9 as. Find how much each earned.

40. If the expense of gravelling a road 30 ft. long be R 1. 8 as., what will be the expense of gravelling another road of equal breadth whose length is 1 mile ?

41. Rs 41 were divided amongst 100 children, each boy getting 8 as., and each girl 4 as. ; how many girls were there ?

42. Sound travels at the rate of 1140 ft. a second. If a gun be fired at a distance of 4 miles 560 yds., how long will it be, after seeing the flash, before the report is heard ?

43. In a box there were Rs 13. 11 as. 1 pice in half-rupees and pice. A man when asked to count the coins, counted them in darkness, and thinking them all to be pice said that they amounted to R 1. 1 a. 3 pice. How many half-rupees and pice were there in the box ?

44. In a box there are Rs 26. 4 as. consisting of half-rupees and pice. If the number of half-rupees is 30 less than the number of pice, find the number of each.

45. I have a certain sum of money to be distributed among a certain number of boys, and I find that if I give Rs 3. 8 as. to each I shall spend Rs 12. 8 as. too little, but that if I give Rs 4. 12 as. to each I shall spend Rs 18. 12 a. too much. How much have I to spend ?

46. The monthly income of *A* is Rs 18. 10 as. and that of *B* is Rs 16. 12 as. In what time will *A* earn Rs 39. 10 as. more than *B* ?

47. Ram is as much older than Syam as he is younger than Hari. The respective ages of Hari and Syam are 28 years 8 months 6 days and 20 years 6 months 4 days. Find the age of Ram.

48. The circumference of the hind-wheel of a carriage is twice that of the fore-wheel. In passing a distance of 1 mile, it was found that the fore-wheel made 880 revolutions more than the hind-wheel. Find the circumference of the fore-wheel.

49. One-third of A's money was equal to $\frac{3}{4}$ of B's money, and was Rs. 25 less than B's money. Find what each had.

50. I have to be at a certain place in a certain time, and I find that if I walk at the rate of 4 miles an hour I shall be $7\frac{1}{2}$ minutes too late, but if I walk at the rate of 5 miles an hour, I shall be 6 minutes too early. What distance have I to go ?

51. Find the value of—

$$\frac{10\text{lbs. } 1\text{ oz.}}{11\text{lbs. } 8\text{ oz.}} \div \frac{£3.\ 7s.\ 9d.}{£4.\ 10s.\ 4d.} \text{ of } \frac{25\text{ yds. } 1\text{ ft. } 5\text{ in.}}{14\text{ yds. } 1\text{ ft. } 8\text{ in.}} \times \cdot 03125$$

of $\cdot 041\dot{6}$ of a day.

52. The sum of the ages of A and B is now 55 years, and 5 years ago A's age was $\cdot 8$ of B's. Find their present ages.

53. A certain sum was divided among A, B, C and D in such a way that A received $\frac{1}{3}$ of the whole, B, $\frac{1}{4}$ of what remained, C, $\frac{1}{5}$ of what still remained, and D the rest. If what D received amounted to Rs. 60, find what A, B and C each received.

54. A and B can do a piece of work in 8 days, A and C in $10\frac{2}{3}$ days, and B and C in $9\frac{3}{5}$ days. Find in what time A alone can do the whole work.

55. A and B can do a piece of work in 6 days, B and C in 7 days, and A, B and C in 4 days. Find in what time A and C can do the work.

56. A can do $\frac{3}{8}$ of a piece of work in 4 hours ; B, $\frac{3}{4}$ of the remainder in 1 hour ; and C the rest in 20 minutes. In what time can they all together finish the work ?

57. What A can do in 4 days, B can do in 6 days ; and what B can do in 10 days, C can do in 8 days. How long will it take C to finish what A can do in 18 days ?

58. A man bequeathed his property among his two sons and one daughter in such a way that the eldest son got $\frac{5}{17}$ of the whole property, the youngest $\frac{1}{3}$ of the remainder and the daughter the rest. Of the sons one got Rs 784 more than the other. Find what the daughter got.

59. If *A*'s 5 months' income be equal to *B*'s 8 months' income, and if *A*'s one month's income be equal to Rs 2. 10 as., what will be the income of both for 6 months ?

60. A woman walks twice as fast as a boy, and a man 3 times as fast as a woman. How long will it take a man to walk the same distance which a boy walks in 15 hours ?

61. *A* and *B* had respectively 15 and 17 pieces of bread for their own consumption. But when *C* came in, they divided each bread equally into three parts and each took a part. *C* gave them a rupee when he went away. Find how the rupee is to be divided between *A* and *B*.

62. A wine merchant buys 180 gals. of wine at 9*s*. 6*d*. a gallon. How much water must he mix with it, so that by selling the mixture at 7*s*. 6*d*. a gallon, he may gain £16 on his outlay ?

63. Divide 1 guinea among *A*, *B*, *C* and *D* in such a way, that *B*'s share may be $\cdot\dot{3}$ more than *A*'s , *C*'s share $\cdot\dot{3}$ more than *B*'s, and *D*'s share $\cdot\dot{3}$ more than *C*'s.

64. In a box there are $2\cdot5$ times as many half-rupees, $\cdot0\dot{7}$ times as many four-anna pieces, and $1\cdot2$ times as many two-anna pieces as there are rupees ; and the whole sum amounts to Rs 217. 12 as. Find the number of coins of each kind.

65. There are 6 classes in a school. In the first class the number of boys is $\cdot25$ of the whole ; in the second class $\cdot45$ of the whole; in the third class $\cdot05$ of the whole ; in the fourth class 40 ; and in the fifth class, half as many as in the sixth class. The whole number of boys is 400. Find the number of boys in each class.

66. *A* and *B* began business with equal capitals. At the end of one year *A* gained Rs400, and *B* lost $\cdot125$ of his capital. *B* then had $\cdot625$ of what *A* had. How much had each at first ?

67. Divide Rs1000 between *A* and *B* in such a manner that 4 times *A*'s share together with 8 times *B*'s share may amount to Rs 6300.

68. Two passengers going to the same place have 7 mds. of luggage between them, and are charged for excess of luggage Rs. 4. 6 as. and Rs. 2. 10 as. respectively ; had the luggage all belonged to one of them, he would have been charged Rs. 9. 10 as. for excess. How much is allowed free ?

69. Mangoes are bought at Rs12 per 100 ; at what price per 100 must they be sold that the gain on Rs300 may be equal to the selling price of 400 mangoes ?

70. If 44 oxen in 5 weeks eat up the grass on a certain field and what grows upon it during the time ; and 26 oxen eat up the same in 11 weeks ; how many oxen will it maintain for 15 weeks, supposing the grass to grow uniformly during the time ?

71. Gold is bought at £4. 5 *s.* $2\frac{1}{2}$*d.* per oz. ; and sold at £4. 5*s.* 5*d.* per oz. What is the largest unit of money in which both prices can be expressed as integers ? and what is the smallest integral number of ounces the value of which can be exactly expressed in £ at both prices ?

72. Pandit Iswar Chandra Vidyasagar died on July 28 of 1891. If he lived 25873 days including the day of his birth and of his death, find the date and year of his birth.

73. Guns are fired at intervals of 15 minutes in a town towards which a railway train is approaching at the rate of 40 miles an hour ; if sound travels at the rate of 1152 feet per second, at what intervals will the passengers hear the reports ?

74. A cistern has a supply-pipe and a waste-pipe attached to it. If both the pipes are opened together, the cistern is filled in 14 hours 40 minutes ; but if the waste-pipe is opened an hour and a half after the supply-pipe, the cistern is filled in 10 hours 40 minutes. In what time can the supply-pipe fill the empty cistern ?

75. A leaky cistern is filled in 9 hours with 40 pails of 3 gallons each, but in 5 hours with 25 pails of 4 gallons each, the pails being poured in at intervals. Find how much the cistern holds, and in what time the water would waste away.

Additional Examples XXXI.—(Square and Cubic Measure.)

1. What length of paper $\frac{7}{8}$ of a yard wide will be required to cover the walls of a room 28 ft. 5 in. long, 20 ft. 7 in. broad, and 15 ft. 6 in. high; and what will it cost at 9 as. per yard?

2. Find the expense of painting the walls and the ceiling of a room 18 ft. 8 in. long, 14 ft. 9 in. broad, and 9 ft. high at 4 as. 6 p. per sq. yard.

3. In a rectangular court, which measures 120. ft. by 80 ft., there are four rectangular grass plots, measuring each 24 ft. 6 in. by 19 ft.; find the cost of paving the remaining part of the court at R1. 11 as. per sq. cubit.

4. A man owns a rectangular plot of land measuring 20 bghs. 6 kths. by 13 bghs. 6$\frac{3}{4}$ kths. Out of it he sells four plots the dimensions of each of which are exactly one-fourth of those of the whole plot. What quantity of land does he retain?

5. A reservoir is 25 ft. 5 in. long by 12 ft. 10 in. wide; how many cubic feet of water must be drawn off to make the surface sink one foot?

6. Find the expense of painting the outside of a cubical iron-chest, whose edge is 2 ft. 7 in. at R 1. 2 as: per sq. yard.

7. A piece of cloth 6 times as long as it is broad costs Rs 147; supposing its price to be Rs 4. 8as. per sq. yard, find the dimensions of the piece.

8. What will be the cost of covering the walls of a room 26 ft. long, 20 ft. broad, and 15 ft. high with half-anna postage stamps each measuring $\frac{7}{8}$ of an inch by $\frac{3}{4}$ of an inch?

9. How many Chunar stones each 2 cubits square will be required for paving a street 45 feet wide, surrounding a square, the side of which is 270 feet?

10. A rectangular plot of land 80 yds. long and 40 yds. broad has a path inside it running along its sides. It has also two paths running through its middle and crossing each other. All the paths are of the same uniform breadth of 8 ft. Find the cost of gravelling the paths at the rate of R1. 9 as. per 100 sq. ft., and of covering the remainder with turf at the rate of 5 as. per 100 sq. feet.

11. Find the expense of lining a cistern, 10 ft. 7 in. long, 5 ft. 5 in. broad, and 5 ft. deep, with lead, at Rs12 a maund, which weighs 8 seers per sq. foot.

12. A lawn 1800 ft. long and 1200 ft. broad is to be raised 2 ft. $1\frac{1}{4}$ in., and for that purpose a moat of a uniform breadth of 45 ft. and of uniform depth is dug all round the lawn. What must be the depth of the moat in order that there may be soil just sufficient for the purpose ?

13. If there be 3456 bricks each measuring 10 in. by $4\frac{1}{2}$ in. by $2\frac{1}{2}$ in. in a rod of brick-work $12\frac{1}{4}$ in. thick, how much mortar is there to a brick ?

14. During a rainfall of an inch and a half, 151 tons, 18 cwt. 2 qrs. $\frac{1}{2}$lb. of water fell on an acre of land. How many ounces would there be in a cubic foot of water ?

15. A square field is bordered on the outside by a path 12 ft. wide, the field and the path together occupying 10 acres. Find the cost of gravelling the path at the rate of R 1 per 100 sq. feet.

16. The cost of levelling a lawn-tennis ground which is half as long again as broad at 10*d.* per sq. yard is £ 248. 1*s.* 3*d.* Find the cost of enclosing it with an iron railing at 7*s.* 8*d.* per yard.

17. A rectangular cistern open at the top, 10 ft. long and 5 ft. broad, is made of sheet lead every square foot of which weighs 16 lbs. If the weight of the cistern be a ton and a half, find approximately the number of gallons of water that the cistern can hold, assuming that a cubic foot of water weighs 1000 ounces.

18. The matting of a room half as long again as broad at 5as. per sq. foot costs Rs 187. 8 as., and the painting of the walls at 9as. per sq. yard costs Rs 100. Find the height of the room.

19. A box with a lid is to be made of plank an inch and a half thick, and the external dimensions are to be 2 ft., 18 in., and 15 in. How many square feet of plank will be required ?

20. Find the cost of whitewashing the ceiling and the inner and outer sides of the walls of a room, 30 ft. long, 20 ft. wide, and 18 ft. high, at 1 pie per sq. foot ; the walls being 2 ft. thick and 4 ft. higher at the outside.

21. A room 35 ft. 6 in. long and 25 ft. 6 in. broad was carpeted with a certain material. If the room had been 4 ft. longer and 2 ft. 6 in. broader, the additional cost would have been Rs. 351. 5 as. How much did it cost to carpet the room ?

22. The outer dimensions of a wooden box are 3 ft. 6 in., 2 ft. 4 in., and 1 ft. 7 in. respectively, and the box is made of wood 2 in. thick. Find the cost of lining the inside with velvet at Rs. 6. 12 as. per sq. yard.

23. A cistern 175 yds. long, 70 yds. broad, and 18 yds. deep, has water in it 6 yds. deep; find how many oblong pieces of stone each 3 ft. 9 in. long, 1 ft. 2 in. broad, and $10\frac{1}{2}$ in. thick, must be thrown into it that the water may rise just to the brim.

24. A cubic foot of gold weighs 19 times as much as water of the same bulk. Find the weight of the gold which a box made of wood an inch and a half thick, and with external dimensions 5 ft. 3 in., 3 ft. 3 in., and 2 ft. 3 in. respectively, can contain, a cubic foot of water weighing 1000 ounces.

25. A hollow rectangular iron pillar 24 ft. high, is made of sheet iron $1\frac{1}{2}$ in. thick; and the hollow part is one foot square at the end. Find the cost of the pillar at the rate of Rs. 7. 8 as. per maund, a cubic inch of iron of the same quality weighing $2\frac{1}{2}$ chataks.

Additional Examples XXXII.—Practice.

1. Find the cost of 19 qrs. 5 bus. 2 pks. of wheat at £2. 12*s.* 10*d.* per quarter.

2. What is the rent of 34 ac. 3 ro. 23 po. at $2\frac{3}{4}$ guineas an acre?

3. Find the dividend on Rs. 3456. .. .. at 13 as. 9 p. in the rupee.

4. Find the price of 12 mds. 16 srs. 10 chts. of milk at Rs. 6. 8 as. per maund.

5. Find the cost of 12 pipes, 24 gals. 3 qts. of wine at £90. 10*s.* a pipe.

6. Find the cost of 4 hhds. 1 bar. 7 qts. of beer at £2. 3*s.* 6*d.* a barrel.

7. A bankrupt can pay 11 as. 10 p. in the rupee. How much will a creditor get whose debt is Rs. 5678. 8 as.?

8. Find the value of 17 oz. 13 dwts. 17 grs. of gold at Rs. 65. 10 as. per ounce.

9. Find the price of 7 oz. 5 drs. 18 grs. of quinine at Rs. 6. 4 as. per ounce.

10. Find the price of 13 kahuns, 11 sollies, 9 pallies, 3 reks, 1 kunki of *dhan* at Rs. 106. 10 as. 8 p. per kahun.

Additional Examples XXXIII.—Ratio and Proportion: Rule of Three.

1. A map is drawn on the scale of 1 inch to 32 miles; in what ratio are lengths diminished?

2. A map is drawn on the scale of 1 cubit to the linear bigha; in what ratio are areas diminished?

3. A model is made on the scale of half an inch to the foot; in what ratio are volumes diminished?

4. What is the ratio of (1) a pound Troy to a pound Avoirdupois, (2) a pound Avoirdupois to a bazar maund, and (3) a bigha to an acre?

5. The length of a room is one-third as much again as that of another, and the breadth of the latter is one-fourth smaller than that of the former; what is the ratio of their areas?

6. The amounts of work done by two men in an hour are as 10 : 12, and the times they work as 15 : 10; what is the ratio of the wages they should receive?

7. Supposing the value of diamonds to vary as the square of their weight, what is the ratio of the price of a diamond weighing 8 rattis to that of another weighing 12 rattis?

8. What is the ratio of a man's net income when the income-tax is 4 pies in the rupee to that when it is 5 pies in the rupee?

9. A bankrupt can pay 10 as. 8 p. in the rupee; find the ratio of his assets to his debts.

10. Can 10 maunds of liquid measure and 12 feet (linear, square, or cubic) have a ratio?

11. Two boxes are of the same shape, and the length of the one is $3\frac{1}{2}$ times the length of the other; what is the ratio of their cubical contents?

12. Two barracks are of the same shape, and the height of the one is $2\frac{1}{2}$ times the height of the other. Supposing a soldier requires 1000 cubic feet of air, what is the ratio of the numbers of soldiers which the two barracks can respectively accommodate?

13. Show that the thickness of an eight-anna piece will be not quite $\frac{4}{5}$ of the thickness of a rupee if they are of the same shape.

14. *A* can walk $17\frac{1}{2}$ miles in 7 hours, and *B* 176 cubits a minute. What is the ratio of their rates?

15. A can walk half as fast again as B, but A's journey is one-third as much longer again as B's; what is the ratio of the times they take on their journeys?

16. One milkman adds 6 seers of water to 16 seers of milk, and another 9 seers of water to 24 seers of milk; compare the quantities of milk in the two mixtures.

17. The rupee weighs 180 grs. Troy, and the ratio of the quantity of pure silver in it to that of alloy is 11 : 1. How many rupees can be coined out of a lump of pure silver weighing 35 lbs. 4 oz. 3 dwts. 18 grs.?

18. The shilling weighs $\frac{1}{66}$ of 1 pound Troy, and contains pure silver and bronze in the ratio of 37 : 3. How many shillings can be coined out of a lump of pure silver weighing 111 lbs.?

19. A mixture weighing 39 gallons contains wine and water in the ratio of 7 : 6; how much water must be added to it that the ratio of wine to water may be 3 : 2?

20. A takes 5 paces for every 6 paces of B, but 4 paces of A are equal to 5 of B; find the ratio of the rates of A and B.

21. Find a fourth proportional to

(1). $\cdot\dot{4}2857\dot{1}$, $\cdot\dot{9}\dot{0}$, and $\cdot 91\dot{6}$.

(2). £123, 891 guineas, and Rs. 149. 7 as. 8 p.

(3). 9 tons, 245 mds., and 1234.

(4). 1541, 1943, and 207 kathas.

(5). 1 dwt., 1 cwt., and 1 hour.

(6). 1 sq. bgh., 1 sq. pole, and 6 srs. 4 chts.

(7). 306 men, 323 men, and 2 dandas 6 pals.

(8). 3 hrs. 45′, 7 prahars 3 dandas 45 pals, and 7 pals 30 vipals.

22. Find a third proportional to

(1). $1\frac{321}{343}$ and $5\cdot\dot{8}5714\dot{2}$.

(2). 1 guinea and 1 moidore.

(3). 15 cwt. 1 qr. 20 lbs. and 1 md. 30 srs.

(4). Half an acre and 11 sq. kathas.

(5). 4900 centigrams and 7 kilograms.

(6). 1 kilometre and 1 inch.

23. Find a mean proportional between

(1). $1\frac{3}{4}$ and ·4375.

(2). 1 R. and 1 a.

(3). 2 powas 1 cht. and 2 srs. 1 powa.

(4). 1 acre and 2 sq. kathas.

(5). 1 pal and 6″.

(6). 10 litres and 1 cubic metre.

24. If $123\frac{1}{2}$ yards of cloth cost £14. 1*s*. $8\frac{8}{18}$ *d*., how many yards can be bought for £11. 1 *s*. $3\frac{3}{8}$ *d*. ?

25. If a farm containing 400 bghs. 17 kths. 13 chts. be let at Rs. 50779 7 as. 8 p. for the year, what is the rent per bigha ?

26. If 12 men or 18 boys can reap field in 26 days, in how many days will 18 men and 12 boys reap another field thrice as large ?

27. How many yards of silk at Rs. 5. 10 as. 8 p. per yard must be given in exchange for $76\frac{1}{2}$ yards of linen at R. 1. 5 as. 4 p. per yard ?

28. An insolvent debtor can pay 10 as. $7\frac{1}{2}$ p. in the rupee ; how much will a creditor lose whose debt is Rs. 1000. 8 as. ?

29. A creditor of an insolvent debtor loses £331. 7*s*. $9\frac{1}{8}$*d*. on a debt of £1200. 10 *s*. How much can the insolvent pay in the pound ?

30. If a rectangular plot of land 203 rasis long and 144 rasis broad cost Rs. 41760000, what will be the price of another rectangular plot 49 rasis long and 37 rasis broad at the same price per square bigha ?

31. If a rectangular plot of land 275 ft. 6 in. long and 56 ft. 3 in. broad cost £39029. 3 *s*. 4 *d*., what must be the breadth of another rectangular plot which is 100 ft. 6 in. long, and which costs £15819. 8 *s*. $10\frac{3}{4}$ *d*. at the same price per acre ?

32. A zemindar employs a manager on Jan. 1, 1800 at a yearly salary of Rs. 821. 4 as. ; he leaves the situation on the 24th May following ; what ought he to receive for his services, the first and the last days being both included ?

33. A person after paying 5 pies in the rupee for income-tax on his income has Rs. 3652. 5 as. 6 p. remaining ; what had he at first ?

34. A man working $7\frac{1}{2}$ hours a day, does a piece of work in 11 days ; how many hours a day must he work to do it in 10 days ?

35. Five horses and 8 cows together find sufficient grass on a certain field; and 7 cows eat as much as 9 horses; what must be the size of a field relatively to the former, which will support 16 horses and 12 cows?

36. *A* alone can do a piece of work in 6 days, and *B* in 7 days, working 13 hours a day; find in what time *A* and *B* can do it together, working 7 hours a day.

37. If a threepenny loaf weighs 18 oz. when wheat is at £4. 12*s.* a quarter, what should it weigh when wheat is at £10. 7 *s.* a quarter?

38. If a threepenny loaf weighs 1 lb. 8 oz. when wheat is at 2·34 *s.* a quarter, what is the price of wheat per quarter, when the threepenny loaf weighs 1 lb. 2 oz?

39. If 14 chairs and 5 tables cost Rs. 207, find the cost of 12 chairs and 6 tables, the cost of 10 chairs being equal to that of 3 tables.

40. If 300 mangoes can be had for Rs. 25. 14 as., and 34 pomegranates for Rs. 8. 10 as., how many pomegranates should he had in exchange for 550 mangoes?

41. In a certain town the population at the beginning of a year was 2679799, and at the end of the year 2680650. Find by how many the population of another town would increase at the end of a year at the same rate, the population at the beginning of the year being 3926803.

42. A merchant in Calcutta bought of a merchant in London goods worth £730, and paid £13 for freight. If a rupee be equal to 1 *s.* 1½*d.*, for how many annas must he sell goods for which he paid 1 *s.* to the London merchant, in order to gain £60 on the whole outlay?

43. If a certain quantity of rice serve 391 men for 18 days at the rate of 17 oz. a day for each man, how many ounces a day will each man get, when the same quantity of rice serves 578 men for the same time?

44. If 256 horses can be kept for a certain sum when grain is at Rs. 2 6 as. 6 p. per maund, how many horses can be kept for the same sum when grain is at Rs. 2. 10 as. 8 p. per maund?

45. If 12 men can do a piece of work in 5 days working 8½ hours a day, how many hours a day must 17 men work to do in 18 days a piece of work three times as great?

46. If 10 men can do a piece of work in 8 days working 7¾ hours a day, in how many days of 8 working hours each can 23 men do another piece of work twice as great?

47. If 5 men can do as much work as 7 boys in the same time, and if a piece of work be done by 18 men and 9 boys in 17 days, how many days would 9 men and 18 boys take to do the same ?

48. If the cost of printing a book of 469 pages, with 23 lines on each page, and on an average 10 words in each line, be 335 Rs., what will be the cost of printing a book of 322 pages, with 25 lines on each page, and 8 words in each line ?

49. If 7 men can do as much work as 9 boys in the same time, and if a piece of work be done by 13 men and 15 boys in 13 days working 5 hours 48 minutes a day, how many men must be associated with 24 boys to finish a piece of work thrice as great in 22½ days of 6¼ working hours each ?

50. A man pays an income-tax of 5p. in the rupee on ⅘ of his income ; at what rate per rupee does he pay on his whole income ?

51. When the income-tax is 3½ p. in the rupee, a person has to pay Rs. 122. 13 as. 9 p. less than when the tax was 5 p. in the rupee ; find his income.

52. When the income-tax is 8½*d.* in the pound, a person has to pay £271. 8 *s.* 8¾ *d.* more than when the tax was 5⅝ *d.* in the pound ; find his income.

53. *A* contracts to perform a piece of work in 48 days and employs a certain number of men ; at the end of 18 days he finds that only one-third of the work has been done ; he then employs 13 additional men and is able to fulfil his contract. How many men did he employ at first ?

54. The area of a triangle varies as the product of its base and its altitude. If the base of a triangle be 3 bghs. 17½ kths. and its altitude 2 bghs. 3⅓ kths., what is the altitude of another triangle of equal area, the base of which is $16 \cdot \dot{9}\dot{0}$ poles ?

55. The volume of a sphere varies as the cube of its radius. Find the volume of a sphere of radius 1 ft. 6 in., supposing the volume of another sphere of radius 2 ft. 3 in. to be 1·76715 cub. yds.

56. A clock which is 7′ too fast at 8 A. M. on Sunday loses 2′ 8″ per day ; what time will it show at 1 P. M. on the following Thursday, and when will it show correct time ?

57. Two clocks of which one gains 1′44″ and the other loses 40″ a day are both set right at 1 P. M. on Sunday : when will the one gain 18′ on the other, and what time will each then show ?

58. A clock which is 4′ 5″ too fast at 1 P. M. on Wednesday is 1′ too slow at 9-30 P. M. on the following Tuesday ; how much did it lose in at day ?

59. A clock gains 3′ 36″ a day ; how must its hands be placed at 11-30 A. M. so as to indicate correct time at 9-30 P. M. ?

60. One watch gains $1\frac{1}{2}$ minutes, and another gains $3\frac{1}{4}$ minutes a day : the first is set right at 1 P. M. on Sunday, and the second at 10 A. M. on the following Monday : when will they indicate the same time ?

61. A clock which shows correct time at 9-30 P. M. on the 1st of June loses 3′ 45″ a day. What is the true time when the clock indicates 3-30 A. M. on the 7th of June following ?

62. A clock is set right at 1 P. M. on Sunday, June 9, 1895, and at 5 A. M. on the following Tuesday it is 3′ 45″ too fast. Supposing its rate regular, when will it again indicate true time ?

63. A clock was 16′ too fast at 6 P. M. 40 days ago, and to-day at the same hour it is 16′ too slow ; when did it show true time, and when will it again indicate true time ?

64. Two watches strike one together on Monday afternoon ; on the next day one of them indicates 9 h. 17′ P. M. when the other indicates 9-30 P. M. How much must the faster watch be put back or the slower put forward, in order that they may strike five together on Wednesday morning ?

65. A watch which was 2·3 min. too fast at a quarter to 9 A. M. on June 5, was 10·8 minutes too slow at 10 P. M. on June 11 ; when was it exactly right ?

66. Two watches are set right at 9 P. M. on Monday. At 1 P. M. on the following Wednesday one of them is found to indicate 4′ 10″ past one, and the other to indicate 12 h. 57′ 5″. Supposing their rates regular, what time will the former indicate when the latter indicates 53′ past 8 P. M. on the following Friday ?

67. A clock set right at 1 P. M. indicates 8 min. to 9 at 9 P. M. What is the true time when the clock indicates 9 o'clock ?

68. Two clocks, one of which gains $5'\ 33\cdot\dot{3}''$ in 20 hours and the other loses $9\cdot1\dot{6}'$ in 44 hours, are both set right at 1 o'clock P. M. on Monday; when will they again strike together?

69. Two clocks, of which one gains and the other loses $1\frac{1}{4}$ minutes in an hour, strike twelve o'clock together; what will be the interval, measured by a correct clock, between their respective striking one?

70. At what time between 10 and 11 o'clock are the hands of a watch (1) together, (2) exactly opposite, (3) at right angles to each other?

71. At what time between 7 and 8 o'clock are the hands of a clock (1) 19 minute-divisions apart, (2) 23 minute-divisions apart?

72. A clock is $6'$ too fast at 1 P. M. and it loses $1'\ 30''$ per hour. What is the true time when its hands are at right angles between 3 and 4 o'clock P. M.?

73. A watch is $8'$ too slow at noon, and it gains $2'$ per hour. What is the true time when its hands are directly opposite for the fifth time afternoon?

74. A clock indicates correct time when its hands are together for the third time afternoon; if it lose $2'\ 12''$ per day, what time will it indicate at 1 P. M. on the following day?

75. A watch, in which the hour-hand has been displaced, shows the time to be $48'$ past 9, and the two hands are together: the time is between 9 and 10 o'clock. Find by how many minute-divisions the hand has been displaced.

76. If the hands of a clock come together every $65\frac{4085}{10571}$ min. (true time), how much does the clock gain or lose in a day?

77. *A* can give *B* 41 yds. 1ft. in a race of 100 yds. and *C* can give *B* 72 yds. in a race of 270 yds. How much can *A* give *C* in a half-a-mile race?

78. In a mile race *A* gives *B* 200 yds. start and beats him by 93 yds. 1 ft. If *A* runs the mile in $7\frac{1}{2}$ min., how long will *B* take?

79. *A* can give *B* 10 yds. in a race of 200 yds., and *B* can give *C* 80 yds. in a race of 380 yds. If *C* runs a mile in 11 min., how long will *A* take to run the same distance?

80. *A* can give *B* 220 yds. and *B* can give *C* 106 yds. 2 ft. in a race of 2 miles; how much should *A* give *C* that they may run a dead heat?

81. In a game of billiards B can give A 35 points out of 323, and C can give B 24 points out of 432. How many can C give A out of 361?

82. In a game of skill A can give to B 666 points out of 1739, and to C 258 points out of 2021. Which is the better player, B or C? How many points can the one give the other in a game of 1681?

83. In a mile race A can give B 220 yds. and B can give C 330 yds.; if A and C run the race, who will win and by how much?

84. A can give B 26 yds. in a race of 350 yds., and B can give C 32 yds. in a race of 288 yds. What is the length of a race-course on which A beats C by 31 yds.?

85. A and B run a mile race, and A wins by 272 yds. A and C run over the same course, and A wins by 31″. B and C run, and B wins by 14″. In what time can A run a mile?

86. In a game of billiards A can give B 18 points and B can give C 75 points out of 270. How many should A give C so as to make an even match?

87. A can give B 640 yds'. and C 320 yds'. start in a three-mile race; C can give B a start of 1′ 17″ in a race of $5\frac{1}{2}$ miles. In what time can each run a mile?

88. A can give C 60 yds. in a mile race; B can give D 966 yds. in a race of 2 miles 183 yds., and C can give D 1729 yds. in a race of 3 miles 401 yds. If A and B run a mile, which will win and by how much?

89. If Rs.26 are equivalent to £2. 9*s.* $3\frac{1}{2}$*d.*, 30*s.* to $37\frac{1}{2}$ francs, and 91 francs to 28·8 rubles, express a ruble in Indian money.

90. If 1541 sheep are worth as much as 2479 goats, 2109 goats as much as 209 buffaloes, 1679 buffaloes as much as 3431 oxen, and 8789 oxen as much as 391 horses, how many horses should be given in exchange for 1311 sheep?

91. If A can do as much work in $4\frac{2}{7}$ days as B in 5 days, B can do as much work in 7 days as C in $8\frac{1}{2}$ days, and C can do as much in 15 days as D in $19\frac{1}{2}$ days, find in how many days A, B and C can together do a piece of work which D alone can do in 13 days.

92. A hare is 100 of her own leaps before a greyhound. She takes 4 leaps for every 3 that he takes, but 5 of the greyhound's leaps cover as much ground as 7 of the hare's; how many leaps will the greyhound have taken before he overtakes the hare?

93. Two trains start at the same time, one from Howrah and the other from Allahabad, and proceed towards each other at the rates of 40 and 24 miles per hour respectively. When they meet, the faster train has run 141 miles more than the other. Find the distance between Howrah and Allahabad.

94. A man can row 141 miles down a stream in 9 hrs. 24 min. and row back in 23 hrs. 30 min. Find the rates of the man and the stream.

95. Two persons are 119 miles apart in 7 hours, when they row from the same place in opposite directions; but they are 19½ miles apart in 13 hours, when they both row down the stream. Find the rate at which each can row per hour.

96. Two trains 44 yds. and 121 yds. long respectively, run at the rates of 40 miles and 20 miles per hour on parallel rails: find how long a person in the first train would take to pass the other train, and how long the two trains would take to pass each other, supposing the trains run (1) in opposite directions, (2) in the same direction.

97. A train 66 yds. long, overtook a person walking in the same direction along the line at the rate of 5 miles an hour, and passed him in 9″; later on, it overtook another person walking in the same direction, and passed him in 8·4375 seconds. At what rate per hour was this second person walking?

98. A monkey, in climbing up a greased pole 136 ft. 4 in. high, ascends 12 ft. 8 in. and slips down 7 ft. 3 in. in alternate minutes: how long will it take him to get to the top?

99. A mail train and a passenger train start from Howrah at the same time; the former travelling at the rate of 48 miles an hour reaches Cawnpore, a distance of 684 miles, 8 hrs. 33′ before the latter. Find the rate of the latter.

100. Two guns are fired from the same place at an interval of 1 hr. 17′ 36″; but a person going towards the place observes that 1 hr. 16′56″ elapse between the reports: if sound travels 1154 ft. per second, find his rate of travelling in miles per hour.

101. A man is to be at a certain place at a certain hour, and finds that if he walks at the rate of 4½ miles an hour, he will reach the place 35 min. too late; and if at the rate of 6 miles an hour, he will be there 20 min. too early. What distance has he to travel?

102. A man rides at the rate of 352 yds. per minute, and stops 6 minutes to change horses at the end of every sixth mile ; how long will he take to go a distance of 108 miles ?

103. If 19 fires burning 8 hours a day consume 114 maunds of coal in 24 days, how many hours a day can 40 fires be kept burning for 18 days with a stock of 225 maunds of coal ?

104. If 16 masons build a wall 60 ft. long, 16 ft. high, and $3\frac{1}{2}$ ft. thick in 18 days working 7 hours a day, what is the height of another wall 400 ft. long and $4\frac{1}{2}$ ft. thick, which 85 masons can build in 48 days working $6\frac{1}{4}$ hours a day ?

105. If 16 compositors set up 18 sheets of 16 pages, each page containing 38 lines of 48 letters, in 19 days of 6 working hours each ; how many compositors will set up 28 sheets of 12 pages, each page containing 54 lines of 56 letters, in 14 days of 7 working hours each ?

106. If 846000 bricks each 13. in. long, $5\frac{1}{4}$ in. broad and $3\frac{1}{4}$ in. thick, are required to build a house, how many bricks which are one-fourth less in every dimension will be required to build another house half as large again as the former ?

107. A ship having a crew of 201 men and provisions for 70 days at 24 oz. a day for each man, encountered a storm after 15 days and lost 8 of her crew ; and on the same day took on board 27 persons from a wreck : what must the daily allowance of each man be reduced to, that the provisions might just last for 67 days more ?

108. If in an asylum 240 beggars be fed 31 days for Rs.950 when rice is at Rs. 4. 12 as. per maund, what is the price of rice per maund when 403 beggars can be fed 30 days for Rs.1665. 10 as. ?

109. If the fourpenny loaf weighs 4·37lbs. when wheat is at $14\cdot902\dot{7}$ *s.* per bushel, what ought the sixpenny loaf to weigh when wheat is at $72\cdot8\dot{3}$ *s.* per quarter ?

110. A contractor agreeing to finish a piece of work in a fixed time employs 4750 men who work 8 hours a day. After $\frac{4}{7}$ of the time has expired, he finds that only $\frac{8}{15}$ of the work has been done, and then 590 men leave his service. How many hours a day must the remaining men work in order that their master may fulfil his contract ?

111. If 235 men can dig a trench 413 ft. long, 5 ft. wide, and 11 ft. 9 in. deep in 59 days of 7 working hours each ; how many men must be

employed for 67 days of 6½ working hours each, to dig a trench 871 ft. long, 6 ft. 6 in. wide, and 7ft. 9 in. deep, each man of the latter set doing $\frac{1}{17}$ part more work in an hour, than each of the former ?

112. If 79 men can dig a well 5 yds. in diameter and 85 ft. deep in 119 days working 8 hours a day, how many hours a day must 158 men work for 204 days in order to dig another well 20 ft. in diameter and 35 yds. deep, the hardness of the soil in the two cases being as 7 : 9 ? (The area of a circle varies as the square of its radius.)

113. If 98 men, or 154 women or 238 boys can do a piece of work in 24 days, working 7½ hours a day, how many men, with the assistance of 121 women and 221 boys will be able to do another piece of work 3½ times as great in 49 days working 6 hours a day ?

114. If 345 horses consume a stack of hay 115 ft. long, 31 ft. broad, and 26 ft. 6 in. high in a week, what is the height of another stack 1085 ft. long, and 39 ft. 9 in. broad, which will supply 1470 mules for 84 days, the voracity of a horse being $\frac{3}{10}$ more than that of a mule ?

115. If a 5½ chatak loaf costs 10as. 8 p. when wheat is at Rs 5. 13 as. 4 p. per maund, what should be paid for a 2·75 chatak loaf when wheat is at Rs 6. 9 as. per maund ?

116. If the expenses of a family of 34 persons for 19 months be Rs. 1530 when rice is selling 6½ seers for the rupee, what will be the expenses of another family of 38 persons for 40 months when rice is selling at the rate of Rs 5. 12 as. per maund, the daily quantities of rice required for a single member of each of the two families being as 5 : 6 ?

117. If 623 almiras each 11 ft. 3 in. long, 3 ft. 9 in. broad, and 21 ft. high can be carried through a distance of 2½ miles for Rs. 116. 13 as. how many almiras each 8 ft. 9 in. long, 3 ft. 4 in. broad, and 17 ft. high can be carried through one league for Rs. 130. 5 as. 4 p. ?

118. A person at the beginning of a year invested a capital of Rs. 8500 in trade ; after 4 months another man joined him in his business with a capital of Rs. 10500. At the end of the year the first man received Rs. 1870 as the share of his gain. What should his partner receive as his share ?

119. If 20 pumps, each making 37 strokes 4½ ft. in length in 7 min., empty a tank in two weeks working 9½ hours a day, how many pumps, each making 27 strokes 2½ ft. in length in 6 min., will empty another tank of double the size in 19 days working 9½ hours a day ?

120. An employer pays his workmen Rs.36. a week when they work 8 hrs. a day : what ought he to pay for 7 hrs. a day, if he finds that in the latter case 14 men can do as much work as 15 men before the change ?

121. If 21 men dig a moat of a uniform breadth and depth of 15 ft. and 21 ft. respectively and surrounding a rectangular field 400 yds. long by 250 yds. broad, in 175 days working $7\frac{1}{2}$ hours a day ; how many hours a day must 44 men work for 135 days in order to dig another moat of a uniform breadth and depth of 18 ft. and 24 ft. respectively and surrounding a rectangular field 550 yds. long by 285 yds. broad, 7 men of the former set digging as much as 6 men of the latter in the same time ?

122. A contract is to be finished in 240 days, and 70 men are employed at once ; at the end of $\frac{5}{8}$ of this time it is found that only $\frac{3}{8}$ of the work is done. How many additional men must be employed to fulfil the contract, if these additional men work 7 hours a day, whilst the first 70 men work only 6 hours a day until the contract is finished ?

123. In a boat race one crew rows 60 strokes to the other's 51 ; but the strokes of the latter are $\frac{1}{17}$ stronger than those of the former. If it takes the first crew 1·05 hr. to row a race of $4\frac{1}{4}$ miles, by how many yards and by how many seconds will they win ?

124. A contractor agreeing to finish a piece of work in 62 days, employed 152 men. They worked at the rate of 8 hours a day, but after 24 days only $\frac{1}{3}$ of the work was done. The contractor then engaged some boys to help the men in finishing the work in the stipulated time. Each of the men could do half as much again as a boy in the same time, and the boys worked only 6 hours a day. How many boys did he employ ?

125. If 247 men earn Rs.3570 in 42 days working $8\frac{1}{4}$ hours a day, how many hours a day should 304 men work to earn Rs5520 in 69 days, at the same rate ?

126. Before the introduction of the income-tax, the yearly income of a gentleman was £16170. What must it be now with an income-tax of 9d. in the pound, that he may still have £16170 a year net ?

127. A zemindar after paying 5 as. 5 p. in the rupee for Government revenue, and 8 p. in the rupee on the remainder for road-cess, has Rs.204470 left. How much does his estate produce annually ?

128. A gentleman after spending 2 as. 9 p. on every rupee of his income for charitable purposes and saving $\frac{1}{4}$ of the whole, can spend Rs.8890 per annum. What does he save annually ?

129. An occupier pays house-rate of 1 a. 5·2 p., water-rate of 10·2p., lighting-rate of 5 p., and police-rate of 5·04 p. in the rupee every year. If the rent and rates amount to Rs.4063, what is the assessed annual value of the house ?

130. A man hires a house at an annual rent of £2160, which is assessed at $\frac{7}{8}$ of the rent ; he agrees to pay the following rates upon it :—a poor s rate of £3. 19 s. 2 d., a church-rate of £3. 4 s. 7 d., a lighting-rate of £3. 10 s. 10 d., a water-rate of £4. 13 s. 9d. and a police-rate of £1. 13 s. 4 d. on every £100 of the assessed value. What is the annual cost of the house ?

131. A bankrupt owes Rs.57000 to his three creditors, and his whole property amounts to Rs.38000. Two of his creditors lose respectively Rs.8333. 5 as. 4 p. and Rs.5666. 10 as. 8 p. on their debts. How much will the remaining creditor lose on his debt ?

132. A bankrupt's assets amount to Rs.33263, and his debts to Rs. 49358. If a certain creditor loses Rs.1170 on his debt, find the amount of his debt.

133. A bankrupt has book-debts equal in amount to his liabilities ; but on £6000, £8500, and £9000 of them he can recover only 14 s., 12 s. and 11 s. in the £ respectively ; the expenses of the bankruptcy are £6 for every £100 of the book-debts ; if he pays 11·4 s. in the £, find the amount of his liabilities.

134. A creditor first gets a dividend of 10 as. 8 p. in the rupee, and then a further dividend upon the deficiency of 2 as. 8 p. in the rupee. What does he get in the rupee altogether ?

135. A bankrupt can pay 14 s. 2 d. in the pound ; if his assets were £4500 more, he could pay 18 s. 4d. in the pound. Find his debts and assets.

136. A bankrupt's assets are Rs.22200. Out of this he pays 13 as. 2 p. in the rupee on half of his debts, and 11 as. 6 p. on the other half. What is the amount of his debts ?

137. Assuming that the square of the time of oscillation of a pendulum varies as its length, find the length of a pendulum which oscillates 70 times in a minute, if the length of one which oscillates once in a second be 39·2 inches.

138. The weight of a body varies inversely as the square of its distance from the centre of the Earth. Assuming the radius of the Earth to be 4000

miles, and the distance of the Moon from the Earth to be 240000 miles, find how much a body, which weighs one maund on the surface of the Earth, would weigh if carried to the Moon.

139. When a body falls from rest, the space described by it varies as the square of the time during which it has been falling: if a body falls through 788·9ft. in 7 seconds, how far does it fall in 10 seconds, and what space does it describe in the 10th second?

140. The square of the time of a planet's revolution varies as the cube of its mean distance from the Sun. Find the time of Jupiter's revolution, assuming the mean distances of the Earth and Jupiter from the Sun to be 92 millions and 480 millions of miles respectively.

Additional Examples XXXIV.—Division into Proportional Parts.

1. Divide £426. 12*s.* 7*d.* among 5 persons so that their shares may be as the numbers 3, $4\frac{3}{8}$, $7\cdot\dot{8}$, $10\cdot91\dot{6}$, and ·13.

2. Gunpowder is composed of 75 parts of saltpetre, 10 parts of sulphur, and 15 parts of charcoal. How many pounds of sulphur are there in 5 cwt. of gunpowder, and what weight of gunpowder can be made with 2 cwt. 1 qr. 3 lbs. of charcoal?

3. A number of mangoes is to be divided amongst 5 persons in shares proportional to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$; what must the number *at least* be, that this may be done without cutting any of the mangoes?

4. Brass is composed of 31·5 parts of copper and 15·5 parts of zinc; what quantity of brass contains 2 mds. 32 srs. more of copper than zinc?

5. Pure water contains $88\cdot\dot{8}$ parts of oxygen and $11\cdot\dot{1}$ parts of hydrogen; find the weight of each in 20 cub. ft. of water, one cubic foot of water weighing 62 lbs. 7 oz. Avoir.

6. A certain sum of money is divided among *A*, *B* and *C* so that *A*'s share : *B*'s share = 4 : 5, and *B*'s share : *C*'s share = 5 : 6; *A* receives Rs. 50 less than *B*. Find the entire sum.

7. The area of a circle varies as the square of its radius. Divide a circle of 18 in. radius into three parts by means of two concentric circles, such that the areas of the parts may be proportional to 4, 21, and 56.

8. There are 399 coins consisting of moidores, sovereigns, half-guineas and half-crowns, and the values of the different coins are as 9 : 8 : 7 : 6. Find the number of each.

9. There are 16082 coins consisting of rupees, half-rupees, quarter-rupees, two-anna pieces, and pice, and the values of the different coins are as 34 : 19 : 9 : 23 : 25. Find the number of two-anna pieces.

10. £1274 is made up of moidores, guineas, sovereigns and crowns; and the numbers of coins are proportional to the numbers, 12, 34, 56, 78. Find the number of coins of each kind.

Additional Examples XXXV.—(Percentage, Profit and Loss, and Average.)

1. The population of a country is one thousand crores; it rises $1\frac{1}{2}$ per cent. for 3 years successively; find the population at the end of the 3 years.

2. One maund of sugar contains 19·9424 seers of oxygen, 17·306 seers of carbon, and the rest hydrogen. Find the percentage composition of sugar.

3. The increase in the number of male and female criminals is 2·5 per cent. The increase in the number of males alone is 5·6 per cent., while the decrease in the number of females is 1·7 per cent. Compare the number of male and female criminals respectively.

4. The price of rice being raised 12·5 per cent., by how much per cent. must a man reduce his consumption of that article so as not to increase his expenditure?

5. Of the candidates at an examination, 5·5 per cent. are females; 46 per cent. of the males and 28 per cent. of the females fail: what percentage of the whole number of candidates pass?

6. A zemindar employs a gomashta to collect rents, and agrees to allow him $2\frac{1}{2}$ per cent. on the net amount collected by him. The rates payable

by the zemindar are $16\frac{2}{3}$ per cent. on the gross rent, and the income-tax is $4\frac{1}{6}$ pies in the rupee on the remainder. If the zemindar gets Rs. 10647, find the gross rent.

7. If a reduction of 20 per cent. of the duty on salt causes an increase of $4\frac{1}{7}$ per cent. on the revenue derived from it, how much per cent. does it cause the consumption of salt to increase ?

8. A building costs Rs. 29900. At how much per week must it be let in order to pay 6 per cent. annually, allowing 8 per cent. of the receipts for annual repairs ?

9. If 56 per cent. of the rupees in circulation are short of weight by $1\frac{1}{4}$ per cent., and this is made good by taking 14 per cent. from the eight-anna pieces, compare the number of rupees in circulation with that of the eight-anna pieces.

10. If 35 per cent. of income-tax returns are so wrong that if corrected the tax would yield 20 per cent. more, how much per cent wrong on the average must they be, the other returns being right ?

11. Living expenses of all sorts are $42\frac{1}{2}$ per cent. dearer in town than in the village, and wages are $42\frac{1}{2}$ per cent. higher. A labourer can save Rs. 570 a year out of his wages in the town. What can the village labourer save ?

12. The wages of a native carpenter are two-thirds of those of a Chinese carpenter, and yet a given amount of carpenter's work in a given time would cost $16\frac{2}{3}$ per cent. more in wages if done by native carpenters than by Chinese carpenters. Compare the efficiency of a native and a Chinese carpenter.

13. After a strike of 8 weeks, labourer's wages are raised 6 per cent. ; during the strike they live upon $\frac{3}{8}$ of what they spend when in work. Supposing that before the strike they spent just what they earned, and that their expenditure remains the same, how many weeks must they work at the increased rate in order to recover what they had to spend during the strike ?

14. A labourer saves $12\frac{1}{2}$ per cent. of his wages ; in consequence of a strike, his wages are raised *2s. 6d.* a week ; but the cost of living being thereby raised $21\frac{3}{7}$ per cent., he finds that he saves £1. *6s.* a year less than he did before. How much were his weekly wages ?

15. Of two candidates at an election, it was calculated that A would be returned by a majority of 140; but as, at the election, A only polled 85 per cent. of his promises, whilst B polled 94 per cent. of his, B was returned by a majority of 7. How many votes had each?

16. If by selling a house for £490 there be a loss of $12\frac{1}{2}$ per cent., what per cent. is gained or lost by selling it for £596. 8*s*.?

17. A book-seller sells 100 copies of a book for Rs. 693. 5 as. 4 p. The expense of sale is 4 per cent. on the cost price, and the profit 25 per cent. on the whole outlay. Find the cost price of a single copy.

18. A wine-merchant bought 6 pipes of wine for Rs. 600; he sold 189 gallons so as to lose 15 per cent.; at what price must he sell the remainder so as to gain 20 per cent. on the whole?

19. A merchant sold 125 yds. of velvet for Rs. 793. 5 as. 4 p., and his profit was the cost price of 15 yds. How much did he gain per cent.?

20. How much per cent. must a tradesman add to the cost price of his goods, that he may make 13·4 per cent. profit after allowing his customers a reduction of 10 per cent. on his bill?

21. A farm is let for Rs. 120 and the value of a certain number of maunds of rice. When rice is at Rs. 2. 6 as. a maund, the whole rent is 15 per cent. lower than when it is at Rs. 3. 8 as. a maund. Find the number of maunds of rice which are paid as part of the rent.

22. A merchant has goods worth Rs. 450; he sells one-third of them at a loss of 5 per cent.; by what increase per cent. must he raise that selling price, in order that by selling the rest at the increased rate, he may gain 5 per cent. on the whole transaction?

23. A person bought a Geneva watch, bearing a duty of 20 per cent., and sold it at a loss of 4 per cent. Had he sold it for Rs. 24 more, he would have cleared 1 per cent. on his bargain. What had the Geneva maker for it?

24. If a milkman buys milk at the rate of $6\frac{1}{4}$ seers for the rupee, and adds water to it, what will be the percentage of milk in the mixture which he can supply to his customers without loss at 2 as. a seer?

25. If the coal owner, the wholesale merchant, and the retail dealer make profits of 20, 15 and 26 per cent. respectively, what is the cost to the coal owner of a quantity of coal which is retailed for Rs90. 9 as.?

26. A manufacturer sells goods at a profit of 70 per cent. If the tradesman becomes insolvent and pays a dividend of 11*s.* 8*d.* in the £, how much per cent. does the manufacturer gain or lose ?

27. A grocer buys 2 mds. of sugar ; the first maund he sells at a profit of 8 per cent., and the second, which cost Rs. 2. 8 as. more, at 12½ per cent. profit. The difference in the retail price being 1 a. 5·28 p. per seer, find the cost price of each maund.

28. A person buys a horse and sells it at a profit of 6 per cent. If he had bought it at 5 per cent. less and sold it for Rs. 2 more, he would have gained 12 per cent. Find the cost price of the horse.

29. Two kinds of tea are mixed in the ratio of 3 : 4 and sold so as to gain 7 per cent. ; again they are mixed in the ratio of 4 : 3 and sold so as to gain 11 per cent. ; the retail price per pound is the same in the two cases. What is the ratio of their cost prices ?

30. If a wholesale dealer sells to a retailer at 7½ per cent. profit, and the retailer to the consumer at 18·75 per cent. profit, in what ratio is the whole profit shared between the wholesale dealer and the retailer ? Find also how much per cent. more than the prime cost the consumers pay.

31. A speculator invested his capital successively in four different speculations. In the first he gained 72 per cent., in the second he lost 12½ per cent., in the third he gained 20 per cent., and in the fourth he lost 25 per cent. How much per cent. did he gain or lose on his original capital ?

32. A man bought a certain number of mangoes at R. 1. 8 as. per hundred, one-third of the number at Rs. 7. 8 as. per hundred, and one-fifth of the number at 20 for the rupee. At what rate must he sell them to gain 15 per cent. on his outlay ? If his total profit be Rs. 9, how many did he buy ?

33. A vintner buys wine at £1. 5*s.* a gallon and brandy at £2. 8*s.* a gallon. In what proportion must he mix them together, so that by selling the mixture at £1. 14*s.* 6*d.* a gallon, there may be a profit of 16 per cent. on the price of the wine and 25 per cent. on the price of the brandy ?

34. By using a false balance a retail-dealer defrauds both the wholesale-dealer from whom he buys and the customer to whom he sells, to the extent of 15 per cent. Supposing he sells at the same rate at which he buys, what per cent. does he gain on his outlay by his dishonesty ?

35. A wine-merchant mixes two kinds of wine at £1. 2*s*. 6*d*. and £1. 15*s*. a gallon respectively in a certain proportion, and sells the mixture at a profit of 10¾ per cent. Had he sold each kind of wine at the same rate as the mixture, he would have gained 20 per cent. and 6⅝ per cent. respectively on the cost price of each. In what proportion did he mix together the two kinds of wine ?

36. The average age of 12 men, 13 women and 1 boy is 49 years, that of the 12 men being 53 years and of the 13 women being 48 years ; find the age of the boy.

37. The average age of 16 boys is diminished 4 months when one of them 14 yrs. 10 mo. old is replaced by a new boy ; find the age of the new boy.

38. The average age of a class of 25 boys is 13 years ; the names of two of the boys are struck off from the register, and 6 new boys are admitted, whose ages are 10, 11, 12, 13, 15 and 16 years respectively. If the average age remains the same as before, find the average age of the two boys whose names are struck off.

39. The average price of a sofa, a chair and a table is Rs. 23, and that of the chair, the table and a book-shelf is Rs. 20. If the sofa be worth ⅜ as much again as the book-shelf, find the price of the sofa.

40. The average temperature for Monday, Tuesday, Wednesday and Thursday is 91°, and the average for Tuesday, Wednesday, Thursday and Friday is 88°. If the ratio of the temperatures for Friday and Monday be 7 : 8, find the temperature on Monday.

Additional Examples XXXVI.—Fellowship.

1. Four partners *A*, *B*, *C*, *D* engage in business with a joint capital of Rs.252450. At the end of a year they receive Rs.400, Rs.460, Rs.350, and Rs.275 respectively. How much capital did *A* put in ?

2. In a joint trade, *A* receives ⅜ of the profits, and *B*, and *C* share the remainder equally. *A*'s income is increased by Rs.90 when the profits rise from 7 per cent. on the capital to 8¼ per cent. Find the respective capitals invested.

3. *A* and *B* are partners in a trading firm in which *A* has 9 as. 4 p. share of the capital, and *B* has 6 as. 8 p. share. *A* being the working

partner receives 4 per cent. of the whole profit, and the rest is divided in proportion to the capital. What does A receive out of a profit of Rs. 2500 ?

4. A starts business with a capital of £5000 on the 18th of March, and on the 4th of June following B joins with a capital of £3000. The profits amount to £2181. 18s. by the 31st of December. What is each person's share ?

5. A and B start a business with capitals as 7 : 9. They withdraw respectively $\frac{1}{6}$ and $\frac{5}{8}$ of their capitals at the end of 3 months. At the end of the year the profit amounts to Rs. 3705. How ought this to be divided ?

6. A and B hired a meadow for 8 months at Rs. 13 per month. A put in 28 cows for 5 months, and B put in a certain number for the remaining 3 months. Of the total rent A paid Rs. 56 and B the remainder. How many cows did B put in ?

7. A and B entered into partnership with Rs. 1400 and Rs. 800 respectively. After 4 months A withdrew $\frac{3}{7}$ of his capital, but after 4 months more put back $\frac{4}{5}$ of what he had withdrawn. B added Rs. 300 to his capital at the end of 6 months. The profits at the end of the year amounted to Rs. 5064. How much of this would each get ?

8. A, B and C start a business, A putting in half as much again as B, and B, one-third as much again as C. At the end of 2 months A withdraws $\frac{1}{4}$ of his capital, but after 5 months more puts back $\frac{1}{4}$ of what he has withdrawn, when B takes out $\frac{1}{4}$ of his capital, and C puts in an additional amount equal to $\frac{1}{8}$ of his original capital. A receives Rs. 4710 as profits at the end of the year ; what do B and C respectively receive ?

Additional Examples XXXVII.—Simple Interest.

1. On the 1st of February 1892 a man borrowed Rs. 2000 at $5\frac{3}{4}$ per cent. per annum, promising to repay as soon as the interest amounted to Rs. 69. When did the loan expire ?

2. In what time will Rs. 5600 amount to Rs. 6912. 8 as. at the rate of $1\frac{1}{4}$ pies per rupee per month How much is this a rate per cent. per annum ?

3. At what rate per cent. per annum will the interest on any sum of money be $\frac{3}{8}$ of the amount in 10 years ?

4. The interest on a sum of money at the end of 8 years is ·28 of the sum itself ; what is the rate per cent. ?

5. In what time will the interest on any sum of money at $5\frac{1}{2}$ per cent. per annum be $\frac{2}{8}$ of the amount ?

6. A money-lender lent a sum of money on the 15th of March at 28·125 per cent. per annum. When the debt was cleared on the 8th of August following, he received Rs. 890. What was the sum lent ?

7. A person borrowed a sum of money at $1\frac{1}{2}$ pice per rupee per month, agreeing to repay the loan at the end of 2 years 5 months. At the end of this time the debt amounted to Rs. 1343. 12 as. What was the sum borrowed ?

8. A sum of money increases by $\frac{1}{16}$ of itself every year, and in 9 years it amounts to Rs. 1600. Find the sum.

9. The sum of £495 increases by ·035 of itself per year ; how long will it take to amount to £581. 12*s.* 6*d.* ?

10. A person borrowed £850 at the beginning of a year at a certain rate of interest, and after 5 months he borrowed £500 more at one-fourth of the previous rate. At the end of the year the whole interest on the two loans was £101. 10*s.* 5*d.* What was the rate of interest at which the first sum was borrowed ?

11. *A* borrowed of *B* at the same time Rs. 2500 at $3\frac{1}{2}$ per cent., and Rs. 4000 at 6 per cent. ; when both these sums amounted together to Rs. 7482. 8 as., he cleared off his debts. How long did the loan continue ?

12. A certain sum of money amounts to £69097. 10*s.* in 7 years at $3\frac{1}{2}$ per cent. per annum. In how many years more will it amount to £74925 ?

13. What sum of money laid out at 5 per cent. will give R. 1 interest a day ?

14. A certain sum amounts to £6050 in 6 years, and the interest is ·21 of the principal. Find the principal and the rate per cent. per annum.

15. The principal and interest for a certain time at $3\frac{1}{2}$ per cent. are together Rs. 7637. 8 as , and the interest is ·175 of the principal. Find the time.

16. If £3750 amounts to £3827. 6*s.* 10½*d.* in 9 months, what sum will amount to £5126. 0*s.* 10*d.* in 11 months at the same rate ?

17. What sum of principal money, lent out at 3½ per cent. per annum will produce in 5 years the same interest as Rs. 5000 lent out at 5 per cent. per annum will produce in 7 years ?

18. *A* invests a certain sum which yields an annual income of Rs. 852. 3 as. 5 p. after deduction of an income-tax of 5 pies in the rupee. If the rate of interest be 3½ per cent. per annum, find the sum invested.

19. A person invests Rs. 20000 at 5¾ per cent. per annum, and spends at the end of each year Rs. 800 more than the annual interest on the sum invested, and thus at the end of a certain time has nothing left. If he had spent Rs. 800 less than the annual interest, show that the sum invested would have been doubled at the end of the same time.

20. A sum of money doubles itself in 16 years 8 months ; in how many years would it treble itself ?

Additional Examples XXXVIII.—Compound Interest.

1. Find to the nearest pie the compound interest on Rs. 1000 for 2¾ years at 3½ per cent. per annum.

2. Find to the nearest penny the compound interest on £500 for 3 years at 4½ per cent. per annum.

3. Find the compound interest on Rs. 3000 for 1½ years at 4 per cent. per annum, interest being payable half-yearly.

4. Find to the nearest penny the compound interest on £5000 for 1 year at 5 per cent., interest being payable quarterly.

5. What sum lent at compound interest will amount to £7114. 15*s.* 3·552*d.* in 3 years at 4 per cent. per annum ?

6. Find the difference between the simple and compound interest on Rs. 16666. 10 as. 8 p. for 5 years at 5 per cent. per annum.

7. The difference between the simple and compound interest on a certain sum of money for 4 years at 4 per cent. is Rs. 3851. Find the sum.

8. A merchant commenced with a certain capital, and gained annually at the rate of 20 per cent. At the end of 5 years he is worth Rs. 77760. What was his original capital ?

9. A person lays aside Rs.2000 at the beginning of each year, and employs the money at 4 per cent. compound interest; what will he be worth at the end of 3 years?

10. A money-lender borrowed a certain sum of money at $4\frac{1}{2}$ per cent. per annum simple interest, and paid the interest at the end of the year. He lent it at 5 per cent. per annum compound interest payable quarterly and received the interest at the end of the year. By this means he gained £2435. 4*s.* $2\frac{3}{4}$*d.* in one year; what sum did he borrow?

11. At compound interest money doubles itself in 12 years; what will be the amount of Rs.1000 in 60 years?

12. If at compound interest the second year's interest is Rs.1040, and the third year's Rs.1081·6, what was the first year's interest?

Additional Examples XXXIX.—Present Worth and Discount.

1. Find the present worth of Rs.2771794. 11 as. due 3 years hence at $3\frac{1}{2}$ per cent. per annum compound interest.

2. Find the discount on £5712. 4*s.* due 4 years hence at 4 per cent. per annum compound interest.

3. If the discount on Rs.411. 4 as. be Rs.61. 4 as. at $3\frac{1}{2}$ per cent. per annum simple interest, when is the sum due?

4. If the discount on £1706. 5*s.* due 5 years hence be £206, 5*s.*, what is the rate of interest?

5. The discount on a certain sum of money due $2\frac{1}{2}$ years hence at $3\frac{3}{4}$ per cent. is Rs.60. 10 as. 8 p. Find the sum.

6. If the interest on Rs.3400 at $3\frac{1}{2}$ per cent. be equal to the discount on Rs.4233 for the same time at the same rate, when is the latter sum due?

7. The interest on a certain sum of money is £1250, and the discount on the same sum for the same time at the same rate is £1041. 13*s.* 4*d.* Find the sum.

8. The interest on a certain sum of money for 5 years is Rs 31972. 8 as., and the discount on the same sum for the same time at the same rate is Rs.26100. Find the sum and the rate per cent. per annum.

9. The difference between the interest and discount on a certain sum for 3 years 4 months at 5 per cent. is Rs. 16. 10 as. 8 p. Find the sum.

10. The interest on £2350 for a certain time at a certain rate is £411. 5*s.*; find the discount on the same sum for the same time at the same rate.

11. The discount on a sum of money due 9 months hence at $3\frac{3}{4}$ per cent. per annum is Rs. 450. Find the present worth of the sum.

12. The discount on £4200 for a certain time at a certain rate is £1200; what is the discount on the same sum for half that time at the same rate?

13. *A* offers for a house Rs. 6400, and *B* offers Rs. 6525 to be paid at the end of 5 months. If the rate of interest be $4\frac{1}{2}$ per cent. per annum, which is now the better offer?

14. A tradesman marks his goods with two prices, one for ready money, and the other for 4 months' credit allowing discount at $4\frac{1}{2}$ per cent. What ratio should the two prices bear to each other? If the credit price be marked at Rs. 25. 6 as., what ought to be the cash price?

15. A grocer buys 676 mds. of sugar for Rs. 8450 payable at the end of 9 months, and on the same day sells them at Rs. 13. 12 as. per maund ready money; what does he gain by the transaction, reckoning interest at $7\frac{1}{2}$ per cent. per annum?

16. A man bought an estate for Rs. 207000 payable at the end of 6 months, interest being reckoned at 7 per cent. per annum. He sold it immediately for Rs. 216550 payable at the end of 4 months, interest being reckoned at 5 per cent. per annum. What did he gain per cent. by the transaction?

17. A bankrupt has book-debts equal in amount to his liabilities, half due in 6 months and the rest in a year; calling up his book-debts at once, he is able to pay 17*s.* 6*d.* in the pound; if interest be reckoned at 5 per cent. per annum, how much per cent. are the expenses of his bankruptcy?

18. A bill of £750 drawn on the 14th of February, and payable 7 months after date, is discounted on 24th April at $6\frac{1}{2}$ per cent. How much does the holder of the bill receive?

19. A bill of Rs. 2575 is drawn on the 15th of September 1891 at 9 months date, and is discounted on the 12th of November following, at 5 per cent. How much more was charged than the true discount?

20. A bill is drawn on the 21st of March at 8 months date, and is discounted on the 1st of July following at $5\frac{3}{4}$ per cent. If the banker charges £2. 12*s.* $10\frac{1}{8}$*d.* more than the true discount, find the amount of the bill.

Additional Examples XL.—Equation of Payments.

1. *A* owes *B* Rs. 3000 whereof Rs. 500 are due 73 days hence, Rs. 1000 146 days hence, and the remainder 219 days hence. When may all these debts be paid together ?

2. *A* owes *B* Rs. 4000 payable after $5\frac{3}{8}$ months. He pays Rs. 600 after 2 months, Rs. 800 after 4 months, and Rs. 1000 after 6 months. When ought he to pay the remainder ?

3. *A* owes *B* a debt of which $\frac{1}{3}$ is payable after 3 months, $\frac{1}{4}$ after 5 months, $\frac{1}{6}$ after 7 months, $\frac{1}{18}$ after 9 months, and the remainder after 13 months. If *A* pays the whole debt by a single payment, when ought he to pay ?

4. A debt is payable after $10\frac{5}{8}$ months, but the debtor pays $\frac{1}{3}$ of it after 4 months, $\frac{1}{6}$ after 6 months, and $\frac{1}{7}$ after 8 months. When ought the remainder to be paid ?

5. On the 1st of January 1892 *A* finds that he owes to *B* the following sums of money :—£1000 payable on the 10th of February, £2000 payable on the 23rd of March, £3000 payable on the 14th of May, and £4000 payable on the 4th of August. It he clears the whole debt by a single payment, on what date will it become due ?

6. On the 21st of March 1891 *A* finds that he owes *B* Rs. 50000 payable on the 27th of October following. He pays Rs. 2000 on the 15th of April, Rs. 6000 on the 4th of June, Rs. 10000 on the 1st of August, and Rs. 14000 on the 12th of October. On what date ought he to pay the remainder ?

Additional Examples XLI.—Stocks.

1. A person receives $3\frac{41}{171}$ per cent. interest on his capital by investing in the $3\frac{1}{2}$ per cents. What is the price of the stock, and how much of the stock can be purchased for Rs. 26750, brokerage being $\frac{1}{8}$ per cent. ?

2. How much money must a broker invest in the funds when consols are at $107\frac{1}{3}$, so as to procure the same income as if he had invested Rs. 16850 when consols were at $105\frac{5}{16}$?

3. A person invests Rs. 60130 in the $3\frac{1}{2}$ per cents. at $107\frac{7}{8}$. Find his half-yearly income after deducting an income-tax of 5 pies in the rupee.

4. Which is the better investment, the $3\frac{1}{2}$ per cents. at $109\frac{7}{16}$ or the 5 per cent. Calcutta Municipal Debentures at 117 ? What sums must be invested respectively in the two stocks to produce the same annual income of Rs. 1960 ?

5. How much in the $3\frac{1}{2}$ per cents. at $106\frac{5}{8}$ (brokerage $\frac{1}{8}$ per cent.) must be sold out to pay a bill of Rs. 1751, 9 months before it becomes due, real discount being allowed at 4 per cent. per annum ?

6. A person invests Rs. 18300 in purchasing a number of Preference shares of the Serajgunge Jute Company which pays a yearly dividend of Rs. 5. 8 as. on each share. After paying 5 p. in the rupee for income-tax, his annual income from the investment is Rs. 803. 8 as. 3 p. What is the value of a share ?

7. If I lay out Rs. 16207 in the purchase of $3\frac{1}{2}$ per cent. consols, when they are at $106\frac{1}{2}$, at what price should I sell out my stock again in order to realize on the whole a gain of Rs. 190, after having paid the usual brokerage of $\frac{1}{8}$ per cent. on each transaction ?

8. A person invested Rs. 35670 in the 5 per cent. Calcutta Municipal Debentures at 23 premium, and after receiving the half-yearly dividend sells out when they have risen $1\frac{1}{2}$ per cent., and invests the entire proceeds in the $4\frac{1}{2}$ per cent. railway stock at $115\frac{9}{32}$. Find the alteration in his income.

9. If Rs. 8570 invested in the $3\frac{1}{2}$ per cent. Government Securities bring in a net annual income of Rs. 272. 11 as. 4 p. after paying an income-tax of 5 p. in the rupee, what is the price of the stock, the usual brokerage of $\frac{1}{8}$ per cent. being allowed ?

10. A person who has Rs. 61355 to invest finds that if he invests the sum in buying the Howrah Mills Ordinary shares at Rs. 170. 8 as. per share paying a yearly dividend of Rs. 6. 8 as. on each share, his annual income will be Rs. 377. 5 as. 4 p. more than if he invests the same in the $3\frac{1}{2}$ per cent. Government Securities. A brokerage of 2 as. per share being allowed in the former case, and the usual brokerage of $\frac{1}{8}$ per cent. in the latter, what is the price of the Securities ?

11. A person transfers his capital from the Central Cachar Tea shares at Rs.89 per share paying a dividend of Rs. 3¾ per share, to the 4½ per cent. Calcutta Municipal Debentures at 119. Does he gain or lose in annual income? If his income be altered by Rs. 91. 8 as., how much was it originally?

12. A man has Rs.9600 which he invests partly in the 3½ per cent. Government Securities at 106, and partly in the 4½ per cent. Calcutta Municipal Debentures at 107. What sums must he invest in the respective stocks to make $3\frac{28}{57}$ per cent. on the whole?

13. A person invests Rs.10920 in the 3½ per cents. When the price of the stock rises in the ratio of 106 : 105, he sells and re-invests in the Bengal Steam Ship shares at Rs.104 per share paying a dividend of Rs3. 12 as. per share. His income is increased by Rs.33. 8 as. At what price did he buy?

14. One-third of a certain capital is invested in the 3½ per cents. at 105, one-fourth in the 4 per cent. Calcutta Municipal Debentures at 109, and the remainder in the India General Steam Navigation Company's shares at Rs.133 per share paying a dividend of Rs.5. 4 as. on each share. If the total income is Rs.1141. 1 a. 8 p., what is the capital?

15. *A* invests in the 3½ per cents. at 112; *B*, who has Rs. 840 less than *A*, in the 5 per cent. Calcutta Municipal Debentures at 125, and obtains the same income. How much money has each?

16. The 3½ per cent. Government Securities are at 105, and the 5 per cent. Calcutta Municipal Debentures at 117½. A person has a sum of money to invest which will give him Rs.100 more of the former stock than of the latter. Find the difference of the income he would obtain by investing in the two stocks.

17. A man invests Rs.10575 in the 3½ per cents. at 105⅝; he sells out ⅓ of his stock when the funds have risen to 106⅞, Rs.3000 stock when they have fallen to 104⅛, and the remainder when they have again risen to 107⅝. The usual brokerage of ⅛ per cent. is allowed both in buying and selling. What sum does he gain? If he invests the proceeds in the 5 per cent. Municipal Debentures at 118, what will be the alteration in his income, no brokerage being allowed in this last case?

18. A man wishes to invest Rs.28000 partly in the 3½ per cents. at 105, and partly in the 4½ per cent. Calcutta Municipal Debentures at 117. How

must he divide his capital between the two stocks so as to derive the same income from each of them ?

19. A person finds that if he invest a certain sum in the 5½ per cent. Calcutta Port Trust Debentures at 126, he will get Rs. 114. 8 as. a year more than if he invest it in the 4½ per cent. Calcutta Municipal Debentures at 113½. How much has he to invest ?

20. A person having to pay a debt of Rs. 3584. two years hence invests a certain sum in the 3½ per cents. at 105 (to accumulate interest till the debt is paid). One year after when the funds have risen 3½ per cent., he invests an equal sum together with the interest on the former. Supposing the price of the funds to remain the same throughout the second year, what must be the sum invested on each occasion so that with its interest there may be just sufficient ready money to pay the debt at the proper time ?

21. The income-tax being raised from 4 pies to 5 pies in the rupee, it is calculated that by transferring from the 5½ per cent. Howrah Jute Mills stock at 159 to the 3½ per cent. Government Securities, the net income is diminished in the ratio of 136 : 141. What is the price of the 3½ per cents. ?

22. The income of a railway company would justify a dividend of 6½ per cent., if all the shares were alike ; but as Rs. 4000000 of the stock consists of preference shares guaranteed 8 per cent., the ordinary shareholders receive only 5½ per cent. What is the whole amount of stock ?

23. *A* and *B* have each the same sum of money. *A* buys equal amounts of the 6 per cent. Raneegunge Coal Company's stock at 101, and of the 5 per cent. Darjeeling Himalayan Railway stock at 104½ : *B* invests his money equally in the purchase of the same stocks. Who receives the larger income ? If their incomes differ by Rs. 284. 10 as. 8 p., how much money did they each invest ?

24. A person invests Rs. 2415 in the 3½ per cents. at 105, and sells part of his stock when they have risen 3½ per cent. and the remainder when they have fallen 1½ per cent. He gains Rs. 45½ by the transaction. How much stock did he sell at first ?

25. One company guarantees to pay 6½ per cent. on shares of 1200 each ; another guarantees at the rate of 5½ per cent. on shares of 850 each ; the price of the former is Rs. 1248, and that of the latter Rs. 952 Compare the rates of interest which they return to the purchaser.

26. A person has stock in the $3\frac{1}{2}$ per cents. which produces him Rs.4900 per annum. He sells out one half at $107\frac{1}{2}$, and invests the proceeds in a Railway stock at 129. What dividend per cent. per annum ought the Railway stock to pay; so that he may increase his annual income by Rs.1050 by the transaction ?

27. A person holds a certain amount of the $4\frac{1}{4}$ per cent. Budge-Budge Jute Mills stock which he sells when they are at $103\frac{1}{4}$, and with the proceeds he buys $5\frac{1}{2}$ per cent. Calcutta Municipal Debentures at 126. After a time when the latter stock rises 3 per cent. he sells out, and with the proceeds purchases the original stock at $88\frac{1}{2}$. His annual income is now increased by Rs. $330\frac{5}{8}$. How much of the Jute Mills stock did he originally hold ?

28. A person having Rs. 10200 stock of the $5\frac{1}{2}$ per cent. North Birbhoom Coal Company calculates that by selling and reinvesting in the 6 per cent. Hooghly Mills stock at 102, he will increase his income by Rs.75 ; but before he makes the transfer, each stock falls 1 per cent. If he now transfers, what would be the alteration in his income ?

29. A fundholder directed his broker to purchase sixteen Rs.1000 shares in a certain mine, quoted at Rs. 3475 per share. To accomplish this he authorized the broker to sell out Rs. 20000 stock of the 5 per cent. Calcutta Steam Navigation Company's stock at $106\frac{1}{8}$, and such an amount of the 8 per cent. Cawnpore Cotton Mills Company's stock at $107\frac{1}{16}$, as just to realise the amount required for the purchase. The broker's charge on each of the three transactions was $\frac{1}{8}$ per cent. How much of the latter stock was sold, and what did the broker receive on the whole ?

30. A man had a certain capital which he invested partly in the 5 per cent. Barrakur Coal Company's stock at 105, and partly in the 6 per cent. Calcutta Flour Mill Company's stock at 104, securing a total annual income of Rs. 588. 12 as. Had he invested the former part of his capital in the latter stock, and the latter part in the former stock, his income would have been Rs. 27. 8 as. less. What was his capital ?

Additional Examples XLII.—Alligation.

1. A grocer buys black tea at 2*s.* 8*d.* per lb. and green tea at 3*s.* 9*d.* per lb. ; in what proportion must he mix them so that by selling the mixture at 3*s.* 2*d.* per lb. he may make a profit of $12\frac{1}{2}$ per cent. ?

2. Tobacco at 4 as. 4 p. per seer and 5 as. 6 p. per seer are mixed together so as to make a mixture of 1 md. 32 seers worth 4 as. 11 p. per seer. How many seers of each kind are taken?

3. A grocer mixes oats at Rs. 2. 12 as. a maund with oats at Rs. 2. 14 as. a maund. How many maunds of each kind must he take to make a mixture of 64 mds., so that by selling it at Rs. 3 a maund he may make a profit of 3 as. 3 p. per maund?

4. A milkman has cow's milk and buffalo's milk by means of which he forms a mixture weighing 1 md. 38 seers and worth Rs 11. 4 as. If the quantities of the two kinds of milk were interchanged, the mixture would be worth Rs. 10. 8 as. Supposing cow's milk costs him at the rate of 6¼ seers per rupee, what quantity of each kind did he take, and what is the price of buffalo's milk per seer?

5. A grocer has rice worth Rs. 4, Rs. 4. 4 as., Rs. 4. 8 as., and Rs. 4. 12 as. a maund respectively: how must he mix them to obtain a mixture worth Rs. 4. 7 as. a maund, using equal parts of the first and the third, and also equal parts of the second and the fourth?

6. A milkman buys cow's milk at 6 seers for the rupee and buffalo's milk at 2 as. a seer. He mixes them in the proportion of 3 : 2, and adds water to it so as to produce a mixture weighing a maund and a half; and by selling the mixture at 2 as. 4 p. per seer he makes a profit of 16⅔ per cent. on his outlay. How much water did he add?

7. A grocer buys teas at 2*s*. 3*d*., 2*s*. 6*d*., and 2*s*. 9*d*. a pound respectively: how must he mix them so as to obtain a mixture worth 2*s*. 8*d*. a pound, using the first two kinds in the proportion of 3 to 4?

8. A grocer buys teas at 2*s*. 6*d*., 3*s*., 3*s*. 6*d*., and 4*s*. a pound respectively. How must he mix them (using the first two kinds in the proportion of 4 to 5, and the last two in the proportion of 6 to 7) so that by selling the mixture at 3*s*. 3*d*. a pound he may make a profit of 8⅓ per cent?

Additional Examples XLIII.—Exchange.

1. Calculate the value in Indian money of £975. 12*s* 8*d*. to the nearest pie, when the exchange is at 1*s* 1*d*. a rupee.

2. The price of Taine's History of English Literature in two volumes is 15*s*. Calculate its price in Indian money to the nearest pie at the rate of 1*s*. 1½*d*. a rupee.

3. The price of a book is Rs 6. 13 as. $8\frac{4}{7}$ p. What is its price in English money, when the exchange is at 1*s.* $1\frac{1}{8}d.$ a rupee ?

4. The price of Ritchie's Natural Rights is 10*s.* 6*d.*, and its price in Indian money is Rs. 10. 1 a. $3\frac{9}{25}$ p. What is the Rate of Exchange ?

5. The price of a book is 11*s.* 8*d.* What is its price in Indian money to the nearest pie, when £1 is equivalent to Rs. 17. 7as. $3\frac{3}{11}$ p. ?

6. The price of Casey's Treatise on Analytical Geometry is 12*s.* What is its price in Indian money to the nearest pie, when the exchange is at 1*s.* $0\frac{3}{4}d.$ a rupee ?

7. I bought one copy of Deutsch's Literary Remains for 9*s.* 6*d.*, one copy of Goldstucker's Literary Remains for 11*s.* 8*d.*, one copy of Goldstucker's Panini for £1. 19*s.* 6*d.*, and Gladwini's Ayini Akbari in two volumes for £1. 13*s.* 10*d.* I had to pay Rs 84 in all. What was the Rate of Exchange ?

8. I bought a complete set of Tennyson's works for £3. 3*s.*, Aristotle's Theory of Poetry by Butcher for 11*s.* 8*d.*, Bendall's Catalogue of Sanskrit and Buddhist Manuscripts for 12*s.*, Darwin's Origin of Species in two volumes for 12*s.*, Pendlebury's Arithmetic for 4*s.* 6*d.*, Smith's Analytical Conic Sections for 7*s.* 6*d.*, and Greaves's Elementary Hydrostatics for 5*s.* What had I to pay altogether to the nearest pie, the exchange being at 1*s.* $1\frac{1}{8}d.$ per rupee ?

9. If 1 cwt. of fish costs £1. 7*s.* $6\frac{3}{4}d.$, find the price of one bazar maund of fish in Indian money, the exchange being at 1*s.* $1\frac{1}{3}d.$ a rupee.

10. A person in London owes another in St. Petersburg a debt of 7990 rubles, which must be remitted through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 23 francs 50 centimes for £1, and between Paris and St. Petersburg 2 francs 40 centimes for one ruble. The remittance is delayed until the rates of exchange are 23 francs 80 centimes for £1, and 7 francs for 3 rubles. What does the broker gain or lose by the transaction ?

11. A merchant in London is indebted to one at St. Petersburg 17000 rubles : the exchange between St. Petersburg and London is 23*d.* per ruble, between St. Petersburg and Amsterdam 37*d.* Flemish per ruble, and between Amsterdam and London 24*s.* 9*d.* Flemish per £ sterling. Is it more advantageous for the London merchant to be drawn upon directly from St. Petersburg or circuitously through Amsterdam ?

12. A merchant in Calcutta owes a sum of money to one in London: which method of payment will be more advantageous to him—a direct exchange, or a circuitous remittance from Calcutta to Vienna, from Vienna to Paris, and from Paris to London, the exchanges being R.1 = 1*s.* 1½*d.*, £1 = 25·6 francs, 15 francs = 7 florins, and 17 florins = Rs.24?

Additional Examples XLIV.—Square Root.

1. Find the least number which must be subtracted from ·00757 to make the remainder a perfect square.

2. Find the least integer by which 50540 must be multiplied to make the product a perfect square.

3. Find the least integer by which 15606 must be divided to make the quotient a perfect square.

4 Find the least square number which is exactly divisible by 32, 90 and 105.

5. Extract the fourth roots of

(1). ·00390625; (2). $1\frac{5085}{7203}$.

6. Find the least number of soldiers in a regiment so that it may be drawn up into hollow squares 30, 40 or 50 deep, and also into a solid square. What will be the number of men in front of the solid square?

7. The whole surface of a cube is 165 sq. ft. 54 sq. in. Find its edge.

8. The length of a rectangular field is to its breadth as 4: 3, and its area is 750000 sq. ft. Find its perimeter.

9, A general wishing to arrange his men, who were 127460 in number, into a solid square, found that there were 11 men over. How many men were there in the front?

10. The whole of the outward surface of a tin box is 12 sq. ft. 54 sq. in., and its length, breadth and height are proportional to the numbers 3, 2, 1. Find its dimensions.

11. A room 2½ times as long as it is broad is carpeted at R. 1. 4 as. per sq. ft., and the walls are painted at Rs. 2. 4 as. per sq. yd., the respective costs being Rs. 2112. 8 as., and Rs. 1365. Find the dimensions of the room.

12. The price of rice in annas per maund is equal to the number of maunds bought for Rs.370. 9 as : what is the price ?

13. A brick is twice as long as it is broad, and its height is $\cdot 10\dot{6}$ times the square of its breadth. The solid content of the brick is $325\frac{25}{48}$ cub. in. Find the dimensions of the brick.

14. The rate of interest being equal to the number of years, the simple interest is ·950625 times the principal ; what is the rate ?

15. A sum of money was lent at compound interest ; the first year's interest was £91. 5*s.*, and the third year's interest was £100. 12*s.* $0\frac{3}{4}$*d.* What was the sum and the rate per cent. ?

16. The discount on Rs. 16303 due a certain number of years hence at a certain rate per cent. per annum which is equal to the number of years is Rs. 5103. What is the rate ?

17. A square field contains 10 acres. Find its diagonal correct to the tenth part of an inch.

18. A room is 40 ft. long, 25 ft. broad, and 20 ft. high. Find its diagonal to the nearest inch.

19. The diagonal of a square is 5 ft. Find its side correct to the hundredth part of an inch.

20. Two persons start from the same place at the same time ; the one goes due west at the rate of 16 miles an hour, and the other due south at the rate of 12 miles an hour. How far are they distant from each other at the end of eight hours ?

Additional Examples XLV.—Cube Root.

1. Find the least number which must be subtracted from ·00186087 to make the remainder a perfect cube.

2. Find the least integer by which 41503 must be multiplied to make the product a perfect cube.

3. Find the least integer by which 74536 must be divided to make the quotient a perfect cube.

4. Extract the sixth root of ·000000117649 ; and the ninth root of $38\frac{227}{512}$.

5. The cost of a cubic mass of metal is Rs. 1984941, at Rs. 3. 5 as. 7 p. a cubic foot. Find the edge of the mass.

6. A piece of gold 28 ft. long, $4\frac{5}{8}$ in. broad and $1\frac{1}{4}$ in. thick is made into a cube. What is the length of its edge ?

7. A wall 4 times as high as it is broad, and 7 times as long as it is high, contains 18634 cub. ft. Find the breadth of the wall.

8. A cube whose edge is $8\frac{1}{2}$ ft. weighs 899 lbs. ; find the edge of another cube of the same material which weighs 1 cwt. 6 oz.

9. A certain sum of money was lent at compound interest. The first year's interest was £3125, and the fourth year's interest was £3515. 4*s*. What was the sum and the rate per cent. ?

10. A cube contains 151 cub. ft. 1216 cub. in. ; find its diagonal correct to the tenth part of an inch. Find also its whole surface.

APPENDIX B.

EXAMPLES ON CHAPTER I.

Sec. I.

Examples I.

1. Express in figures the following :—

(1). Sixteen thousand and nine.

(2). Nineteen hundred thousand, seven hundred and five.

(3). Six millions, seven hundred and four thousand, three hundred and seventy-one.

(4). Five billions, four hundred thousand millions, seven hundred and nine thousand, eight hundred and nine.

(5). Two quadrillions, three hundred and forty-five thousand six hundred and seventy-eight trillions, nine hundred and twelve thousand three hundred and forty-five billions, six hundred and seventy-eight thousand nine hundred and twelve millions, three hundred and forty-five thousand six hundred and seventy-eight.

2. Express in words the following :—

(1). 987654321. (2). 10200040506. (3). 6006006006.

(4). 555000555. (5). 5789123456789I2.

3 Express in Roman numerals the following :—

(1). 203. (2). 5492. (3). 27060. (4). 3874.

(5). 100000.

4. Express in figures the following :—

(1). XXIV. (2). LXX. (3). XCVI. (4). MII.

(5). DCCCCXXXVII.

SEC. II.

Ex. II.

1. Add together 2, 3, 4, 5, 6, 7, 8, 9.

2. Add together 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

3. Add together 300, 421, 530, 670, 780, 820, 959, 1024.

4. Add together 999888, 777666, 555444, 333222, 111000.

5. Add together 987654321, 123456789, 896547321, 892134567, 234567891, 124356879, 213564798, 986754213, 432198765.

6. Add together the values of 1+1+1+1, 2+2+2+2, 3+3+3+3, 4+4+4+4, 5+5+5+5 and 6+6+6+6.

7. Add together the values of 1+2+3+4, 2+3+4+5, 3+4+5+6, 6+7+8+9 and 9+8+7+6.

8. In an Alms-house 502 beggars are supplied with their food in the morning, 9785 at noon and 7625 in the evening. How many beggars are supplied with their food daily there?

9. Syam keeps with you 1020304 Rs., Gopal 90708025 Rs., Ram 56000 Rs., and Hari 875002039 Rs. How many rupees have they all kept with you?

10. Find the sum of 23 repeated 5 times; 25 repeated 7 times; 27 repeated 4 times; and 31 repeated 3 times. Find also the sum of these

SEC. III.

Ex. III.

1. From 9223	2. 998877	3. 9080706050
take 7509	665544	8171615141

4. By how much is the number nine thousand nine hundred and ninety-nine greater than seven thousand and seven?

5. In a garden there were forty-nine thousand six hundred and ninety-eight trees altogether; of these, thirty thousand were cut down by the owner

on several occasions ; ninety-nine were dried up on accouut of their extreme age ; and five were destroyed by lightning. How many trees were left in the garden ?

6. Find the difference between the sum and difference of 92325 and 70619.

7. An Assistant Surgeon whose fee is Rs. 4, had 1050 calls in a certain month ; his expenditure in that month was 520 Rs. What sum did he save in that month ?

8. A Pleader gets on an average 6000 Rs. annually ; he spends 150 Rs. a month. What amount can he place in the Savings Bank in a year ?

9. Simplify :—

(1). $(5403+9780)-(2301+5019)$.

(2). $(9999+8888)-(9999-8888)$.

10. A man was 93 years old when he died in 1886 ; in what year was he born ; and how old was he at the time when his first son was born in 1820 ?

SEC. IV.

Ex. IV.

1 Multiply 95768023 by 871209.

2. Find the product of the sum and difference of 97029 and 53021.

3. Find the continued product of 4, 5, 6, 7, 8 and 9.

4. Find the value of $16^2+15^2+14^2\times 5^2$.

5. Multiply the difference between 9009 and 6006 by that between 8008 and 5005.

6. The slow passenger train of the East Indian Railway Company runs at the rate of 20 miles an hour ; what distance does it travel in 5 days and 9 hours ?

7. Sound travels at the rate of 1139 feet per second ; how far does it go in 2 days 2 hours 2 minutes and 2 seconds ?

8. A goods-train consists of 56 carriages, each of which contains 4545 bags of sugar, each bag weighing 3 maunds, and in each maund there are 40 seers ; what is the total weight of sugar in the train in seers ?

9. In a garden there is a mango-tree consisting of 12 branches ; in each branch there are 7 secondary branches ; in each secondary branch there are 9 bunches of fruits ; in each bunch there are 5 mangoes ; how many mangoes are there in the tree ?

10. The number of English-teaching schools in Calcutta, is said to be 54 ; supposing the average number of boys attending each school is 320, what is the number of boys learning English in all the schools, and what would be the total collection of the schooling fees, if the boys are charged at the uniform rate of 4 rupees each ?

SEC. V.

Ex. V.

1. Divide 123456789 by 2, 3, 4, 5, and 6.
2. Divide 23456789 by 7, 8, 9, 10, and 11.
3. Divide 3456789 by 12, 13, 14, 15, and 16.
4. Divide 987654321 by 17, 18, 19, and 20.
5. Divide 990880770660550044 by 3302201110.
6. Divide 9876876576546543 by 543210.

7. If the multiplier be 227788 and the product 151160301842O4, what is the multiplicand ?

8. If the divisor be twice the quotient, and the quotient seven times the remainder, find the dividend when the remainder is 9.

9. Divide the continued product of 4, 5, 6, 7 and 10 by the continued product of 4, 5 and 6.

10. The diameter of the Earth is 7963 miles, and the distance of the Sun from the Earth is 23984 times the diameter of the Earth. How long would a man take to reach the Sun if he can manage to travel at the rate of 4 miles per second ?

11. An army consisted of 84000 men and was divided into 210 detachments. How many men were there in each detachment ?

12. A railway line is 500 miles long ; the total cost of constructing it is 29925000 Rupees ; at what rate is that per mile ?

SEC. VI.

Ex. VI.

1. Find the G. C. M. of—

(1). 32 and 72. (2). 24 and 64.

(3). 32 and 144. (4). 126 and 144.

(5). 336 and 1280. (6). 324 and 5526.

(7). 10332 and 6804. (8). 15068880 and 11861064.

(9). 54, 72 and 96. (10). 64, 160 and 288.

(11). 56, 72. 168 and 140. (12). 42, 24, 66 and 198.

2. Find the L. C. M. of—

(1). 2, 6, 8, 12 and 16. (2). 2, 3, 4, 5, 6, 7, 8 and 9.

(3). 3, 4, 5, 6, 7, 8 and 9. (4). 2, 4, 6, 8, 10, 12, 14 and 16.

(5). 5, 10, 15, 20, 25 and 30. (6). 4, 8, 16, 32, 64, and 128.

(7). 7, 14, 21, 28, 35, 42 and 49.

(8). 3, 27, 54, 72, 90, and 270. (9). 8, 9, 12, and 18.

(10). 3, 9, 7, 15, 28 and 42. (11). 6, 15, 27 and 35.

(12). 8, 18, 28, 36, 54, 72 and 90.

3. Seven men fire at a target at intervals of 2, 5, 7, 10, 12, 14 and 16 minutes respectively. After what time will they first all fire simultaneously, and how many times will each man have fired by that time ?

4. An island is 468 miles in circumference, and three persons *P*, *Q* and *R*, all start together to travel the same way round it ; *P* goes 40 miles a day, *Q* 48 and *R* 60 ; when will they all be together again ?

5. The product of two numbers is 6517, and their least common multiple is 931 ; find their greatest common measure.

EXAMPLES ON CHAPTER II.

(DIVISION I.)

SEC. I.

Ex. VII.

1. Express as fractions :—

(1). The integer 4 having 3, 5 and 7 for the denominator.

(2). The integer 6 having 4, 8, 12 and 15 for the denominator.

(3). The integer 9 having 15, 17, 19 and 24 for the denominator.

(4). The integer 11 having 8, 9, 11, 12, 13, 14 and 19 for the denominator.

(5). The integer 12 having 12, 13, 15 and 20 for the denominator.

(6). The integer 30 having 5, 6, 7, 8, 9, 10, 11 and 12 for the denominator.

(7). The integer 45 having 2, 3, 4, 5, 6, 7 and 9 for the denominator.

(8). The integer 55 having 20, 30, 40, 50, 60, 70, 80 and 90 for the denominator.

2. Convert the following fractions into their equivalent integers :—

(1). $\frac{12}{4}$. (2). $\frac{15}{3}$. (3). $\frac{24}{8}$. (4). $\frac{38}{19}$.

(5). $\frac{25}{5}$. (6). $\frac{51}{17}$. (7). $\frac{225}{15}$. (8). $\frac{361}{19}$.

(9). $\frac{400}{25}$. (10). $\frac{480}{16}$. (11). $\frac{729}{27}$. (12). $\frac{2025}{45}$.

SEC. II.

Ex. VIII.

1. Express the following improper fractions as mixed or whole numbers :—

(1). $\frac{5}{3}$. (2). $\frac{7}{5}$. (3). $\frac{8}{3}$. (4). $\frac{11}{5}$.

(5). $\frac{18}{7}$. (6). $\frac{25}{8}$. (7). $\frac{34}{12}$. (8). $\frac{45}{9}$.

(9). $\frac{50}{7}$. (10). $\frac{63}{23}$. (11). $\frac{100}{33}$. (12). $\frac{1000}{333}$.

(13). $\frac{105}{16}$. (14). $\frac{210}{20}$. (15). $\frac{1005}{200}$. (16). $\frac{725}{99}$.

(17). $\frac{220}{30}$. (18). $\frac{999}{333}$. (19). $\frac{2222}{1111}$. (20). $\frac{788888}{100000}$.

2. Reduce the following mixed numbers to the form of simple fractions :—

(1). $1\frac{2}{3}$. (2). $2\frac{3}{4}$. (3). $3\frac{4}{5}$. (4). $4\frac{5}{6}$.

(5). $5\frac{6}{7}$. (6). $6\frac{7}{8}$. (7). $7\frac{8}{9}$. (8). $8\frac{9}{10}$.

(9). $9\frac{10}{11}$. (10). $10\frac{11}{12}$. (11). $11\frac{12}{13}$. (12). $12\frac{13}{14}$.

(13). $13\frac{14}{15}$. (14). $14\frac{15}{16}$. (15). $15\frac{16}{17}$. (16). $105\frac{7}{8}$.

(17). $1591\frac{1}{3}$. (18). $20205\frac{8}{11}$. (19). $999\frac{1}{11}$. (20). 10000

3. Reduce the following compound fractions to simple fractions in their lowest terms :—

(1). $\frac{2}{3}$ of $\frac{4}{5}$. (2). $\frac{5}{7}$ of $\frac{8}{9}$. (3). $\frac{9}{10}$ of $\frac{3}{4}$.

(4). $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{9}{11}$. (5). $1\frac{1}{5}$ of $\frac{5}{7}$ of $2\frac{5}{8}$. (6). $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{9}{10}$.

(7). $1\frac{1}{2}$ of $2\frac{2}{3}$ of $3\frac{3}{4}$ of $4\frac{4}{5}$ of $\frac{1}{7}$.

(8). $2\frac{1}{2}$ of $3\frac{1}{3}$ of $4\frac{1}{4}$ of $5\frac{1}{5}$ of $6\frac{1}{6}$ of $7\frac{1}{7}$.

(9). $5\frac{1}{2}$ of $6\frac{1}{2}$ of $7\frac{1}{2}$ of $8\frac{1}{2}$ of $9\frac{1}{2}$ of $10\frac{1}{2}$ of $11\frac{1}{2}$.

(10). $10\frac{1}{3}$ of $11\frac{1}{4}$ of $12\frac{1}{5}$ of $13\frac{1}{6}$ of $14\frac{1}{7}$ of $15\frac{1}{8}$.

(11). $\frac{2}{7}$ of $7\frac{1}{5}$ of $\frac{3}{5}$ of $9\frac{1}{10}$ of $\frac{2}{11}$ of $11\frac{1}{12}$.

(12). $3\frac{1}{3}$ of $9\frac{1}{6}$ of $4\frac{1}{4}$ of $8\frac{1}{3}$ of $5\frac{1}{5}$ of $7\frac{1}{7}$ of $6\frac{1}{6}$.

4. Reduce the following complex fractions to simple fractions in their lowest terms :—

(1). $\dfrac{1\frac{1}{4}}{2\frac{2}{3}}$. (2). $\dfrac{1\frac{1}{3}}{1\frac{1}{4}}$. (3). $\dfrac{4\frac{1}{2}}{5\frac{1}{4}}$. (4). $\dfrac{6\frac{1}{7}}{3\frac{1}{3}}$.

(5). $\dfrac{24}{5\frac{1}{3}}$. (6). $\dfrac{7\frac{1}{3}}{22}$. (7). $\dfrac{44\frac{1}{4}}{15\frac{3}{8}}$. (8). $\dfrac{25\frac{1}{2}}{2\frac{1}{3}}$.

(9). $\dfrac{\frac{1}{2} \text{ of } \frac{1}{3}}{6}$. (10). $\dfrac{\frac{2}{3} \text{ of } \frac{4}{5}}{\frac{5}{6} \text{ of } \frac{6}{7}}$. (11). $\dfrac{\frac{1}{3} \text{ of } \frac{1}{4}}{\frac{1}{30}}$. (12). $\dfrac{9\frac{1}{10}}{\frac{1}{3} \text{ of } \frac{7}{10}}$.

(13). $\dfrac{\frac{1}{4} \text{ of } \frac{3}{5} \text{ of } \frac{4}{5}}{\frac{1}{5} \text{ of } \frac{3}{8} \text{ of } \frac{5}{7}}$. (14). $\dfrac{25\frac{1}{2} \text{ of } 2\frac{1}{10} \text{ of } \frac{1}{30}}{15\frac{1}{3} \text{ of } 2\frac{1}{3} \text{ of } \frac{1}{12}}$.

5. Reduce the following fractions to their lowest terms :—

(1). $\dfrac{147}{198}$. (2). $\dfrac{555}{999}$. (3). $\dfrac{3094}{3042}$. (4). $\dfrac{3444}{3556}$.

(5). $\dfrac{5565}{8533}$. (6). $\dfrac{13478}{16701}$. (7). $\dfrac{11050}{3558}$. (8). $\dfrac{714285}{999999}$.

(9). $\frac{109005}{10000000}$. (10). $\frac{135795}{222210}$.

6. Reduce the fractions in each of the following sets to equivalent fractions having the least common denominator :—

(1). $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{9}{10}$. (2). $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{35}$ and $\frac{5}{21}$.

(3). $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{8}$. (4). $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$ and $\frac{8}{9}$.

(5). $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{9}$. (6). $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{40}$, $\frac{1}{50}$, $\frac{1}{60}$ and $\frac{1}{70}$.

(7). $\frac{1}{2}$, $\frac{2}{3}$, $\frac{2}{5}$ and $\frac{9}{13}$. (8). $1\frac{1}{3}$, $2\frac{1}{5}$ and $3\frac{1}{7}$.

(9). $\frac{4}{5}$, $2\frac{5}{6}$ and $3\frac{2}{11}$. (10). $\frac{1}{3}$, $\frac{1}{9}$, $\frac{5}{12}$ and $\frac{7}{18}$.

(11). $\frac{3}{7}$, $\frac{9}{10}$, $\frac{19}{21}$ and $\frac{25}{28}$. (12). $\frac{2\frac{1}{3}}{7}$, $8\frac{1}{4}$, $\frac{9+\frac{1}{11}}{\frac{9}{11}}$ and $16\frac{7}{8}$.

7. Compare the values of—

(1). $\frac{1}{5}$, $\frac{3}{8}$ and $\frac{3}{7}$. (2). $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{7}{10}$.

(3). $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{12}$ and $\frac{4}{15}$. (4). $\frac{5}{6}$, $\frac{9}{10}$, $\frac{7}{12}$ and $\frac{8}{30}$.

(5). $\frac{1}{10}$, $\frac{2}{20}$, $\frac{3}{30}$, $\frac{4}{40}$ and $\frac{5}{50}$. (6). $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{6}{7}$.

(7). $\frac{1}{2}$ of $\frac{4}{5}$, $\frac{5}{12}$ and $\frac{4}{15}$.

(8). $\frac{1}{10}$, $\frac{2}{100}$, $\frac{3}{1000}$, $\frac{4}{10000}$ and $\frac{5}{100000}$.

(9). $\frac{3}{5}$, $\frac{2}{3}$, $\frac{5}{9}$ and $\frac{7}{10}$. (10). $\frac{7}{9}$, $\frac{9}{11}$ of $\frac{11}{13}$.

(11). $\frac{3}{5}$, $4\frac{1}{2}$, $\frac{2}{3}$ of $\frac{4}{5}$ and $\frac{4}{7}$ of $3\frac{1}{2}$. (12). $\frac{4}{5}$ of $\frac{5}{6}$, $7\frac{2}{3}$ and $\frac{1}{4}$ of 5.

(13). $\frac{9}{13}$ of $\frac{1}{2}$, $\frac{5}{6}$ and $\frac{15}{16}$. (14). $\frac{5}{9}$, $\frac{12}{18}$, $\frac{5}{20}$, $\frac{6}{24}$, $\frac{4}{27}$ and $\frac{10}{36}$.

(15). $\frac{4}{5}$, $\frac{7}{9}$ of $13\frac{2}{5}$, $\frac{5}{6}$ of $\frac{3}{4}$, $\frac{4}{13}$, $9\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{4}{5}$.

SEC. III.

Ex. IX.

1. Add together—

(1). $\frac{13}{25}$ of $\frac{128}{165}$ of $\frac{7}{9\frac{3}{5}}$ and $\frac{14}{15}$ of $\frac{25}{28}$ of $\frac{4\frac{11}{18}}{6\frac{12}{13}}$.

(2). $\frac{3}{16}$, $\frac{13}{60}$, $\frac{31}{250}$ and $\frac{9}{625}$. (3). $\frac{7}{8}$ of $3\frac{1}{3}$, $\frac{5}{7}$ of $1\frac{10}{11}$ and $\frac{7}{365}$.

(4). $4\frac{1}{3}$, $\frac{3}{4}$ of $\frac{7}{8}$, $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{6}$. (5). $\frac{2\frac{1}{4}+3\frac{1}{28}}{3\frac{13}{36}+12\frac{2}{5}}$ and $\frac{6\frac{2}{3}+3\frac{19}{36}}{12\frac{19}{28}+2\frac{1}{2}}$.

(6). $\frac{7}{36}$, $\frac{5}{12}$, $\frac{3}{44}$, $\frac{13}{28}$, $\frac{11}{56}$.

2. Find the value of

(1). $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36}$. (2). $2\frac{3}{4}\div 3\frac{2}{3}$ of $2\frac{7}{8}$ of $\frac{5}{7}+\frac{1}{5}$ of $3\frac{1}{3}$.

(3). $\frac{17}{12}+2\frac{1}{6}+\frac{15}{9}+3\frac{1}{2}+4\frac{1}{6}+\frac{5}{6}$. (4). $\frac{19}{21}+\frac{2}{3}+\frac{1}{2}+\frac{33}{35}+\frac{19}{42}$.

(5). $\frac{3}{10}+\frac{5}{21}+\frac{9}{140}+\frac{14}{33}+\frac{15}{64}+\frac{3}{55}$.

3. Simplify :—

(1). $\frac{1\frac{3}{8}}{3+\frac{1}{3\frac{1}{8}}}+\frac{1\frac{2}{3} \text{ of } 4\frac{2}{7}}{1\frac{2}{5} \text{ of } 8\frac{4}{7}}+\frac{5\frac{3}{8} \text{ of } 7\frac{3}{5}}{8\frac{7}{24}+3\frac{5}{12}}$.

(2). $\frac{3}{5}$ of $\frac{5}{8}$ of $11\frac{1}{2}+1\frac{3}{4}$ of $3\frac{5}{8}$ of $\frac{1}{28}+2\frac{5}{7}$ of $4\frac{3}{8}$ of $\frac{1}{152}$.

(3). $1\frac{1}{3}+\frac{2}{3}$ of $\frac{4}{21}+\frac{4}{5\frac{1}{10}}$.

(4). $14\frac{3}{4}+\frac{2}{3}$ of $\frac{5}{6}$ of 8. (5). $\frac{3}{7}+4\frac{1}{3}+\frac{3}{5}$ of 2.

(6). $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}+\frac{1}{5}$ of $\frac{2}{3}$ of $\frac{15}{8}+\frac{5}{16}$ of $\frac{1}{5}$ of $\frac{28\frac{1}{2}}{2}$.

(7). $\frac{3}{5}$ of $\frac{5}{7}+9+\frac{2\frac{4}{5}}{7}+\frac{1\frac{2}{3}}{2\frac{1}{3}}$.

SEC. IV.

Ex. X.

1. Find the difference between—

(1). $\frac{17}{24}$ and $\frac{11}{12}$. (2). $56\frac{3}{8}$ and $35\frac{4}{7}$. (3). 45 and $35\frac{7}{8}$.

(4). $75\frac{1}{3}$ and $48\frac{5}{6}$. (5). $\frac{1}{3}$ of $\frac{3}{4}$ of 5 and $\frac{4}{5}$ of $\frac{2}{5}$ of 56.

(6). $\frac{2}{3}$ of $5\frac{4}{7}$ of $1\frac{2}{5}$ and $\frac{3\frac{1}{2}}{\frac{3}{4}}$. (7). $\frac{4\frac{1}{3}}{\frac{32}{5}}$ and $\frac{\frac{4}{7} \text{ of } \frac{3}{11} \text{ of } \frac{7}{2}}{\frac{3}{8} \text{ of } \frac{3}{11} \text{ of } 6}$.

2. (1). By how much does $\frac{3}{17}+\frac{3\frac{8}{21}}{5}+2$ exceed $\frac{22}{33}$ of $3\frac{3}{17}$ of $\frac{22}{32}$.

(2). Simplify $\left(\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}+\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}}\right)-\left(\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}+\frac{1}{5}}-\frac{\frac{1}{5}-\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}}\right)$.

(3). What number added to the sum of $\frac{4}{5}$, $6\frac{5}{8}$ and $\frac{12}{35}$ will make the sum total equal to 12 ?

(4). Simplify $\frac{\frac{1}{4}+\frac{1}{5}}{1\frac{3}{4}}-\frac{1}{3}+\frac{1}{2\frac{4}{5}}-\frac{\frac{1}{3}}{2\frac{11}{12}}+7$.

(5). What number must be added to the sum of $\frac{3}{4}$ and $\frac{5}{8}$ in order that the sum may be equal to the difference between $9\frac{1}{4}$ and $6\frac{1}{7}$?

Sec. V.

Ex. XI.

1. Multiply—

(1). $\frac{1}{3}$ of $\frac{2\frac{3}{4}}{5\frac{7}{8}}$ by $\frac{7\frac{2}{7}}{5\frac{1}{6}}$ of $\frac{4\frac{3}{4}}{7\frac{1}{2}}$ of $\frac{3}{8}$.

(2). $\frac{3\frac{1}{4}}{10\frac{3}{8}}$ of $\frac{4\frac{1}{8}}{13}$ of $\frac{11\frac{3}{5}}{29}$ by 6 of $20\frac{3}{4}$ of $5\frac{1}{3}$.

(3). $\frac{1}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $25\frac{5}{8}$ by $\frac{1}{3}$ of $\frac{5}{6}$ of $\frac{3}{5}$ of $\frac{1}{8}$ of 75.

(4). $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{9}{7}$ of $13\frac{4}{15}$ by $2\frac{3}{4}$ of $5\frac{1}{6}$ of $\frac{3}{7}$ of $23\frac{5}{8}$.

(5). $\frac{2\frac{1}{7}}{3\frac{1}{3}}$ by $\frac{2\frac{1}{3}}{3\frac{1}{2}}$. (6). $\frac{2}{3\frac{3}{16}}$ of $\frac{6\frac{1}{14}}{8}$ by $\frac{2}{3}$ of $8\frac{1}{4}$ of $\frac{38}{55}$.

2. Find the continued product of—

(1). $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$ and $\frac{8}{9}$. (2). $1\frac{1}{2}$, $2\frac{2}{3}$, $3\frac{9}{11}$, $4\frac{4}{5}$ and $5\frac{5}{6}$.

(3). $13\frac{5}{8}$, $22\frac{4}{5}$, $\frac{16}{35}$, $\frac{14}{15}$ and $\frac{5}{8}$.

(4). $\frac{1}{3}$ of $\frac{4}{5}$ of $25\frac{7}{8}$, $\frac{16}{27}$, $24\frac{3}{4}$ and $\frac{5}{6}$ of $15\frac{5}{8}$.

(5). $2\frac{1}{3}-1\frac{3}{4}$, $\frac{1}{3}$ of $\frac{4}{5}+2\frac{3}{5}$ and $\frac{5}{8}$ of $56-\frac{1}{3}$ of 24.

(6). $7\frac{4}{5}$ of $23\frac{1}{6}-4\frac{1}{3}$, $\frac{4}{7}$ of $2\frac{3}{4}$ of $4\frac{7}{8}$ and $\frac{1}{2}$ of $45+\frac{2}{3}$ of 36.

3. Find the value of—

(1). $\frac{81}{91}\times(1-\frac{84}{91})+\frac{84}{88}\times\frac{1}{3}\times(\frac{3}{8}+\frac{5}{12})$.

(2). $\frac{9}{35}$ of $\frac{1}{3}\times\frac{50}{51}\times\frac{78}{85}$ of $1\frac{3}{12}$.

(3). $\dfrac{\frac{3\frac{1}{3}}{7\frac{1}{5}}-\frac{1}{2\frac{4}{7}}}{4\frac{1}{3} \text{ of } \frac{2\frac{7}{8}}{6\frac{3}{5}}}$ of $\frac{11\frac{1}{5}}{17}\times 52\frac{4}{11}$. (4). $\frac{7\frac{1}{2}}{6\frac{1}{3}}+\frac{11\frac{1}{3}-2\frac{2}{5}}{11\frac{1}{2}+2\frac{2}{5}}\times 10\frac{9}{13}-7\frac{1}{6}$.

(5). $\frac{1}{3}\times\frac{17}{3}$ of $\frac{27}{55}+\left(2\frac{1}{3}+\frac{1}{3+\frac{1}{4}}\right)\times\frac{39}{103}$. (6). $(49\frac{11}{16}\times 50\frac{1}{16})+\frac{1}{256}$.

(7). $1\frac{5}{8}$ of $\dfrac{\frac{1}{5}+\frac{1}{3}+\frac{1}{4}}{2\frac{1}{3}-3\frac{1}{3}+4\frac{1}{4}}+\dfrac{\frac{2\frac{1}{5}}{3\frac{1}{7}}+\frac{\frac{5}{16}}{1\frac{5}{8}}}{\frac{3}{4\frac{1}{2}}+\frac{4\frac{1}{3}}{3}}$.

Sec. VI.

Ex. XII.

Divide—

(1). $\frac{3}{7}$ by $\frac{3}{5}$. (2). $\frac{5}{8}$ by $\frac{4}{5}$. (3). $\frac{4}{11}$ by $\frac{9}{13}$.

(4). $\frac{10}{3}$ by $\frac{8}{15}$. (5). $2\frac{1}{5}$ by $3\frac{2}{5}$. (6). $10\frac{4}{5}$ by $13\frac{5}{7}$.

(7). $17\frac{2}{3}$ by $7\frac{11}{12}$. (8). $\frac{7\frac{2}{7}}{40\frac{5}{6}}$ by $\frac{17\frac{1}{3}}{73}$.

(9). $\frac{2}{5}$ of $\frac{5}{8}$ of $18\frac{4}{5}$ by $\frac{5}{8}$ of $\frac{7}{8}$ of $\frac{2}{5}$.

(10). $\frac{3}{5}$ of $5\frac{1}{2}$ of $\frac{4}{5}$ by $\frac{1}{3}$ of $\frac{5}{6}$ of $\frac{6}{7}$.

(11). $(1+\frac{1}{15}+\frac{1}{14}+\frac{20}{105})$ by $(\frac{9}{12}-\frac{20}{32})$.

(12). $\dfrac{4\frac{2}{7}\times 8\frac{6}{11}}{\frac{20}{35}\div 10\frac{1}{2}}$ by $\dfrac{6\frac{1}{3}\text{ of }4\frac{4}{5}}{4+2\frac{1}{3}}$.

(13). $\left\{\left(\dfrac{2\frac{3}{8}}{3\frac{9}{12}}+\dfrac{7\frac{5}{8}}{2\frac{4}{24}}\right)-\left(\dfrac{\frac{1}{4}\text{ of }\frac{13}{8}}{4\frac{9}{10}}+\dfrac{\frac{1}{3}\text{ of }\frac{2}{3}\text{ of }\frac{3}{8}}{\frac{10}{3}}\right)\times\dfrac{12\frac{9}{12}}{\frac{7}{8}}\right\}$

by $\dfrac{\frac{3}{8}\text{ of }\frac{3}{4}}{2\frac{3}{24}}$.

(1). Divide the difference between $2\frac{1}{4}$ and $\frac{2}{3}$ of $2\frac{5}{8}$ by the sum of $\frac{1}{5}$ of $3\frac{1}{3}$ and $\frac{13}{16}$.

(2). Simplify $\dfrac{\dfrac{48\frac{4}{7}}{1085\frac{7}{10}}\div\dfrac{7\frac{3}{11}}{174\frac{3}{17}}}{\frac{13}{14}}$.

(3). What number multiplied by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ will give $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{7}{8}$?

(4). What number must be multiplied by the continued product of the reciprocals of 6, 7, 8 and 9 to give the reciprocl of 10 ?

(5). Divide the product of 10 and $\frac{1}{15}$ by the quotient of 1 by $\frac{1}{10}$.

(6). Simplify $\left(\dfrac{1\frac{1}{3}\div 1\frac{1}{4}}{1\frac{1}{4}\div 1\frac{1}{3}}\div\dfrac{1\frac{2}{7}\div 1\frac{1}{7}}{1\frac{1}{8}\div 1\frac{1}{6}}\right)\div\left(\dfrac{\frac{1}{3}\div\frac{1}{4}}{\frac{1}{4}\div\frac{1}{3}}\div\dfrac{\frac{2}{7}\div\frac{1}{7}}{\frac{1}{8}\div\frac{1}{6}}\right)$.

(7). Simplify $\dfrac{5\frac{3}{8}-2\frac{3}{4}}{11\frac{1}{4}+4\frac{5}{8}}\div\dfrac{7\frac{11}{12}\text{ of }2\frac{7}{15}}{5\frac{3}{8}\text{ of }6\frac{4}{5}}$.

EXAMPLES ON CHAPTER II.

(Division II).

Sec. VII.

Ex. XIII.

1. Express according to the decimal notation :—

(1). Three-tenths. (2). Nine-tenths.

(3). Twenty-three hundredths. (4). Fifty-seven hundredths.

(5). Seven-thousandths. (6). Sixty-five millionths.

(7). Two hundred and sixty-five thousandths.

(8). Eight hundred and sixty-five millionths.

2. Express as decimals :—

(1). $\frac{3}{10}$. (2). $\frac{170}{10}$. (3). $\frac{32}{100}$. (4). $\frac{1845}{1000}$.

(5). $\frac{48}{1000}$. (6). $\frac{65}{100000}$. (7). $\frac{87}{1000000}$. (8). $\frac{8}{100}$.

(9). $\frac{1888}{10000}$. (10). $\frac{2}{1000000}$. (11). $\frac{7}{100^2}$. (12). $\frac{9}{1000^4}$.

(13). $\frac{1}{1000000}$. (14). $\frac{1}{10}$.

3. Express as vulgar fractions in their lowest terms :—

(1). ·8. (2). ·35. (3). ·08. (4). ·035.

(5). ·02. (6). ·002. (7). ·0005. (8). ·01.

(9). 2·15. (10). 2·015. (11). 8·27. (12). ·565.

(13). 1000·0005. (14). ·000665. (15). ·00456. (16). 4·00052.

4. Multiply :—

(1). ·4 by 10, 100, 1000 and 10000.

(2). ·100 by 100, 1000, 10000 and 100000.

(3). ·002 by 100, 1000, 10000 and 100000.

(4). 30·00030 by 1000 and 10000.

(5). ·123 by 10000 and 100000.

5. Divide :—

(1). ·200 by 100 and 10000. (2). 2·134 by 100 and 1000.

(3). 2546·1 by 10 and 10000.

(4). 12345·007 by 100 and 1000.

(5). 7·131400 by 100 and 10000.

Sec. VIII.

Ex. XIV.

1. Add together—

(1). 889·554, 542·0054, 546·32, 312·105 and 64·0005.

(2). 13·041, 564·8746, 456·707, ·0747, and ·3254.

(3). 103·34, 552·840, 53·005, 88·345 and 57·758.

(4). ·195, 3·086, 12·87, ·0051 and 729·54.

(5). 360·826, 36·0826, 3·60826 and ·360826.

2. Find the value of—

(1). 100·002 + 234·678 + 77.0008 + 45·05.

(2). ·01 + ·002 + ·0003 + ·00004 + ·123456.

(3). ·002 + 200·003 + 3000·0004 + ·00005.

(4). 47·253 + 478·364 + 589·456 + ·01.

(5). 111·111 + 222·222 + 333·333 + 444·444 + 555·555.

3. Find the sum of—

(1). Four-tenths ; twenty-one hundredths, five-thousandths ; fifty-six-millionths ; and one-hundred fifty-six thousandths.

(2). Twenty-six and forty-three hundredths; three hundred, and sixty-five hundredths ; fifty-two and twenty-five thousandths ; four hundred and five and four hundred and five thousandths.

(3). Seven tenths ; seven hundredths ; seven thousandths ; and one hundred, and sixty-two millionths.

(4). Forty-five thousandths ; seventy-seven hundredths ; and eighty-eight millionths.

(5). Two hundred, and thirty thousandths ; three million and three millionths ; and sixty thousand and sixty thousandths.

SEC. IX.

Ex. XV.

1. Subtract—

(1). 1·53 from 3·09. (2). 3·64 from 4·78.

(3). 4·75 from 5·89. (4). 1·234 from 2·345.

(5). 40·00379 from 600000.

2. Find the difference between—

(1). Three and three tenths. (2). Four and four hundredths.

(3). Seven and seven thousandths. (4). Two and two millionths.

(5). Eight tenths and eight thousandths.

3. Find the value of—

(1). 44·86 − ·926. (2). 43·26 − ·005.

(3). 104·006 − ·605. (4). 646·75 + 26·345 − 105·35.

(5). 105·64 + 236·006 + 135·0068 − 75·0036.

SEC. X.

Ex. XVI.

1. Multiply—

(1). 35·16 by 11·4. (2). 120·23 by 10·45.

(3). ·0715 by 00·57. (4). 35·14 by 76·05.

(5). 4·6186 by 21·04.

2. Find the product of—

(1). One thousand and one thousandth.

(2). Eight thousandths and eight millionths.

(3). Fifty-six and sixty-six thousandths.

(4). Four and four hundredths.

(5). Nine thousandths and nine millionths.

3. Find the continued product of—

(1). 3, ·3, ·03, ·003 and 3000.

(2). 14, 1·4, ·014, 1400 and 52600.

(3). 1·8, 18, 1800, ·018 and 526000.

(4). 24·7, 2·14, ·0065 and 72·035.

(5). 45·062, ·0705, ·715 and 12·62.

SEC. XI.

Ex. XVII.

1. Divide—

(1). 1·54 by ·11. (2). 154 by ·07.

(3). 15·4 by ·77. (4). 12150138 by 2·023.

(5). 6400 by ·04096. (6). ·06735 by ·125.

(7). 4·8 by ·0016. (8). 73·8 by ·0018.

(9). 1215013·8 by 2·023. (10). ·000072072 by ·000012

(11). 10·01 by 390·625. (12). 9·614 by ·0000019.

2. Simplify—

(1). $(\cdot 18988 \times 8\cdot 08) \div \cdot 0235 - (8\cdot 08 \times 8\cdot 08)$.

(2). $\cdot 0000072 \div \cdot 012$. (3). $367402\cdot 32 \div \cdot 35601$.

(4). $33 \div \cdot 00011$. (5). $189\cdot 3978 \div 4\cdot 3$.

3. Find the value of—

(1). $\cdot 4 \div \cdot 000255$. (2). $\cdot 0128 \div 81\cdot 92$. (3). $\cdot 015625 \div 26$.

(4). $4350\cdot 5 \div 259\cdot 86$. (5). $214\cdot 86 \div \cdot 048$.

4. What number multiplied by 2·4 will produce 76·8 ?

5. What number multiplied by ·024 will produce ·0000768 ?

6. What number multiplied by the quotient arising from the division of 8 by ·5 will produce 3.52 ?

SEC. XII.

Ex. XVIII.

1. Convert into decimals the following vulgar fractions :—

(1). [illegible] ; [illegible] ; [illegible]. (2). [illegible] ; [illegible] ; 3[illegible].

(3). [illegible] ; [illegible] ; 15[illegible]. (4). $\frac{1}{2}$ of $2\frac{1}{2}$; $\frac{1}{2}$ of [illegible] of [illegible].

(5). [illegible] of $1\frac{1}{2}$; [illegible] ; [illegible].

2. Reduce the following vulgar fractions to decimals correctly to 5 places of decimals :—

(1). $\frac{24}{75}$; $\frac{18}{80}$; $\frac{29}{32}$. (2). $\frac{27}{1250}$; $\frac{23}{36}$; $7\frac{333}{6250}$.

(3). $\frac{17}{2500}$; $\frac{389}{62500}$; $\frac{60}{250}$; $\frac{5}{11}$. (4). $\frac{300}{374}$; $\frac{88}{117}$; $\frac{60}{140}$; $\frac{55}{165}$.

(5). $\frac{1}{60}$; $\frac{72}{584}$; $\frac{184}{384}$; $\frac{49}{63}$.

3. Reduce the following vulgar fractions to recurring decimals :—

(1). $\frac{1}{3}$; $\frac{2}{3}$; $\frac{7}{9}$. (2). $\frac{5}{6}$; $\frac{8}{9}$; $\frac{10}{111}$; $\frac{1}{37}$.

(3). $\frac{23}{33}$; $\frac{8}{11}$; $\frac{4}{111}$. (4). $\frac{1016}{3333}$; $\frac{71}{111}$; $\frac{31749}{37037}$.

(5). $\frac{3412}{9901}$; $\frac{4}{111111}$; $\frac{20468}{111111}$.

4. Convert the following recurring decimals into vulgar fractions :—

(1). $\cdot\dot{6}$; $\cdot\dot{7}$; $\cdot\dot{8}$. (2). $\cdot\dot{2}\dot{7}$; $\cdot0\dot{9}\dot{3}$; $\cdot1\dot{8}\dot{9}$; $\cdot2\dot{8}\dot{1}$.

(3). $\cdot00\dot{2}\dot{1}$; $\cdot01\dot{5}\dot{4}$; $\cdot223\dot{5}6\dot{8}$.

(4). $21\cdot923\dot{4}56\dot{7}$; $36\cdot\dot{6}\dot{7}$; $853\cdot\dot{5}9\dot{3}$.

(5). $1234\cdot5\dot{6}78\dot{9}$; $221\cdot33\dot{4}4\dot{5}$; $0\cdot02\dot{3}$.

Sec. XIII.

Ex. XIX.

1. Write the following decimals retaining only 6 places of decimals, so as to be approximately correct :—

(1). ·00123456 ; ·09587217. (2). ·983217785 ; ·000087879.

(3). ·123456789 ; ·1020304O5. (4). ·7654321 ; ·987654321.

(5). ·09080706 ; ·5544667788.

2. Find the value (correct to 5 places of decimals) of—

(1). $53\cdot\dot{4}\dot{3}+\cdot\dot{7}\dot{2}+\cdot54\dot{6}+\cdot03\dot{8}$.

(2). $\cdot002\dot{3}+\cdot5\dot{4}6\dot{9}+11\cdot56\dot{7}\dot{8}+\cdot02\dot{5}$.

(3). $\cdot0000123+\cdot99887\dot{7}+2\cdot33445\dot{5}+\cdot0\dot{2}$.

(4). $\cdot 972\dot{3}4\dot{5} + \cdot 789\dot{1}2\dot{3} + \cdot 112233445$. (5). $3\cdot 85\dot{6}\dot{4} - \cdot 038\dot{2}$.

(6). $\cdot 12\dot{7} + \cdot\dot{1}4285\dot{7} + 2\cdot 3\dot{2}\dot{7} + 2\cdot 0\dot{5} + 44\cdot\dot{6}\dot{3} + \cdot 00\dot{8}49713\dot{3}$.

(7). $27\cdot 2\dot{3} + \cdot 3\dot{7} + 6\cdot\dot{5}\dot{2} + \cdot 29\dot{7} + \cdot\dot{9}7\dot{3} + 9 + 4\cdot 7\dot{5} + 82\cdot 0367 + 22\cdot 31$.

(8). $\cdot 4 - \cdot\dot{0}\dot{8}$; $\cdot\dot{0}\dot{5} - \cdot 00\dot{6}5812\dot{7}$. (9). $7 - 6\cdot\dot{1}3174\dot{6}$: $\cdot 05\dot{3} - \cdot 0\dot{4}\dot{7}$.

(10). $7\cdot\dot{7}\dot{2} \times \cdot 2\dot{9}\dot{7}$; $36\cdot 2\dot{3} \times \cdot 2\dot{6}$. (11). $74\cdot 0367 \times 4\cdot 7\dot{5}$; $3\cdot\dot{9}7\dot{3} \times 8$.

(12). $7 \div \cdot\dot{1}4285\dot{7}$; $\cdot 042 \div \cdot 03\dot{6}$. (13). $\cdot 3\dot{1}70\dot{5} \div 5\cdot\dot{4}8\dot{3}$.

(14). $(25\cdot 4)^2 + (24\cdot 6)^2 - 12\cdot 7 \times 98\cdot 4 + (\cdot 6)^2$.

EXAMPLES ON CHAPTER III.

Sec. II.

Ex. XX.

1. Reduce

(1). Rs. 25. 11 as. 3p. to pies ; 10000 pies to rupees.

(2). Rs. 159. 15 as. 6p. to pies ; 4050 annas to rupees.

(3). £32. 12*s*. 8*d*. to pence ; 50000*d*. to pounds.

(4). £145. 13*s*. 7*d*. to pence ; 3974*d*. to pounds.

(5). 445 half-sovereigns to pence ; 11266*d*. to half-guineas.

(6). 1587 half-crowns to pounds ; £156. 15*s*. to crowns.

(7). 3 lbs. 4 oz. 3 dwts. to grains ; 2468 grains to ounces.

(8). 24 lbs. 10 oz. 14 dwts. to pennyweights.

(9). 493656 grains to pounds.

(10). 7 tons 9 cwt. 2 qrs. to pounds.

(11). 62 mds. 37 seers 9 chts. to kanchas.

(12). 100000 tolas to seers.

(13). 192 miles 7 furlongs 14 poles to yards.

(14). 200000 yds. to miles.

(15). 2 ac. 3 ro. 30 sq. poles to sq. yds.

(16). 300080 sq. yds. to acres.

(17). 5 cub. yds. 19 cub. ft. to cub. inches.

(18). 5 kroses 90 dhanus to hats.

(19). 31356 hats to kroses.

(20). 35 bighas 18 kathas to chataks.

(21). 4200 kathas to bighas.

2. How many pice are there in 45 rupees, 35 half-rupees, 25 four-anna pieces, and 15 two-anna pieces together?

3. Find the number of seconds in the months of September, October and November together.

4. How many pence are there in £10000, and how many pounds are there in 9600*d.*?

5. How many acres are there in 2560 sq. miles, and how many bighas in 5860 kathas?

6. The value of a dollar being 2 Rs. 4 as., how many rupees are there in 56972 dollars?

7. A box contains 10 gold mohurs 20 Rs. 12 as. 9 p.: another contains 20 gold mohurs Rs. 250. 10 as. and 6 pies. How many pies are there in these sums taken together?

8. If 11728 persons travel in 3rd class carriages 500 miles, each paying a pic a mile, what does the whole fare amount to?

SEC. III.

Ex. XXI.

1. Add together

	£	s.	d.		£	s.	d.
(1).	5	2	9	(2).	12	15	11
	4	3	5		9	5	3
	10	10	10		25	10	10
	7	9	3		102	5	9
					95		

(3).

Rs.	as.	p.
225	4	9
50	15	8
1520	7	3
2	1	1
109	9	9

(4).

Rs.	as	p.
777	7	7
888	8	8
999	9	9
1020	10	10
12345	15	11

(5).

Tons.	cwt.	qrs.	lbs.
15	15	3	17
25	16	2	16
59	17	1	19
62	18	3	18

(6).

Mds.	seers.	chts.
42	37	12¼
125	38	13
[illegible]	39	14
1234	36	15¾

(7).

Days.	hours.	mins.
5	9	20
20	6	23
77	11	19
6	6	6¾
7	7	7¼

(8).

Bighas.	kathas.	chts.
52	19	14
500	18	12
79	17	11½
999	9	9½
704	4	4

(9).

Ac.	ro.	po.	yds.	ft.	in.
26	2	27	25	7	84
35	3	35	28	8	102
47	1	39	27	6	96
58	2	16	21	5	110
69	3	24	17	2	120

(10).

Weeks.	days.	hrs.	mins.	seconds.
2	5	9	30	20
5	6	10	20	30
7	4	12	40	40
10	2	10	10	10
15	3	23	52	59

2. A gentleman owes as follows :—to Ram Rs. 687. 12 as. 9 p. ; to Hari Rs. 432. 13 as. 4 p. ; to Upendra Rs. 352. 12 as. 1 p. ; to the Bengal

Banking Corporation Rs. 7682. 13 as. 11 p. What is the amount of his debts ?

3. A sarkar submits the following account of his purchases in the market : Fish Rs. 25. 8 as. 7 p. ; vegetables Rs. 32. 6 as. 9 p. ; sweetmeats Rs. 15. 14 as. 3 p. ; different kinds of oil and tobacco Rs. 15. 4 as. 9 p. ; ghee Rs. 5. 6 as. 6 p. ; pictures and photographs Rs. 18. 13 as. 6 p. ; and paper, pen and pencils and other writing materials Rs. 10. 8 as. 3 p. What was the amount spent ?

4. In furnishing a house the following expenses were incurred :— Cabinet-ware Rs. 6432. 8 as. 6 p. ; glass-ware Rs. 532. 14 as. 6 p. ; matting Rs. 240. 11 as. 6p. ; silver-plate Rs. 658. 15 as. 9 p. ; a good English clock Rs. 655. 13 as. 3 p. ; and gas-fittings Rs. 570. 12 as. 9 p. What did the expenses amount to ?

5. In the balance sheet of a shop-keeper's account, the following items appear :—Sugar Rs. 255. 4 as. 8 p. ; rice Rs. 635. 12 as. 9 p. ; ghee Rs. 1279 15 as. 6p. ; flour Rs. 251. 3 as. 3 p. ; salt Rs 109. 1 a. 2p. ; oil Rs. 9230 5 as. 7 p. ; and spices Rs. 220. 13 as. 11p. What is the amount of his stock ?

6. In an iron foundry the following articles were found :—20 screws weighing in all 60 mds. 15 srs. 7 chts. ; a boiler weighing 13 mds. 12 srs. 4 chts. ; an anvil weighing 3 mds. 14 srs. 5 chts. Find the total weight of all the articles.

Sec. IV.

Ex. XXII.

1. Perform the following subtractions :—

(1).

Rs.	as.	p.
23	10	8
13		

(2).

Rs.	as.	p.
45	14	$7\frac{1}{2}$
12		$5\frac{3}{4}$

(3).

Rs.	as.	p.
74	0	$6\frac{3}{4}$
13	8	$4\frac{1}{2}$

(4).

Rs.	as.	p.
89	15	7
74	11	9

(5).

£.	s.	d.
486	13	4
236	15	6

(6).

£.	s.	d.
256	12	9
186	9	10

(7).

£.	s.	d.
27	11	3
12	6	2

(8).

£.	s.	d.
35	15	2
15	11	3

(9).

Mds.	srs.	chts.
928	25	14½
720	15	7¼

(10).

Tons.	cwt.	qrs.	lbs.
756	17	3	25
525	19	2	21

(11).

Bghs.	kths.	chts.
886	19	13¾
201	16	13½

(12).

Ac	ro.	po.	sq. yd.
25	3	38	28½
19	2	35	14½

(13).

Hrs.	min.	sec.
112	56	40½
60	5	30½

(14).

Weeks.	days.	hrs.	min.
77	6	9	57
60	4	18	58

(15).

Mi.	fur.	po.
9020	7	39
109	6	35¼

(16).

Lbs.	oz.	dwts.	grs.
25	9	18	20
22	7	15	11

2. Ram lends Gopal Rs. 5000. How much does Gopal owe to Ram after Ram has taken goods from him to the value of Rs. 2913. 13 as. 6 p. ?

3. A young graduate has served 4 years 8 months and 20 days of his 6 years' apprenticeship; what time has he yet to serve?

4. A tradesman's effects amount to Rs. 71311. 13 as. 6 p. He owes to James Rs. 11729. 5 as. 0 p; to John Rs 18973. 11 as. 10 p.; to William Rs. 8773. 5 as. 6 p.; to Edward Rs. 491. 10 as. 6 p. What money will remain in hand after paying his debts?

5. Which is the greater £490 or 480 guineas, and by how much ?

6. Prabhash has in his pocket Rs. 25. 9 as. 4 p. and Satis Rs. 23. 13 as. 4 p. If Prabhash gives to Satis Rs. 19. 5 as. 8 p., and Satis gives to Prabhash Rs. 17. 14 as. 4 p., who will have more and by how much ?

SEC. V.

Ex. XXIII.

1. Multiply—

(1). Rs 28. 4 as. $4\frac{1}{2}$ p. by 3, 5 and 9.

(2). Rs. 151. 13 as. 4p. by 4, 7 and 8.

(3). £28. 5*s.* 3*d.* by 4, 6, 7 and 11.

(4). £48. 12*s.* $9\frac{1}{2}$*d.* by 5, 8, 10 and 12.

(5). 23 mds. 6 srs. 2 chts. by 3, 5, 7 and 8.

(6). 2 tons. 15 cwt. 1qr. 12 lbs. by 5, 7, 11 and 12.

(7). 24 bghs. 6 kths. 10 chts. by 9, 10, 12 and 16.

(8). Rs. 22. 11 as. 10 p. by 25 and 28.

(9). £25. 15*s.* $4\frac{3}{4}$*d.* by 21 and 48.

(10). 62 ac. 2 ro. 32 po. by 44 and 63.

(11). 75 yds. 2 ft. 3 in. by 25 and 96.

(12). Rs. 15. 8 as. 9 p. by 19 and 26.

(13). £46. 13*s.* $9\frac{1}{4}$*d.* by 41 and 46. (14). £66. 11*s.* 8*d.* by 396.

(15). £96. 13*s.* $4\frac{1}{2}$*d.* by 5645.

2. What is the price of 5 mds. of sugar, at Rs. 17. 13 as. per maund ?

3. What will be the cost of making 70 glass-cases, at Rs. 26. 8 as. 6 p. each ?

4. How much will 50 yds. of cloth come to at Rs. 2. 8 as. per yd. ?

5. If a person puts in the Savings Bank Rs. 1000 a year after spending Rs. 110. 4 as. 3 p. a month, what is his income ?

6. If the daily income of a trader be Rs. 5. 6 as. 8 p., what is his annual income ? [1 year=365 days.]

7. The Government pays at the rate of Rs. 7. 8 as. per month for each native police-constable : what will be the cost of maintaining the police in a town where there are 18 police stations, and in each station there are 75 native constables ?

8. A piece of ground has been estimated to contain 428 bghs. of land fit for cultivation : what is its value, at the rate of Rs. 30. 4 as. per katha ?

Sec. VI.

Ex. XXIV.

1. Divide :—

(1). Rs. 913. 15 as. 6 p. by 12 and 16.

(2). Rs. 29731. 7 as. 6 p. by 28 and 46.

(3) £913. 5*s.* 5½*d.* by 14 and 15.

(4). £95719. 11*s.* 6*d.* by 75 and 246.

(5). 25696 mds. 26 srs. 8 chts. by 73 and 345.

(6). 5842 mi. 6 fur. 19 po. by 44 and 105.

(7). 4980 days. 12 hrs. 54 min. 48 sec. by 36 and 215.

(8). 2345 tons. 14 cwt. 3 qrs. 3 lbs. by 75 and 433.

(9). Rs. 23745. 12 as. 8 p. by 10 and 100.

(10). Rs. 23674. 14 as. 5 p. by 100 and 1000.

(11). Rs. 2744. 11 as. 9 p. by Rs. 26. 9 as. 6 p.

(12). 5697 mds. 14 srs. 3 chts. by 26 mds. 5 srs. 3 chts.

2. The yearly expense of a benevolent institution was Rs. 58967. 14 as. There was a grant of Rs. 460 per month by a rich man, and an endowment, which yielded Rs. 4600 a year. The rest was made up by local subscription among the neighbouring inhabitants, of whom there were 438 in number. What did each of them pay on an average ?

3. An estate containing 3601 bghs. 6 kths. of land, was divided among 30 persons ; what was the share of each ?

4. The education expenses of 112 charity boys came up to Rs. 2200 annually ; what was the expense for each boy ?

5. A party of 58 boys went on a feasting, and it was found that each boy's share of the expense was Rs. 3. 8 as. 4 p.; but three boys of the number being poor, were exempted from paying their shares. How much had each to pay to meet the expense?

6. A field containing 25500 bighas of land, one-third of which is under cultivation, produces 2150000 mds. of rice; how many maunds does each bigha yield?

7. If Rs. 965. 14 as. 10 p. be divided among 29 persons, how much will each receive?

8. What sum multiplied by fifty-seven will give Rs. 87325. 6 as. 6 p.?

9. What quantity of coal, at 7 as. 6 p. a maund, can be purchased for Rs. 8670?

10. A shop-keeper buys 60 srs. of ghee at Rs. 2 per seer, and 76 srs. of ghee at Rs. 2. 4 as. per seer; what will he sell the mixture at, that he may gain Rs. 50?

11. A merchant buys 138 mds. of sugar at Rs 5 per maund; he keeps for his own use 18 mds. At what price must he sell the remainder, so that he may not suffer any thing for the quantity used by himself?

12. If in every box there be silk worth Rs. 58. 8 as., how many pieces of cloth must be given in exchange for 17 such boxes, if the price of each piece of cloth be Rs. 6. 6 as.?

EXAMPLES ON CHAPTER IV.

Sec. I.

Ex. XXV.

1. Find the value of —

(1). [illegible] of Rs 5. 8 as. 6 p.	(2). [illegible] of Rs. 8. 12 as. 8 p.
(3). [illegible] of Rs. 12. 14 as. 6 p.	(4). [illegible] of R. 1. 4 as. 3 p.
(5). ·3 of Rs. 8. 9 as. 6 p.	(6). ·4 of Rs. 10. 10 as. 10 p.
(7). ·5 of Rs. 15. 15 as. 10 p.	(8). ·6 of Rs. 20. 10 as. 5 p.
(9). [illegible] of £16. 16s. 8d.	(10). [illegible] of £14. 12s. 6d.
(11). [illegible] of £20. 19s. 2d.	(12). [illegible] of £25. 16s. 1d.

(13). ·7 of £5. 5s 10d.

(14). ·8 of £7. 6s. 8d.

(15). ·9 of £2. 4s. 2d.

(16). ·25 of £4. 8s. 4d.

(17). $\frac{3}{4}$ of 14 cwt. 3qrs. 8 lbs.

(18) $\frac{3}{5}$ of 16 mds. 15 srs. 3 chts.

(19). $\frac{3}{5}$ of 28 days, 12 hrs. 30 min.

(20). ·4 of 116 yds. 2 ft. 10 inches.

2. Reduce—

(1). 12s. to the fraction of a £.

(2). 44s. to the decimal of a £.

(3). 8s. 4d. to the fraction of £12. 16s.

(4). 6 as. 6 p. to the decimal of a Rupee.

(5). 4 ft. 10 in. to the fraction of 5 yds.

3. What fraction of R. 1. is 4 as. 6 p. ; and what decimal of £1. is 6s. 4d. ?

4. What decimal of a week is 15 hrs. ; and what fraction of a maund is 14 seers 2 chts. ?

5. What part of Rs. 20 is Rs. 4. 8 as. ; and what part of Rs. 1000 is Rs. 60. 4 as. ?

6. Compare the values of—

(1). $\frac{5}{8}$ of R. 1, $\frac{1}{3}$ of 14 as. and $\frac{1}{12}$ of R. 1. 4 as.

(2). $\frac{1}{4}$ of £1, $\frac{3}{8}$ of 15s. and $\frac{3}{5}$ of a crown.

(3). $\frac{1}{8}$ of a md., $\frac{1}{4}$ of 14 srs. and $\frac{5}{6}$ of 3 srs. 6 chts.

(4). $\frac{4}{7}$ of 21 yds., $\frac{3}{8}$ of 11 ft. and $\frac{2}{3}$ of 7 ft.

(5). $\frac{3}{8}$ of 5 days, $\frac{4}{9}$ of 20 hrs. and $\frac{1}{3}$ of 59 mins.

SEC. II.

EX. XXVI.

1. Find the value of—

(1). £$\frac{1}{2}$ + $\frac{3}{4}$s. + $\frac{3}{5}$ crown + $\frac{1}{4}$ guinea.

(2). R. $\frac{3}{4}$ + $\frac{3}{8}$ of Rs. 12. 8 as. + $\frac{1}{3}$ of 15 as.

(3). £·4 + ·$\dot{3}$s. + 4·$\dot{4}$s. + £$\frac{3}{8}$.

(4). $2\frac{3}{4}$cwt. + $\frac{1}{5}$ of $\frac{1}{3}$ qr. + $6\frac{1}{2}$lbs.

(5). $2\frac{3}{4}$ guineas + £$\frac{31}{24}$ + $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{8}$ of 5s.

(6). $\frac{3}{4}$ of $\frac{7}{8}$ of a year + $\frac{23}{35}$ month + $\frac{6}{9}$ hr.

(7). $15\frac{3}{8}$ bghs. + $\frac{1}{3}$ of $\frac{4}{5}$ of a katha + $6\frac{1}{3}$ chts.

(8). $\frac{1}{3}$ of 2s. 6d. + $\frac{1}{8}$ of a guinea + $\frac{4}{13}$ of £1 + $\cdot 01\dot{3}$ of £15s.

(9). $\frac{5}{73}$ of a year + $\frac{1}{56}$ of a week + $\frac{5}{12}$ of an hour.

(10). $\cdot\dot{3}$ of guinea + ·125 of a pound + $\cdot 208\dot{3}$ of a shilling + ·5 of a penny.

Sec. III.

Ex. XXVII.

Find the difference between—

(1). $\frac{1}{3}$ of Rs. 6 and $\frac{3}{4}$ of Rs. 2.
(2). $\frac{1}{4}$ of Rs. 10. 8 as. and $\frac{1}{5}$ of Rs. 5. 10 as.
(3). $\frac{1}{6}$ of Rs. 8. 4 as. and $\frac{1}{12}$ of Rs. 6. 6 as.
(4). $\frac{2}{3}$ of £3 and $\frac{1}{4}$ of £2.
(5). $\frac{3}{4}$ of £12. 6s. and $\frac{3}{5}$ of £10. 4s.
(6). $\frac{2}{3}$ of £41 and $\frac{1}{3}$ of $\frac{4}{5}$ of a guinea.
(7). $\frac{1}{4}$ of $\frac{2}{3}$ of 5 days and $\frac{2}{3}$ of $\frac{7}{8}$ of 5 hours.
(8). $\frac{2}{3}$ of $\frac{5}{8}$ of a maund and $\frac{25}{36}$ of a seer.
(9). $\frac{1}{2}$ of a rupee + $\frac{3}{4}$ of an anna and $\frac{1}{3}$ of $\frac{3}{5}$ of a rupee.
(10). $\frac{1}{3}$ of $\frac{5}{8}$ of a ton and $\frac{2}{5}$ of $\frac{1}{10}$ of a cwt.

Sec. IV.

Ex. XXVIII.

Find the value of—

(1). R. 1. 5 as. 7 p. × $\frac{3}{4}$. (2). Rs. 15. 13 as. 6 p. × $\frac{5}{8}$.
(3). Rs. 28. 7 as. 7 p. × ·15. (4). £3. 5s. 6d. × $\frac{2}{5}$.
(5). £57. 14s. 10d. × $\frac{7}{8}$. (6). £25. 15s. 9d. × ·45.
(7). ($\frac{3}{5}$ md. + $\frac{3}{7}$ seer) × $\frac{4}{5}$. (8). ($\frac{11}{16}$ cwt. + $\frac{3}{4}$ qr.) × $\frac{2}{3}$.
(9). (£·3 + ·3s.) × ·015. (10). ($\frac{3}{4}$ yd. + $\frac{1}{6}$ ft.) × $\cdot\dot{4}$.

SEC. V.

Ex. XXIX.

Find the value of—

(1). £20. 15*s*. 10*d*. ÷

(2). £35. 10*s*. 5*d*. ÷ $\frac{5}{18}$.

(3). Rs. 15. 5 as. ÷ $6\frac{1}{4}$.

(4). Rs. $25\frac{1}{2}$ ÷ Rs. 4. 4 as.

(5). Rs. 10. 4 as. ÷ ·05.

(6). 19 ft. 10 in. ÷ $4\frac{1}{8}$.

(7). 24 mds. 15 seers ÷ $2\frac{1}{8}$.

(8). 19 cwt. 2 qrs. ÷ 4 cwt. 3 qrs.

(9). 120 bghs. 5 kths. ÷ 2·9.

(10). ·05 hrs. 2 mins. ÷ 10′ 20″.

SEC. VI.

Ex. XXX.

Convert—

(1). 1575 Sicca Rs. to current rupees.

(2) 2520 Sicca Rs. to current rupees.

(3) 5000 Sicca Rs. to current rupees.

(4). Rs. 302. 4 as. to Sicca rupees.

(5). Rs. 125. 5 as. to English money (R. 1 = 2*s*.)

(6). £46. 10*s*. to Indian money (£1 = Rs. 10. 8 as.)

(7). 3 mds. 37 srs. 8 chts. to Factory weight, and also to Avoirdupois weight.

(8). 220 bighas 5 kathas to acres.

(9). 45 hrs. 45 mins. to dandas.

(10). 42 sq. miles to bighas.

EXAMPLES ON CHAPTER V.

SEC. I.

Ex. XXXI.

1. Find the areas of the following rectangles :—

(1). 27 ft. 4 in. × 18 ft. 3 in.

(2). 14 ft. 6 in. × 12 ft. 7 in.

(3). 25 ft. 7 in. × 7 ft. 10 in.

(4). 16 ft. 5 in. × 12 ft. 11 in.

(5). 12 ft. 8 in. × 5 ft. 6 in.

2. Find the volumes of the following rectangular parallelopipeds :—

(1). 3 ft. 2 in. × 2 ft. 6 in. × 6 in.

(2). 6 ft. 3 in. × 3 ft. 2 in. × 1 ft. 2 in.

(3). 3 ft. 4 in. × 1 ft. 8 in. × 6 in.

(4). 9 ft. 4 in. × 2 ft. 3 in. × 1 ft.

(5). 8 ft. 2 in. × 4 ft. 1 in. × 2 ft. 3 in.

3. The diagonal of a rectangle is 200 cubits and its length is 160 cubits ; find its area.

4. The sum of the four sides of a rectangle is 33 yds.; what would be its area, if its breadth be half that of its length ?

5. The perimeter of a rectangle, whose length is 140 cubits and breadth 80 cubits, is equal to the sum of the four sides of a square garden. Calculate the value of the garden at Rs. 64 per bigha.

6. A reservoir whose length is 50 cubits and breadth 20 cubits, is filled with water ; what quantity of water must be drawn off from it, so as to make the surface sink 3 cubits ?

7. A tank measuring 50 ft. by 30 ft. has an opening of 1 square inch, near the bottom of it, and has 3 ft. of its depth filled with water ; how many feet per hour must water run through it if the tank be emptied in 12 days ?

8. A man cutting grass on one of the slopes of a rail road, passes from one telegraph post to another, distant 1000 ft. in 50 mins. ; if he can cut the grass of an acre in one hour, find the breadth of the slope.

Sec. II.

Ex. XXXII.

1. Find by Cross Multiplication the areas of the following rectangles :—

(1). 5 ft. 6 in. × 4 ft 3 in. (2). 12 ft. 6 in. 4 sec. × 8 ft. 6 in. 5 sec.

(3). 10 ft. 4 in. × 4 ft. 4 in. (4). 7 ft. 3 in. × 6 ft. 3 in.

(5). A room which is 25 ft. 6 in. long and 15 ft. 6 in. broad.

(6). A square garden whose side is 45 ft. 10 in.

(7). 36 ft. 4 in. × 20 ft. 6 in. (8). 36 ft. 6 in × 28 ft. 8 in.

(9). 80 ft. 6 in. × 44 ft. 4 in.

2. How many yards of matting 2 ft. wide will cover the floor of a room 40 ft. 6 in. by 15 ft. 9 in. ?

EXAMPLES ON CHAPTER VI.

Ex. XXXIII.

1. Find the value of—
   (1). 102 things at 1*s*. 8*d*. each.
   (2). 162 things at £1. 3*s*. 4*d*. each.
   (3). 400 things at £2. 0*s*. 1¼*d*. each.
   (4). 320 things at £7. 4*s*. 7½*d*. each.
   (5). 111 things at £11. 11*s*. 4*d*. for every 11.
   (6). 243 mds. at Rs. 2. 8 as. 6 p. per maund.
   (7). 74yds. of silk at Rs. 10. 7 as. 3 p. per yard.
   (8). 1109 cwt. at Rs. 32. 14 as. 6 p. per cwt.
   (9). 5ac. 2 ro. 4 po. 5½ yds. at Rs. 10. per rood.
   (10). 60 cwt. 3 qrs. 12 lbs. at £7. 13*s*. 6*d*. per cwt.

2. Find by Practice the cost of replacing a cistern, to weigh 8 cwt. 2 qrs. 14 lbs., at the rate of £2. 0*s*. 6*d*. per cwt., if the plumber allows £1. 11*s*. 6*d*. per cwt. for the lead of the old one which weighs 6 cwt. 1 qr. 10 lbs.

3. Find the value of 9 tons 17 cwt. 2 qrs. 24 lbs. at Rs. 125. 6 as. 8 p. per ton.

4. What is the cost of 16 mds. 25 srs. 12 chts. of sugar at Rs. 10. 8 as. 6 p. per maund?

5. Find the value of 19 cwt. 3 qrs. 27½ lbs. at £999. 19*s*. per cwt.

6. Find by Practice the cost of travelling 6 miles 7 fur. 32½ po. at 5*s*. 10*d*. per mile.

7. Find by Practice the time of building a wall 54 yds. long and 12 ft. high, of which one square yard is built in 3 hrs. 18 mins. 45 secs.

EXAMPLES ON CHAPTER VII.

SECS. I AND II.

Ex. XXXIV.

1. Find a fourth proportional to
   (1). 2, 3 and 6. (2). 3, 7 and 9.
   (3). 3, 4 and 6. (4). Rs. 10, Rs. 5 and Rs. 6.

(5). Rs. 12. 8 as., Rs. 3. 2 as. and 32 yds. 4 ft.

2. Find a third proportional to

(1). 3 and 9. (2). 5 and 20. (3). 96 and 12.

(4). £2 and 10*s*. (5). Rs. 5 and 5 as.

3. If a score of sheep cost £45, what would be the price of 4560 sheep ?

4. If 8 yds. of cloth cost Rs. 10, what would be the price of 32 yds. of cloth of the same kind ?

5. If the rent of 10 bighas of land be Rs. 34. 6 as., what would be the rent of 19 bighas and 16 kathas ?

6. If a mechanic, working 40 days, can earn Rs. 381, how many days must he work to earn Rs. 238. 2 as. ?

7. If a pole which is 70 ft. long, when placed vertically, at 11 A. M. cast a shadow 84 ft., what length of shadow will a pole 105 ft. in length cast when similarly placed at the same hour ?

8. A person buys a piece of land, containing 75 bighas for Rs. 54000. At what rate per katha must he sell it so as to gain Rs. 6000 on the whole ?

9 The estimated rental of a parish amounts to £3500, and a rate is levied of £65. 13*s*. What is the rate in the pound ?

10. If a watch gain 6 seconds in every 5 hours, how much will it gain in 2 weeks ?

11. A bankrupt's assets are £450 out of which he pays 5*s*. in the pound on half his debts, and 4*s*. on the other half ; find the amount of his debts.

12. A clock is 12 minutes too fast on Saturday at 7 P. M., and it gains 2′ 5″ a day. What will be the time by the clock, at half-past 10 o'clock A. M. on Friday following ?

13. If a person can travel 120 miles in 4 days walking 9 hrs. a day, how many days will he take, walking 7 hours a day, to travel 386 miles ?

14. If 12 men can do a piece of work in 20 days, how many men will do a piece of work 4 times as great in 4 days ?

15. If a money-lender, with a capital of Rs. 10000, gain Rs. 1000 in 7 months, in what time, with a capital of Rs. 3850 will he gain Rs. 605, at the same rate ?

16. Twelve masons build 18 yds. of garden wall, which is to be made 54 yds. long in 9 days, working 10 hours a day, how many must be employed to finish the wall, in 6 days of 9 hours each ?

17. If the wages of 18 labourers for a month amount to Rs. 118. 2 as. when rice is 24 seers per Rupee, what ought the daily pay of a labourer to be in proportion, when the price of rice is Rs. 2. 10 as. 8 p. per maund ?

18. If 4 men or 6 women can do a piece of work in 20 days, in how many days will 3 men and 5 women do it ?

19. If the wages of 25 men amount to £76. 13*s*. 4*d*. in 16 days, how many men must work to receive £103. 10*s*., the daily wages of the latter being one-half of those of the former ?

20. If 12 pumps can discharge from the hold of a leaky ship 64800 gallons of water in 6 hours, what quantity of water would be discharged by 10 pumps in 8 hours ?

21. If 12 men working 8 hours a day, cut a ditch of uniform depth 4 ft. wide and 20 yds. long in 5 days, how many hours a day must 220 men work in order to cut a ditch of the same depth, 5 ft. wide and half a mile long in 18 days ?

EXAMPLES ON CHAPTER IX.

Ex. XXXV.

1. Find the Simple Interest and the Amount of—

(1). Rs. 275. 10 as. for 1 year at 5 per cent. per annum.

(2). Rs. 5494. 10 as. 2 pice for 3¼ years at 3 per cent. per annum.

(3). Rs. 382. 7 as. 3 pice for 4 years at 10 per cent. per annum.

(4). Rs. 1905. 3 as. 3 pice for 2 years and 5 months at 7 per cent. per annum.

2. Find the Simple Interest on—

(1). £651 for ¼ of a year at 3½ per cent. per annum.

(2). £120. 5*s*. for 2 years at 3¾ per cent. per annum.

(3). £ 172. 18*s*. 9*d*. for 3 years at 4 per cent. per annum.

(4). Rs. 9784. 3 as. 6 pice for 2 years and 4 months at $6\frac{1}{2}$ per cent.

(5). Rs. 3415. 15 as. 11 p. for 3 years and 7 months at 9 per cent.

3. In what time will—

(1). Rs. 1500 amount to Rs. 2000 at 9 per cent. per annum ?

(2). Rs. 2000 amount to Rs. 2780 at $6\frac{1}{2}$ per cent. per annum ?

(3). A given sum double itself at 12 per cent. per annum ?

(4). Rs. 6,000. 12 as. 8 p. amonnt to Rs. 10,000 at 12 per cent. per annum ?

(5). Rs. 5,675 amount to Rs. 7,500 at 12 per cent. per annum ?

(6). £2,656 amount to £3,000 at $4\frac{1}{4}$ per cent. per annum ?

4. At what rate of Simple Interest will—

(1). £120 amount to £160 in 2 years ?

(2). £225 amount to £300 in $2\frac{1}{2}$ years ?

(3). Rs. 12000 amount to Rs 15000 in 8 years ?

(4). Rs. 320 amount to Rs. 350 in 5 years ?

(5). £8900 amount to £10680 in 2 years ?

5. What sum will amount to—

(1). Rs. 1560. 8 as. in 4 years at 5 per cent. Simple Interest ?

(2). Rs 10656. 10 as. 6 p. in 6 years at $4\frac{1}{2}$ per cent. Simple Interest ?

(3). Rs. 3486. 12 as. 6 p. in $2\frac{1}{2}$ years at 6 per cent. Simple Interest ?

(4). £756. 10*s*. 4*d*. in 3 years at $4\frac{1}{2}$ per cent. Simple Interest ?

(5). Rs. 96458. 11 as. 6 p. in 4 years at 10 per cent. Simple Interest ?

6. A landed property consisting of 750 acres of land, which pays an average rent of £1. 12*s*. 6*d*. per acre, is burdened with a mortgage of £2500, for which interest is paid at the rate of 4 per cent. per annum ; what is the clear rental of the property ?

7. An annuity of £50 is payable annually ; what will it amount to at the end of the seventh year after the first payment, allowing 5 per cent. simple interest ?

8. What principal, put out for 6½ years at 4½ per cent. Simple Interest will amount to £1002. 19*s*. 7⅛*d*. ?

9. What is the rate of simple interest when £315. 6*s*. 8*d*. will amount to £359. 9*s*. 7⅛*d*. in 4 years ?

10. In what time will the interest upon £320. 12*s*. 6*d*. be £70. 10*s*. 9*d*. at 4 per cent. Simple Interest ?

EXAMPLES ON CHAPTER X.

Ex. XXXVI.

1. Find the Present Worth of—

(1). Rs. 275 due 1 year hence at 6 per cent.

(2). Rs. 1200 due ½ year hence at 8 per cent.

(3). Rs. 5130 due 9 months hence allowing 6 per cent. discount.

(4). Rs. 6000 due in 91 days allowing 6 per cent. discount.

(5). £250. 17*s*. 6*d*. due in 8 months, allowing 6 per cent. discount.

(6). £572 due 8 months hence, at 3¾ per cent. interest.

(7). £1242. 6*s*. 8*d*. due 245 days hence at 3½ per cent.

(8). £150 due 17 months hence, interest being at 4 per cent.

(9). £340 due 5 months hence, at 8 per cent. interest.

(10). £101236. 7*s*. 2*d*. due 3 years hence at 6 per cent. interest.

2. *A* bought a quantity of goods for ready money for £150 and sold them for a bill for £200 payable 9 months hence. If this bill were at once fairly discounted, at 4½ per cent. interest, what would be the ready money gain on the transaction ?

3. If on a debt of £16992. 3*s* 9*d*. due 4 years hence, the present value were £14648. 8*s*. 9*d*., at what rate was Simple Interest calculated ?

4. Find the Discount on—

(1). Rs. 2786 due 5 months hence at 12 per cent.

(2). Rs. 4000. 13 as. 4 p. due 7 months hence at 7 per cent.

(3). Rs. 5680. 12 as. 11 p. due 8 months hence at 10 per cent.

(4). Rs. 1000. 10 as. 6 p. due 10 months hence at 6 per cent.

5. The present value of a debt, due 9 months hence is £16. 10s. 4d.; what is the debt, interest being at 4 per cent. ?

6. What is the discount on a note of hand for Rs. 2500 due 110 days hence at 6 per cent. per annum ?

7. What discount should be allowed on £420 paid 9 months before due, simple interest being calculated at 5 per cent. ?

8. Find the discount on £243. 10s. due 5 months hence at $3\frac{3}{4}$ per cent. interest.

9. What is the discount on £2240, due 16 months hence at 5 per cent. per annum ?

10. What is the discount on £1250 due 9 months hence at $5\frac{1}{4}$ per cent. ?

11. If the discount on £678. 8s. which is due at the end of a year and a half, be £38. 8s., what is the rate per cent. of simple interest ?

12. Prove that the difference between the interest and discount on a given sum for a given time is equal to the interest on the discount.

EXAMPLES ON CHAPTER XI.

Ex. XXXVII.

1. What quantity of stock may be purchased by investing—

(1). Rs. 2200 in the 4 per cents. at $12\frac{1}{4}$ discount.

(2). Rs. 5000 in the 4 per cents. at 10 discount.

(3). Rs. 67155 in the 5 per cents. at 6 premium.

(4). Rs. 10275 in the 5 per cents. at $6\frac{7}{8}$ premium.

(5). Rs. 8460 in the 4 per cents. at $5\frac{3}{8}$ discount.

2. A man lays out Rs. 4739 in the purchase of 3 per cent. stock at $84\frac{1}{2}$; at what price must he sell them so as to gain Rs. 1085 on his whole outlay, brokerage in each case being $\frac{1}{8}$ per cent. ?

3. A person invests Rs. 1037. 8 as. in the 3 per cents. at 83, and when the funds have risen 1 per cent. he transfers his capital to the 4 per cents. at 96 : find the alteration in his income.

4. If I lay out Rs. 2400 in 4 per cent. stocks at 96, and sell them when the funds have fallen 2 per cent., and then invest the proceeds in the purchase of stocks at 17½ per cent. premium which bears 10 per cent. interest ; what quantity of the latter stock shall I hold and what will be the difference of my income ?

5. A person invested £6940 in the 3 per cent. consols at 86⅝ and sold out when they had risen 3¼ per cent. ; what was his gain, the brokerage on each transaction being ⅛ per cent. ?

6. A person possesses £3200 stock in the 3 per cents. which he sells at 99¾ : he invests the proceeds in railway shares at £56 a share which shares pay 5 per cent. interest on £45, the amount paid on each share. How much is his income altered by the transaction ?

7. What sum must a person invest in the 3 per cents. at 90, in order that by selling out £1000 stock, when they have risen to 93¼, and the remainder when they have fallen to 84½, he may gain £6. 5s by the transaction ? If he invests the produce in 4 per cent. at par, what will be the difference in his income ?

EXAMPLES ON CHAPTER XV.

Ex. XXXVIII.

1. Find the square roots of—

(1). 4425. (2). 27225. (3). 99856.

(4). 97535376. (5). 15241578750190521.

2. Find the square roots of—

(1). 365·2564. (2). ·00001. (3). 1·6.

(4). ·16. (5). ·016.

3. Find the square roots of—

(1). ¼. (2). 36¼. (3). $\frac{3\frac{1}{4}}{4\frac{1}{2}}$.

(4). $\dfrac{\frac{1}{4} \text{ of } \frac{2}{3} \text{ of } \frac{5}{8}}{2\frac{1}{2}}$. (5). $\dfrac{5 \cdot 64}{\cdot 021}$.

4. Find the number of which ·0101 is the square root, and find the number which when squared gives ·010164.

5. A certain number of persons agreed to subscribe as many guineas each as there were subscribers in all ; the whole subscription being £1047901. 1*s*., how many subscribers were there ?

6. The Ochterloney monument in Calcutta is about 180 ft. high ; what will be the length of a rope from the top to a peg driven into the ground, exactly 120 ft. from the base of the monument ?

7. A ladder 55 ft. long placed in a street reaches a window 44 ft. high on one side, and, the foot of the ladder being retained in the same position, reaches to another window 40 ft. high on the other side of the street ; what is the width of the street ?

APPENDIX C.

UNIVERSITY EXAMINATION PAPERS.

ENTRANCE EXAMINATION PAPERS.

CALCUTTA.

1858.

1. Multiply R18957. 13*a.* by R568. 11⅜*a.*, and divide the same sum by the same sum. Shew that one of these operations is absurd and impossible, and perform the other.

2. Find the value of the decimal ·16854, and deduce the rule arithmetically or algebraically.

3. Extract the square roots of 3 and of ·3 to 7 decimal places, and explain the rule that in integers the pointing off of the periods begins from the right hand, and in decimals from the left.

4. A plate of metal is beaten to the thickness of ⅛ of an inch, and the weight of a circular medal cut from it, whose diameter is 1⅛ inches, is 1¼ oz. Troy. If the same plate be beaten to the thickness of ⅓ of an inch, what will be the weight of a medal cut out of it of the diameter of 1⅓ inches, (the areas of circles being proportional to the squares of their diameters)?

1859, A.

1. What do you mean by a *prime number*, a *factor*, a *ratio*? Resolve 30 and 132 into their prime factors, and find their ratio in its simplest terms.

2. How much muslin at 1 rupee 5*a.* 8*p.* per yard, is equal in value to 143 yards of cambric at 3 rupees 13*a.* 8*p.* per yard?

3. Whether is the product of 2¾ and 3⅛ or the product of 2⅛ and 3¾ the greater? Extract the square root of the difference.

4. If a person get a bequest of ⅘ of an estate of 2000 acres, and sell ⅜ of his share, how many acres does he retain?

Simplify the expression

$$\frac{1}{10+\frac{1}{2+\frac{1}{10}}}.$$

5. Find by Practice the rent of 586 acres 1 rood 31 poles, at £4. 1*s.* 10¼*d.* per acre.

6. A piece of land is 11·916 poles broad; how long must it be to contain an acre? Divide accurately 0·063 by $0·\dot{3}\dot{6}$.

7. How much must be paid for £1250 stock when it sells at 108 per cent.?

1859, B.

1. A man can count at the rate of 100 a minute,—how long will it take him to count five hundred lacs?

2. A shop-keeper purchased $250\frac{2}{3}$ yards of cloth for R900 and paid expenses amounting to R103: what must he charge per yard in order to make a profit of 50 per cent.?

3. Reduce ·005 of a pound to the fraction of a penny, and extract the square root of ·00006241.

4. Add together $\frac{2\frac{1}{2}}{3\frac{1}{3}}$, $\frac{1\frac{1}{4}}{3}$, 9 and $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{5\frac{3}{5}}{7}$.

5. State the rules for pointing in the multiplication and division of decimals; and multiply ·256 by ·0025 and divide ·0036 by ·4 and 4 by ·00001.

1860.

1. If the price of bricks depends upon their magnitude, and if 100 bricks, of which the length, breadth, and thickness, are 16, 8 and 10 inches respectively, cost R2. 9*a*., what will be the price of 921600 bricks, which are one-fourth less in every dimension?

2. Explain the method of pointing in extracting the square roots of whole numbers and decimals. Find the square root of 57214096, and also the square root of ·5 to four places of decimals.

3. Simplify $(1+\frac{5}{8}+\frac{7}{8}+\frac{11}{12})\div(\frac{3}{4}-\frac{5}{8})$

and $\frac{7\frac{1}{2}}{6\frac{1}{2}}+\frac{11\frac{1}{3}-2\frac{5}{6}}{11\frac{1}{3}+2\frac{5}{6}}\times 10\frac{9}{18}\div 6\frac{42}{73}$.

4. A tea-dealer buys a chest of tea containing 2 maunds and 16 seers, at R4. 2*a*. per seer, and two chests more each containing 3 maunds and 24 seers at R4. 10*a*. per seer: at what rate per seer must he sell the whole in order to gain Rs. 576?

1861.

1. Express as a decimal fraction:—

$$\frac{4\frac{2}{3}\times 8\frac{9}{11}}{\frac{5}{8}\div 10\frac{1}{4}}\times\frac{6\frac{1}{4}\text{ of }4\frac{4}{5}}{4+2\frac{1}{3}}$$

2. Reduce 3*s.* 6*d.* to the decimal of £5, and $\cdot 02\dot{3}\dot{4}$ to a vulgar fraction.

3. If an estate be worth £2374. 16*s.* per annum, and the land tax be assessed at 1*s.* 11½*d.* in the £, what will be the net annual income?

4. How much land may be rented for £1716. 10*s.* 6*d.*, if 3 acres are rented for £4. 13*s.* 4*d.*?

5. Extract the square root of ·00099856.

1862.

1. What is the difference between

$$\frac{4\frac{2}{3}}{5\frac{1}{8}} - \frac{99}{310} \text{ and } \cdot 06 \text{ ?}$$

2. Reduce ·14 of a pie to the fraction of a Rupee, and find the value of ·0875 of a pound sterling.

3. If the wages of 18 coolies for a month amount to R85 when rice is 24 seers per Rupee, what ought the daily pay of a coolie to be in proportion when the price of rice is R2. 10*a.* 8*p.* per maund?

4. *A* and *B* run a race. *A* has a start of 40 yards, and sets off 5 minutes before *B*, at the rate of 10 miles an hour. How soon will *B* overtake him if his rate of running is 12 miles per hour?

5. Extract the square root of $\frac{1}{1000}$ to 5 places of decimals.

1863.

1. Find the value in vulgar and decimal fractions of :—

$$\frac{15\frac{3}{8} + 6 - \frac{3}{4}}{7\frac{1}{6} \times \frac{3}{12}}.$$

2. Find the fractional value of :—

$$(2 \cdot 37979 + 4 \cdot 22) \div (3 \cdot 041 - \cdot 937).$$

3. The weight of five casks of coffee being 31 cwt. 3 qrs. 13 lbs., calculate the price at 90 shillings per cwt.

4. If a man can perform a journey of 170 miles in 4½ days of 11 hours each, in how many days of 8¾ hours will he perform a journey of 470 miles?

5. Extract the square root of :—

$$964 \cdot 226704.$$

6. What sum of money will produce £43 interest in 3¼ years, at 2½ per cent. simple interest?

1864.

1. How many paving stones, each measuring 14 inches by 12 inches, are required to pave a verandah 70 feet long and 9 feet broad?

2. Add together $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{11}$, $\frac{13}{15}$ and $\frac{8}{77}$; and simplify :—

$$\frac{2\frac{1}{4}}{2\frac{2}{3}}+\frac{2\frac{1}{3}+5\frac{1}{8}}{3\frac{1}{3}+9\frac{1}{2}}+\frac{1}{2}+\frac{8}{9}\text{ of }\frac{3}{70}.$$

3. Find the value of 17 cwt. 3 qrs. 22 lbs., at £4. 6*s.* 7½*d.* per cwt.

4. Add together ·0125 of a pound, ·0625 of a shilling, and ·5 of a penny; and reduce 11*s.* 9½*d.* to the decimal of a pound.

5. Extract the square root of ·000196, and divide the result by 140.

6. A company guarantees to pay 5 per cent. on shares of R1000 each; another guarantees to pay 4⅝ per cent. on shares of R75 each: the price of the former is R1245, and of the latter R85. Compare the rates of interest which the shares return to purchasers.

1865.

1. Find the value of $11\frac{3}{8}+14\frac{5}{8}+21\frac{7}{16}+32\frac{13}{32}$, both by vulgar fractions and by decimals, showing that the two results coincide; and reduce 25° 36′ 45″ to the decimal of 75°.

2. Find the product of the sum and difference of ·0421 and ·0029, and divide one-tenth of the square root of that product by ten times the continued product of ·02, ·03, and ·07.

3. How many yards of matting 3·5 feet wide will cover the floor of a room 85·3 feet long, and 40·5 feet broad? and how much will it cost, at R2. 10*a.* 8*p.* per *square* yard?

4. If the wages of 25 men amount to R766. 10*a.* 8*p.* in 16 days, how many men must work 24 days to receive R1035, the daily wages of the latter being one-half those of the former?

5. What principal in 3 years 73 days will amount to R100. 15*a.* at 6¼ per cent. simple interest? A bill for R5035. 4*a.* drawn on September 12th at 5 months, was discounted on 16th January at 4 per cent.; what was the discount charged?

1866.

1. Reduce 3° 45′ 36″·25 to the decimal of 36°.

Simplify—

$$\left(\frac{1}{3}+\frac{\frac{3}{7}}{1\frac{3}{5}}+\frac{1}{1\frac{1}{4}}+\frac{5}{6}-1\right)\div\frac{5}{6}\text{ of }\frac{3}{8}\text{ of }2\frac{4}{5}.$$

2. Find the value of 6 cwt. 2 qrs. 7 lbs. at £3. 4*s.* 6½*d.* per cwt.

3. Find the square of 0·0204 and the square root of 81·757764; and divide one-tenth of the latter result by one hundred times the former.

4. Divide 0·1001 by 0·000390625, and 10·01 by 390·625.

5. What is the expense of paving a rectangular verandah whose length

is 42 feet and breadth 15 feet with Burdwan paving stones, 18 inches square, and which cost R15 per score?

6. The 3 per cents. are at $85\frac{5}{8}$; what price should the $3\frac{1}{2}$ per cents. bear, that an investment may be made with equal advantage in either stock? And what interest would be derived by so investing £5000?

1867.

1. The driving wheel of a locomotive is 226 inches in circumference, and makes 91 revolutions per minute; at what rate per hour is the engine travelling?

2. Divide the least common multiple of 156, 260, 720, and 429 by their greatest common measure, and find the square root of the quotient.

3. If a butcher buy 10 cwt. of beef at 44s. 4d. per cwt. and sell it at the rate of $4\frac{1}{2}d.$ per lb., how much does he lose or gain?

4. Find the value of the following expressions:—

$$\frac{5\frac{1}{3}\times\frac{3}{7}\times 9\frac{1}{2}\times 3\frac{1}{8}}{1\frac{3}{8}\times 67} \text{ and}$$

$$\frac{0{\cdot}625 \text{ of } £143.\ 12s.\ 0d. + 0{\cdot}625 \text{ of } £71.\ 16s.\ 0d.}{\frac{5}{8} \text{ of } 5175}$$

5. Reduce £1. 5s. 6d. to the fraction of £1000 and 5s. $1\frac{3}{8}d.$ to the fraction of £150. 10s., and express the results both as vulgar and decimal fractions.

6. If £450 amount to £523. 10s. in 1 year 8 months, calculate the rat per cent.

1868.

1. Find the difference between $1{\cdot}6$ of $3{\cdot}4$ of $£1{\cdot}125$ and $\frac{1}{6}$ of $3{\cdot}\dot{6}$ of $£9{\cdot}1125$, and find the value of—

$$\frac{6{\cdot}27\times 0{\cdot}5}{(\frac{1}{2} \text{ of } \frac{1}{4})\times 8{\cdot}36} \div \frac{(\frac{1}{4} \text{ of } \frac{7}{10})\times(\frac{3}{4} \text{ of } 21\frac{1}{3})}{(\frac{2}{3} \text{ of } \frac{5}{8})+1{\cdot}\dot{4}}$$

2. Extract the square root of $153{\cdot}140625$, and of $3{\cdot}3$, each to three places of decimals.

3. If one man walks 165 miles in 6 days, how far will another man walk in 15 days if the first man walks $3\frac{3}{4}$ miles in the same time that the other man walks 4 miles?

4. Three equal glasses are filled with a mixture of spirits and water: the proportion of spirits to water in each glass is as follows:—in the first glass as 2 : 3, in the second as 3 : 4, and in the third as 4 : 5. The contents of the three glasses are poured into a single vessel: what is the proportion of spirits to water in it?

5. Find the interest on £350 from 3rd March to 28th December, at $4\frac{1}{2}$ per cent. per annum.

6. How many yards of carpet, 25 inches wide, will be required for a room 19 feet 7 inches long, and 18 feet 9 inches wide?

1869.

1. Simplify $\dfrac{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{1}{12}}{\frac{1}{3}+\frac{1}{6}}\div\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$;

and reduce 4 hrs. 1 min. $10\frac{1}{2}$ sec. to the decimal of a week.

2. Add together ·062435 of 100*l.* + 7·4375 of 10*s.* + 1·35$\dot{6}$ of 7*s.* 6*d.* + 2·784 of $2\frac{1}{2}d.$, and reduce the result to the fraction of 29*l.* 10*s.* $7\frac{1}{2}d.$

3. Divide ·0007 by ·035 and by 3500, and extract the square root of each quotient to four decimal places.

4. A room is 37 ft. 2 in. long, 25 ft. 8 in. broad, and 22 ft. 6 in. high: find the cost of covering its four walls with paper $1\frac{1}{4}$ yard wide, at 1*s.* $1\frac{3}{4}d.$ a yard.

5. In what time will £563. 13*s.* $5\frac{1}{2}d.$ amount to £901. 17*s.* $5\frac{4}{8}d.$ at $3\frac{3}{4}$ per cent.?

1870.

1. Find the cost of matting a room whose floor is 8 yards long by $7\frac{1}{2}$ yards wide, with mats 2ft. wide and $9\frac{1}{2}$ft. long, at the rate of 9 annas 2 pies per mat.

If the same room be $15\frac{1}{2}$ft. high, find how many cubic feet it will contain.

2. Distinguish between a vulgar fraction and a decimal fraction. Multiply 999[illegible] by 999.

State the rule for the multiplication of decimals, and apply it to point the products in (1) 1·23 × ·0011 and (2) 29000 × ·01.

Divide $\cdot\dot{3}\dot{7}$ by $\cdot\dot{1}4\dot{8}$, and shew that $\dfrac{\cdot\dot{1}2\dot{3}}{\cdot\dot{4}\dot{1}}=\dfrac{123123}{414141}$.

3. Find the square root of $19740\frac{1}{4}$ and of $4\frac{1}{60}$, the latter to four places of decimals.

4. Two gangs of six men and nine men are set to reap two fields of 35 and 45 acres respectively. The first gang complete their work in 12 days; in how many days will the second gang complete theirs?

5. Find which is the better investment, $3\frac{1}{2}$ per cent. stock at $98\frac{1}{4}$, or $3\frac{5}{8}$ per cents. at 105.

6. Find how many rupees are equivalent to 200*l.* at the rate of 1*s.* $11\frac{5}{16}d.$ per rupee.

1871.

1. 6625 railway tickets were sold at a station, $\frac{3}{5}$ ths of which were 9 annas each and the rest 5 annas each; what was the amount received for the tickets?

2. Find the greatest and least of the fractions $\frac{1}{7}$, $\frac{4}{25}$, $\frac{6}{35}$, $\frac{25}{105}$. Add together $2\frac{2}{9}$ of £2. 13*s.* $6\frac{1}{4}$*d.* and (£3. 15*s.* $9\frac{1}{2}$*d.*) $\div 6\frac{4}{7}$ and simplify—

$$\frac{2\frac{1}{2}-1\frac{3}{4}}{4 \text{ of } 1\frac{1}{3}+6\times\frac{1}{2}}\times\frac{5-\frac{5}{6}}{\frac{1}{3}+\frac{3}{8}}+\frac{5}{6}.$$

3. Divide ·027 by 14·4 and 1208·04 by $\cdot 01\dot{7}$.

Find the value of 11·1375 of R6. 8*a.* − ·56 of R7. 8*a.*, and reduce 8*a.* 6*p.* to the decimal of R3. 7*a.*

4. If the carriage of $9\frac{3}{4}$ mds. for a distance of 80 miles be Rs. 3, how many miles should 130 mds. be carried for R27. 8*a.*?

5. What sum of money will amount to R3761. 14*a.* in $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. per annum simple interest?

1872.

1. A merchant bought goods which cost him R9810. In the first day he sold to the amount of R992. 8*a.* 6*p.*; in the second to that of R1992. 8*a.* 3*p.*; and in the next three days to an amount equal to twice the two former. Finding that he had one-fourth of the goods left, he calculated his profits in the five days. How much were they?

2. What fraction of R10 is R6. 10*a.* 8*p.*?

Find the value of—

$\frac{3}{8}$ of R2. 8*a.* + $\frac{5}{8}$ of R4. 11*a.* + 2·05 of R5.

Simplify $4\frac{1}{8}$ of $\frac{9}{10}-\dfrac{\frac{5}{7}}{3\frac{1}{4}}+\dfrac{5\frac{1}{8}-4 \text{ of } \frac{2}{3}}{3\frac{1}{4}-2\frac{1}{3}}$.

3. Divide 274·72 by ·0544; find the value (correct to six places of decimals) of (i) $\dfrac{\cdot 00\dot{3}\times\cdot 05}{\cdot 0022}$, (ii) $6\cdot\dot{0}4\dot{5}-5\cdot 3\dot{6}7\dot{8}$; and extract the square root of 951·1056.

4. Find by Practice the cost of 15 mds. 25 seers 11 chts. of oil at R12. 10*a.* 3*p.* per maund.

5. If the interest of R1000 in 5 years be R250, what will be the interest of R3500 for 1 year and 6 months?

1873.

1. Find the value of (i) $\dfrac{1+2\frac{1}{3}+3\frac{1}{4}}{\frac{1}{1\frac{1}{2}}+\frac{2}{2\frac{1}{3}}+\frac{3}{3\frac{1}{4}}}\times\dfrac{55\frac{3}{8}\div 11}{1\frac{4}{11} \text{ of } 13\frac{3}{8}}$.

(ii) $24\frac{5}{8}$ of R103. 7*a.* 6*p.*

If $\frac{4}{15}$ of a maund is worth R45, what is the price of $\frac{1}{3}$ of a maund?

2. Reduce $\frac{1}{308}$ to a decimal; $\cdot 01\dot{9}$ to a vulgar fraction;

and $\frac{4\cdot\dot{2}-3\cdot\dot{1}\dot{4}}{1\cdot\dot{3}+2\cdot 1\dot{0}\dot{2}}$ of $\frac{1\cdot 3 \text{ of } 4}{\cdot 37 \text{ of } 8\cdot\dot{8}1}$. to its lowest terms.

3. What is the expense of matting a room 31 ft. 5 in. long by 20 ft. 4 in. wide, the mat costing 14*a.* per 12 square hath (linear hath = 18 in.)?

4. In what time will R8500 amount to R15767. 8*a.* at $4\frac{1}{2}$ per cent. per annum?

5. A person owes the sums of R31500 and R8500: and his property only amounts to R14125. How much is he able to pay in the rupee; and what is the loss upon the second debt?

1874

1. What fraction of $\frac{7}{8}$ of a rupee is $\frac{4}{9}$ of R5; and what proportion does their difference bear to their sum?

Divide 999·666 by ·30036; and $2\cdot 357142\dot{8}$ by $10\cdot\dot{2}14285\dot{7}$.

2. When rice is 10 seers the rupee, nine persons can be fed for 30 days at a certain cost. For how many days can six persons be fed at the same cost when rice is 14 seers the rupee?

3. A wooden box 3 ft. 8 in. long, 2 ft. 3 in. high, and 2 ft. 4 in. wide, is made of board one inch thick. Find the quantity of wood used; and the cubical content of the box.

4. It is said that 240000 letters are posted in Berlin daily, $16\cdot\dot{6}$ per cent. of which are town letters This gives one letter for every three persons in Berlin; what is its population?

5. What sum will amount to a lakh of rupees in ten years at 5 per cent. simple interest?

Find the discount on R1308 due two years hence at $4\frac{1}{2}$ per cent. per annum.

1875.

1. Simplify $$\frac{17}{7+\frac{3}{4-2\frac{5}{8}}}\times\frac{2021}{2193}\div\left(1\frac{37}{48}-\frac{15}{16}\right).$$

Find the value of $\frac{3}{21}$ of R17. 6*a.* 4*p.* + $3\frac{3}{7}$ of R12. 5*a.* $11\frac{1}{4}$*p.* + R5·49583; and extract the square root of $\cdot 0\dot{4}9$ to four places of decimals.

2. A person received on the death of his aunt $\frac{5}{16}$ of her property and spent $\cdot\dot{5}\dot{4}$ of it in paying off his debts; what fraction of his aunt's property did he then possess?

3. A room is 30ft. long, 22ft. wide, $18\frac{1}{2}$ ft. high, and has 5 doors and 3 windows; find the expense of colouring the walls at 3*a.* per sq. yd., deducting 30 sq. ft. for each door and window.

4. Find the present worth of R19021 due 4 years hence at $3\frac{3}{4}$ per cent.

5. If R16430 be invested in the Government $4\frac{1}{2}$ per cent. loan at 106, what is the monthly income thence derived?

Supposing that the loan is paid off at par in 10 years, what would be the rate of simple interest (per cent. per annum) on the sum invested?

1876.

1. Simplify $\dfrac{5\frac{2}{7} - 3\frac{3}{8} + 4\frac{1}{14}}{3\frac{1}{8} + \dfrac{1+\frac{1}{6}}{2-\frac{5}{8}}}$.

Find the value of $\frac{5}{16}$ of R16. 14*a.* $- 1\cdot1\dot{4}$ of R5. 0*a.* 3*p.* $+ 1\frac{1}{8}$ of R9. 6*a.* 6*p.*

Reduce $(16\cdot0\dot{5} - 6\cdot2\dot{5})$ of a rupee to the decimal of R22. 1*a.*

2. An equal number of men, women, and boys earned R39. 6*a.* in seven days. Each boy received 2*a.* a day, each woman 3*a.* 6*p.* and each man 4*a.* 6*p.* How many were there of each?

Find the square root of 531·065 to five places of decimals.

3. How many yards of matting 2ft. 4 in. wide will be required for a square room, whose side is 9 ft. 4 in.? And what will be the price of it at 2*a.* 3*p.* per yard?

Find the value of 33 cwt. 3 qrs. 7 lbs. at £6. 7*s.* 8*d.* per cwt.

4. If 4000 men have provisions for 190 days, and if after 30 days 800 men go away, find how long the remaining provisions will serve the number left.

5. At what rate per cent. simple interest, will R1462. 8*a.* amount to R1725. 12*a.* in 4 years?

1877.

1. Simplify $\dfrac{\frac{1}{4}+\frac{1}{8}}{4-\frac{5}{6}\text{ of }5\frac{1}{3}} \div \dfrac{\frac{3}{8}+\frac{1}{4}}{\frac{7}{10}\text{ of }4\frac{1}{3}-2\frac{1}{2}}$, and find the value of $\frac{4}{35}$ of 16*s.* 11*d.* + $\frac{3}{13}$ of £1. 1*s.* 4*d.* + £3·23.

2. Find by Practice the value of $739\frac{1}{2}$ maunds of sugar at R1231. 4*a.* per hundred maunds?

3. Find the discount on £453. 15*s.* due 6 years hence at $3\frac{1}{2}$ per cent. per annum.

4. A man sells 3 per cent. stock at 75, and invests the proceeds in the 5 per cents.; at what rate must he buy them in order that his income may be the same as before?

5. If 7 men and 5 boys can reap 168 acres in 18 days, how many days will 15 men and 5 boys take to reap 700 acres, one man being able to do three times as much work as a boy?

6. In a rectangular area, 100 yards long and 50 yards broad, there are two paths crossing one another, each parallel to one side of the rectangle, and each 4 yards broad. Find the cost of paving the area with stone at 12*a.* per square yard, and of covering the paths with gravel at 6*a.* per square yard.

1878.

1. Calculate to three places of decimals the value of $\frac{180 \times 36}{3 \cdot 14159}$.

2. Calculate to five places of decimals the square root of $1 + (\cdot 067)^2$.

3. Reduce R483. 12*a.* 6*p.* to the decimal of R1290. 1*a.* 4*p.*

4. Find the simple interest on R757. 4*a.* 3*p.* for 343 days at $3\frac{1}{2}$ per cent. per annum.

5. Add together $\frac{1}{120}$, $\frac{1}{3040}$, $\frac{1}{10120}$, $\frac{1}{720}$. Express your answer as a decimal.

6. Find, by Practice, the value of 99 cwt. 3 qrs. 27 lbs. at £5. 2*s.* 6*d.* per cwt.

1879.

1. What is the local value of each of the figures composing the number 456·654?

2. R49 was divided amongst 150 children, each girl had 8*a.* and each boy 4*a.*; how many boys were there?

3. Simplify—

(*a*) $8 - 8 \times \dfrac{2\frac{1}{3} - 1\frac{2}{7}}{2 - \dfrac{1}{6 - \frac{1}{6}}}$.

(*b*) $\frac{1}{5} + \frac{1}{6} \div \frac{1}{7} - \frac{1}{7} \times \frac{1}{3} - \frac{1}{3}$.

(*c*) $\cdot 15\dot{9}\dot{0} \times \cdot 47\dot{2} \div 2 \cdot \dot{7}$.

(*d*) What decimal of £4. 3*s.* 4*d.* is $\frac{1}{1300}$ of £5. 8*s.* 4*d.*

4. A tank 75 yards long, 50 yards broad, and 11 ft. deep, is full of water: how many times can each of 16 water-carts, length 5 ft., breadth 4 ft., and depth 27 inches, be filled from the tank before the water in it falls 6 inches?

5. If 17 men can build a wall 100 yards long, 12 ft. high and $2\frac{1}{2}$ ft. thick in 25 days, how many will build a wall of twice the size in half the time?

6. Find the change of income when a person transfers £2616. 5*s.* from the 5 per cents. at $95\frac{1}{8}$ to the 4 per cents. at 83, brokerage as usual.

7. In a game of skill *A* can give *B*, and *B* can give *C*, 10 points out of a game of 50; how many should *A* give *C*?

1880.

1. Express each of the figures composing the number 123·456 as a multiple or sub-multiple of 10.

What fraction must be added to $2\frac{1}{3} + \dfrac{3\frac{1}{4} - \frac{1}{6}}{3\frac{1}{8} + \frac{1}{6}} - 2\frac{6}{7}$ of $\frac{4}{15}$ that the sum may be equal to 3?

2. (*a*) What fraction of $\frac{2}{3}$ of R187. 5*a*. is R28. 8*a*.?

(*b*) Of what sum of money will ·325 be £13?

(*c*) Extract the square root of 7·0225.

3. Divide £127. 8*s*. among 2 men, 3 women, and 7 boys, giving each of the boys one-third of what a woman receives, and each of the men twice as much as a woman.

4. A leaky cistern is filled in 5 hours with 30 pails of 3 galls. each, but in 3 hours with 20 pails of 4 galls. each, the pails being poured in at intervals. Find how much the cistern holds, and in what time the water would waste away.

5. A race-course is half a mile long. *A* and *B* run a race, and *A* wins by 10 yards; *C* and *D* run over the same course and *C* wins by 30 yards; *B* and *D* run over it and *B* wins by 20 yards; if *A* and *C* run over it, which should win, and by how much?

6. A tradesman puts two prices on his goods, one for ready money, the other for 6 months' credit, interest being calculated at $12\frac{1}{2}$ per cent. per annum. If the credit price of an article be Rs. 26. 9*a*., what is its cash price?

1881.

1. What do you mean by *Multiplication*? Define *quotient*, *factor*, *power*, *expression* and *dimension*.

2. Add together $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$:

and simplify $\dfrac{\frac{6}{7} - \frac{5}{6}}{\frac{6}{7} + \frac{5}{6}}$ of $2\frac{11}{14} \div \dfrac{4}{13 - 3\frac{5}{6}} + 3\frac{11}{16} - \dfrac{3}{3 - 1\frac{11}{12}}$.

3. What decimal of R45 is R35. 2*a*. 6*p*.? Find the value of

$$\frac{1\cdot074}{\cdot0015} \text{ of } 8\tfrac{1}{4} \text{ annas.}$$

4. Express $37\cdot8\dot{4}6\dot{3}$ as an improper vulgar fraction in its lowest terms; and find, correct to 4 places of decimals, the result of dividing the square root of this number by the square root of 11.

5. A man who has a certain capital calculates that if he invest it in $3\frac{1}{2}$ per cent. stock at 91, his income will be £25 more than if he invest it in 3 per cent. stock at 88. What is his capital?

1882.

1. The quotient arising from the division of 6739546 by a certain number is 1559 and the remainder is 3107: find the divisor.

2. Subtract $\frac{3}{5}$ of $\frac{5}{8}$ of $\frac{4}{15}$ of £31. 5*s.* from $\frac{4}{7}$ of $\frac{2}{11}$ of $\frac{49}{84}$ of £100. 16*s.* 8*d.*, and express the remainder as the decimal of £10. 8*s.* 4*d.*

3. Seven bells begin to strike simultaneously and strike at intervals of 2, 3, 5, 15, 21, 65, 77 seconds respectively. After what time will they again strike simultaneously, and how often will each have struck?

4. (i) Simplify $\dfrac{2\frac{3}{5}+5\frac{7}{6}}{1\frac{1}{2}-\frac{4}{6}} \div (\frac{3}{26} \text{ of } \frac{7}{42} \text{ of } \frac{21}{66})$.

(ii) Find the value of—

$$\frac{\sqrt{15}+\sqrt{13}}{\sqrt{15}-\sqrt{13}}$$ to five places of decimals.

5. A besieged garrison consists of 300 men, 120 women, and 40 children, and has provisions enough for 200 men for 30 days. If a woman eats $\frac{3}{4}$ as much as a man, and a child $\frac{1}{4}$ as much, and if after 6 days 100 men with all the women and children escape, for how long will the remaining provisions last the garrison?

6. A person begins to speculate with a certain sum of money; in his first transaction he loses $\frac{1}{5}$th of this sum: in his second he gains 10 per cent. on his investment: in his third he loses $\frac{9}{11}$ths of the sum invested: in his fourth he gains $66\frac{2}{3}$ per cent. If he then has R10000, with what sum did he start?

1883.

1. Divide $2\frac{1}{3}+8\frac{6}{11}-\frac{1}{6}$ of $(7\frac{1}{2}-3\frac{1}{3})$ by $11+\cfrac{1}{1-\cfrac{1}{1+\cfrac{1}{8\frac{1}{11}}}}$.

2. Divide the square root of 122·257249 by ·3685$\dot{6}$, and multiply the quotient by the square root of ·000625.

3. What decimal of a square yard is 9 square inches? Add together 1·032 of R5, ·64 of R1·25 and ·08 of half a rupee. What is the value of £10·541$\dot{6}$?

4. Find by "Practice" the value of 6 tons 3 cwt. 21 lbs. 14 oz. at £3. 10*s.* per ton.

5. If it costs R200 to build a wall 6 ft. high by 1 ft. 3 in. broad by 166 ft. 8 in. long, what will be the cost of building a wall $3\frac{1}{2}$ ft. by $1\frac{1}{4}$ ft. by 115 ft.?

6. When will the interest amount to the principal at $3\frac{1}{3}$ per cent. per annum? What will the interest on Rs. 150 at one anna per rupee per month amount to in 5 years, and how much is that rate per cent. per annum?

1885.

1. Of what number is $2\frac{1}{6}$ the $\frac{5}{8}$th part?

By what fraction must $\frac{1\frac{1}{4}}{1\frac{1}{12}}$ of $\frac{3}{8}+\frac{2\frac{1}{2}-1\frac{5}{8}}{\frac{1}{4}+1\frac{3}{8}}-\frac{8\frac{1}{6}}{7\frac{1}{3}}$ be divided in order to give a quotient $=\frac{3}{8}$?

2. Simplify $\frac{\cdot 12 \text{ of } (\cdot 0104 - \cdot 002) + \cdot 36 \times \cdot 002}{\cdot 12 \times \cdot 12}$;

and express your result as a fraction of ·6. Reduce $\frac{5}{8}$ of 16*s.* $4\frac{1}{2}$*d.* to the decimal of £1. 9*s.* $10\frac{1}{2}$*d.*

3. What circulating decimal multiplied by $\frac{2\frac{3}{8}}{4\frac{1}{3}}$ will give 2 for a product?

If $\cdot\dot{4}2857\dot{1}$ of a barrel of beer be worth ·72 of £2. 10*s.*, what is the value of ·625 of the remainder?

4. Find the price of 10 lbs. 11 oz. 16 dwts. 16 grs. of gold at £3. 17*s.* $10\frac{1}{2}$*d.* per oz.

Extract the square roots of $9\frac{7}{8}$ and $\frac{1}{12\cdot 5}$ to 4 places.

5. If 27 men can perform a piece of work in 15 days, how many men must be added to the number that the work may be finished in three-fifths of the time?

I buy a horse for £40 and sell it for £45 at a credit of 8 months. What do I gain per cent., reckoning money worth 6 per cent. per annum?

6. Which is the better investment, bank stock paying 10 per cent. at 319 or 3 per cent. consols at 96?

What will be the cost of £1500, 3 per cent. consols at $89\frac{7}{8}$, brokerage being $\frac{1}{8}$ per cent? What rate of interest will such investment obtain?

1886.

1. Divide $\frac{1\frac{4}{7}\div 1\frac{7}{8}}{1\frac{4}{5}\div 1\frac{5}{6}}\div\frac{1\frac{6}{7}\div 1\frac{7}{8}}{1\frac{8}{9}\div 1\frac{9}{10}}$ by $\frac{\frac{1}{7}\div\frac{1}{9}}{\frac{1}{11}\div\frac{1}{13}}\div\frac{\frac{1}{17}\div\frac{1}{18}}{\frac{1}{19}\div\frac{1}{20}}$.

2. Simplify $\frac{3\cdot 12\dot{5}}{2\cdot 16}$ of $\frac{\cdot 2\dot{4}}{\cdot 125}\div\frac{2\cdot\dot{2}}{1\cdot 5}$ of $\frac{187\cdot 5}{3\cdot 42}$.

3. Reduce £1. 11*s.* $10\frac{1}{2}$*d.* to the fraction of £7. 18*s.* $6\frac{1}{2}$*d.* What fraction of £10 must be added to £16. 10*s.* 3*d.* to make it £20?

4. What decimal of 9mds. 20 seers is $\frac{5}{8}$ of 7 mds. 5 seers? Reduce $5\frac{1}{2}$ sq. yds. to the decimal of an acre.

5. Find the value, by Practice, of 2 tons 15 cwts. 35lbs. at £13. 6*s.* 8*d.* per ton.

6. What sum of money at 4 per cent. simple interest will secure the same income as ₨2547·5 at $4\frac{1}{2}$ per cent.?

7. If a rupee is equivalent to 1*s.* $6\frac{3}{4}$*d.*, what is the price of a sovereign in rupees? If after buying 250 sovereigns at this price, I sell them again when the rupee is equivalent to 1*s.* 6*d.*, how much shall I gain or lose by the transaction?

1887.

1. Simplify :—

(*a*) $(4\frac{2}{3} - 1\frac{1}{3}) \times (3\frac{1}{2} - \frac{3}{8}) \div (13\frac{1}{3} + 7\frac{1}{2})$ of $\frac{3\frac{1}{3}}{1\frac{1}{9}}$.

(*b*) $\frac{1\cdot8\dot{3} + 2\cdot041\dot{6} + \cdot\dot{3} - 3\frac{1}{3}}{1\cdot0025 + \cdot0625 - 1\frac{1}{16}}$.

2. Express $\frac{3}{8}$ of 7*s.* 6*d.* + 1·25 of 5*s.* − $\cdot5\dot{4}\dot{5}$ of 9*s.* 2*d.* as a decimal fraction of £10.

3. (*a*) Find by Practice the value of 5 tons 5 cwt. 2 qrs. $17\frac{1}{2}$ lbs. at £3. 6*s.* 8*d.* per ton.

(*b*) Find the income on which the income-tax at 5 pies per rupee is ₨52. 1*a.* 4 pies.

4. If 50 men can do a piece of work in 12 days, working 8 hours a day, how many hours a day would 60 men have to work in order to do another piece of work twice as great in 16 days?

5. If Rs. 450 amount to ₨540 in 4 years at simple interest, what um will amount to ₨637. 8 annas in 5 years at the same rate?

6. Extract the square root of 177·1561, and of $\cdot\dot{2}$ to 3 decimal places.

1888.

1. Simplify :—

$$\frac{\frac{2}{7}(\frac{5}{16} \text{ of } 3\frac{1}{5} - \frac{1}{8} \text{ of } 2\frac{1}{5})}{\frac{1}{38} \times 1\frac{9}{17} \times 1\frac{15}{19} - \frac{7}{38} \div 2\frac{1}{3}} \div \frac{\frac{1}{6} + \frac{5}{13} - \frac{7}{13}}{\frac{1}{3} \times \frac{1}{4} - \frac{1}{17} \times 1\frac{4}{13}}.$$

2. Divide 16·016 by ·00143, and extract the square root of 1440·9616.

3. Add together $55\cdot500\dot{2}$, $3\cdot\dot{1}\dot{7}$, $4\cdot\dot{5}0\dot{6}$ and $75\cdot2\dot{7}\dot{1}$, and find the value of the following: ·7365 of £3. 6*s.* 8*d.* + ·504 of £15. 12*s.* 6*d.* + $2\cdot10208\dot{3}$ of £5.

4. Find by "Practice" the value of 2 tons 7 cwt. 3 qrs. 11 lbs. at £21. 12*s.* 6*d.* per cwt.

5. A man can walk 600 miles in 35 days, resting 9 hours each day: how long will he take to walk 375 miles if he rests 10 hours each day, and walks $1\frac{1}{4}$ times as fast as before?

6. If the interest on money be one pie per rupee per month, what is the rate per cent. per annum?

A man holds $15\frac{1}{4}$ shares of a bank, and receives £19. 1*s.* 3*d.* per quarter. If the interest he receives be 5 per cent. per annum, find the value of a share.

1889.

1. Multiply ·0069347 by 7439·6.

2. Divide 2100·006983 by 243·5846 correct to five places of decimals.

3. Find in any way the value of 1347 cwt. 3 qrs. and 21 lbs. at £3. 17*s.* $10\frac{1}{2}$*d.* per cwt.

4. Extract the square root of $1+(\cdot0634)^2$ to six places of decimals.

5. Find in English money the value of R100000 at 1*s.* $4\frac{11}{32}$*d.* per rupee.

1890.

1. Simplify $2\frac{2}{7}$ of $\dfrac{13\frac{1}{2}-9\frac{3}{8}}{15\frac{1}{6}-11\frac{7}{30}}\div3\frac{2}{7}+\dfrac{1\frac{7}{15}}{9\frac{2}{5}-8\frac{2}{3}}$, and find by Practice the value of 3049 articles at R7. 13*a.* 7*p.* each.

2. Divide 27·03 by ·0037, and reduce $\cdot\dot{7}\dot{5}-\cdot\dot{1}0\dot{2}-\cdot2\dot{7}$ to a vulgar fraction.

3. Find the cost of putting a fence round a square field whose area is 13·225 acres at R1. 12*a.* per yard.

4. A piece of work can be done in 72 days by 17 men working together. If after 9 days of work these are joined by 4 others, in how many days will the work be finished?

5. Find the price of $4\frac{1}{2}$ per cent. Government Promissory Notes when an investment of R59422. 8*a.* produces a monthly income of R213. 12*a.*

1891.

1. Simplify the following expressions :—

(*a*) $\dfrac{\frac{3}{8}-\frac{2}{7}}{\frac{5}{8}-\frac{1}{5}}\div\dfrac{\frac{5}{7}-\frac{13}{18}}{\frac{1}{3}-\frac{1}{11}}$.

(*b*)

$$4-\cfrac{\;}{2-\cfrac{\;}{1-\frac{6}{15}}}$$

2. Find the value of $2·4607 \times ·06 - 3·75 + ·012 \times 2·163 \div 1·03$.

3. Find the value of 15 cwt. 3 qrs. 9 lbs. at R25. 12*a.* 7*p.* per cwt.

4. If a man, walking at the rate of $3\frac{1}{2}$ miles an hour, walks to a place in 4 hours 20 minutes, how long will it take a man, walking at the rate of $3\frac{1}{4}$ miles an hour, to walk there and back?

5. A man invests a certain sum in $4\frac{1}{2}$ per cent. Government Paper at 104. The price falling to 101, he sells out and loses Rs. 600 in the transaction, exclusive of brokerage. Find the sum invested.

6. *A* gives *B* 10 yards start and *C* 15 yards in a race of 100 yards; how much should *B* give *C* in 150 yards?

1892.

1. Simplify $\dfrac{3\frac{5}{9}-1\frac{7}{9}\text{ of }\frac{24}{35}}{11\frac{2}{3}\text{ of }\frac{9}{14}\text{ of }\frac{2}{5}}-\dfrac{4\frac{1}{3}-7\frac{5}{9}+3\frac{2}{5}}{\frac{5}{6}\text{ of }12}$,

2. Find, to the nearest integer, the value of $\dfrac{39·37 \times 760 \times 13·596}{1·293 \times 12}$.

3. Find the square roots of ·097344, of ·009604, and of ·996004.

4. Find the interest on 10 lakhs of rupees for 10 days at $4\frac{1}{2}$ per cent. per annum.

4. £3000, which I had in the four per cents. was sold for me when they were at $82\frac{5}{8}$ by a broker whose commission is $\frac{1}{8}$ per cent.; and the proceeds were reinvested by him in the four and a half per cents. at $98\frac{3}{8}$. What amount of the latter stock did he purchase?

1893.

1. Simplify :—

(1) $1+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$;

(2) $\dfrac{8\frac{1}{7}-1\frac{1}{6}}{\frac{3}{8}+1\frac{1}{2}}-\dfrac{1}{5\frac{3}{4}-1\frac{1}{5}}$.

2. Divide 1.84626 by 23·4.

Express $·\dot{4}5\dot{6}$ and $·\dot{6}5\dot{4}$ as vulgar fractions reduced to their lowest terms, and their sum as a circulating decimal.

3. Find the cost of 73 cwt. 3 qrs. 14 lbs. at £4. 13*s.* 6*d.* per cwt.

4. Distinguish between true discount and banker's discount.

Find the the former in the case of a bill for R3486. *6a.* *8p.* due 16 months hence, the rate of interest being $5\frac{1}{4}$ per cent. per annum.

5. A man invests R163000, part in Government 4 per cent. stock at 108, and the remainder in Municipal 5 per cent. debenture stock at $109\frac{1}{4}$. Find how much he must invest in each in order that he may have an equal income from the two sources.

1894.

1. In a compound metal containing tin and copper only, the proportion of tin to copper is 7·75 to 92·25. Find to the nearest penny the value of 8 cwt. 3 qrs. of it. Tin costs 140*l.*; copper 80*l.* per ton.

2. A rectangular court is 50 yards long and 30 yards broad. It has paths joining the middle points of the opposite sides of 6 feet in breadth and also paths of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 1*s.* 8*d.* per square foot and the turf 3*s.* per square yard, find the cost of laying out the court.

3. Find the value of $\cdot 26\dot{7}187\dot{5}$ of £3 in shillings, pence, and decimal of a penny.

4. Find the square root of $1-(\cdot 0678)^3$ to four places of decimals.

5. At a cricket match, a contractor provided luncheon for 24, and fixed the price to gain $12\frac{1}{2}$ per cent. on his outlay. Three persons were absent. The remaining 21 paid the fixed price, and the contractor lost 2 rupees. What was the charge?

1895.

1. Find the square root of $1+\frac{1}{3}(\cdot 0345)^3$ correctly to four places of decimals.

2. Find the sum of money which put out at simple interest at $2\frac{7}{8}$ per cent. per annum will in 134 days exactly produce R124. *10a.* 1[illegible]*p.*

[A year contains 365 days.]

3. If one pound sterling be worth twenty-five francs and sixty centimes; and also worth six thalers and twenty silber groschen; how many francs and centimes is one thaler worth?

[N. B. one thaler = 30 silber groschen.
one franc = 100 centimes.]

4. Simplify

$$\frac{1\frac{1}{4}-\frac{6}{17}}{1\frac{1}{4}+\frac{6}{17}}+\frac{7}{6} \text{ of } \frac{9\times 5}{14\times 3}-\frac{11\frac{1}{4}}{15}.$$

5. I invest R12805 in the four per cents. at $98\frac{1}{4}$, and when they have risen to $102\frac{5}{8}$ I sell out and invest in the $4\frac{1}{2}$ per cents. at $105\frac{5}{8}$: what is the change in my income? (Brokerage $\frac{1}{8}$ per cent. on all transactions).

Or convert [illegible] into a decimal fraction, pointing out accurately the recurring portion (if any).

ENTRANCE EXAMINATION PAPERS.

BOMBAY.

1859.

1. What is the value of a chest of tea weighing 2 cwt. 1 qr. 19 lbs. at R2. per lb. ?

2. What is the price of a silver cup weighing 1 lb. 7 oz. 13 dwts. at R2. 8*a*. per oz. for the metal, and $7\frac{1}{2}a$. per oz. for the workmanship?

3. Define a fraction; and explain the effect on the value of a fraction of adding the same number to the numerator and the denominator. Why do you bring fractions to the same common denominator before adding them together?

Add together $\frac{5}{8}$, $7\frac{5}{8}$, and $\frac{1}{2}$ of $\frac{3}{4}$.

4. Divide $\dfrac{48\frac{4}{7}}{1085\frac{7}{10}}$ by $\dfrac{7\frac{3}{11}}{174\frac{6}{17}}$.

5. Express $\frac{5}{6}$ of $1\frac{1}{4}$ rupees as the fraction of $\frac{1}{2}$ rupee.

6. A general after sending $\frac{1}{3}$ of his men to forage in one direction, and $\frac{1}{6}$ of them in another, had 700 remaining. How many did he command?

7. If 72 men can do a certain piece of work in 63 days, how long will it take 42 men to do the same?

8. Define a decimal, and reduce 14 minutes to the decimal of a day.

9. Extract the square roots of 2·5, ·0625, and 1020304030201.

10. The top of a tank is a rectangle, whose sides are 9 feet and 15 feet; it is of the same horizontal section throughout its depth. What must be its depth in order that it may contain 12960 gallons of water, one gallon containing 277·274 cubic inches?

11. Find the interest on R100000 for four years, at 3 per cent. compound interest.

12. The sum of R6000 is to be divided among 24 men, 36 women and 72 children, so that the shares of 2 men shall be equal to those of three women, and each woman's share to the shares of 2 children. What will be the share of each?

1860.

1. State the distinction between Direct Proportion and Inverse Proportion; and find how much land, at 27*s*. per acre should be given in exchange for 480 acres, at 35*s*. per acre.

2. Find the greatest number which is contained exactly in 378, 462 and 693; and find the least fraction which, added to the sum of $\frac{3}{4}$, $\frac{7}{8}$ and $\frac{11}{12}$, shall make the result an integer.

3. In multiplication of decimals how do you determine the position of the decimal point in the product? State the reason of the rule.

4. Simplify $(\cdot 18 + \cdot 009) \div \cdot 016$; and reduce £1. 13*s.* $6\frac{3}{8}$*d.* to the decimal of £3.

5. Divide ·00432 by 240 and 43200 by ·024; and extract the square root of ·0002359296.

6. Of what sum of money does the half exceed the fifth part by £216?

7. *A* buys 200 shares in the G. I. P. Railway at R1000 each, and when they are paying 2 per cent., sells them at R460 each, and invests the proceeds in the $4\frac{1}{2}$ per cent. Government loan at 92. Find the effect on his income.

8. Find the value of 537 articles at R3. 7*a.* $2\frac{1}{2}$*p.* each (by Practice).

9. What will 3650 rupees amount to in 4 years and 2 months at R3. 6*a.* 8*p.* per cent. per annum at simple interest? In what time would a sum of money double itself at the above rate?

10. If a cubic foot of marble weigh 2·716 times as much as a cubic foot of water, find the weight of a block of marble 9 ft. 6 in. long, 2 ft. 3 in. broad and 2 ft. thick, supposing a cubic foot of water to weigh 1000 oz.

11. The surface of a cube is 346·56 square feet; what is the length of an edge?

1861.

1. Express in words 11603700160, and write in Roman numerals 4960 and 10684.

2. Multiply R1875. 13*a.* 8*p.* by 27, and find how many times R1. 0*a.* $4\frac{1}{2}$*p.* are contained in R2. 12*a.* $3\frac{1}{2}$*p.*

3. If 4 candies 2 maunds 7 seers of sugar cost R36. 3*a.*, what is the price of 3 maunds 14 seers?

4. A bankrupt pays 17*s.* 6*d.* in the pound; how much does he pay in R267. 6*a.* 8*p.*? (Practice).

5. Reduce $\dfrac{3\frac{1}{2} - 2\frac{1}{6}}{\frac{1}{2} \text{ of } (\frac{1}{6} + \frac{1}{7})} \div 15\frac{1}{4}$ to its simplest form.

Find a sum of money which shall be the same fraction of R61. 9*a.* 1*p.* that 2 cwt. 2 qrs. 10 lbs. is of 36 cwt. 1 qr.

6. Reduce 13*a.* $6\frac{3}{4}$*p.* to the decimal of (1) R1, (2) R1000 and (3) R·00001.

Divide 1255 by 1·004 and hence deduce the quotient of 12·55 by 1004 and ·012550 by 1004000.

7. How is it that the value of a decimal fraction is not altered by adding on the right hand any number of ciphers?

8. What sum must *A* bequeath to *B* so that *B* may receive R10000 clear, after deducting a legacy duty of 10 per cent.?

9. Find the simple and compound interest of £625 in 2 years at 4 per cent.

1862.

1, What is the fundamental principle in our system of Arithmetic? Write the number three millions four hundred and fifty two thousand one hundred and sixty-seven in an algebraical form, using x to denote ten.

How would the Romans have written the numbers which are expressed in our notation by 1948, 1231, 1262, 1862?

2. Divide 31 by $\cdot124869$ and $\cdot124869$ by 31. Give the *reason* of the rule for placing the decimal point in the quotient.

3. The Hindu year consists of 365 days 6 hours $12\frac{1}{2}$ minutes, the Mahomedan of 354 days 8 hours 48 minutes. After what length of time would the accumulated difference between them amount to the tropical year of 365 days 5 hours 48 minutes $49\cdot7$ seconds?

4. A bag contains a certain number of rupees, half as many again two-anna pieces, and 4 times as many pice, and the value of the whole is R300: find how many rupees, how many two-anna pieces and how many pice are there?

5. How many times does $(\frac{2}{3}+\frac{3}{8}-\frac{1}{18})$ contain $(\frac{2}{7}+\frac{1}{3}-\frac{1}{12})$?

6. What decimal of 1 bushel 1 pint is $\frac{2}{3}$rds of 3 gallons 2 pints?

7. Divide accurately $\cdot0\dot{3}2\dot{4}$ by $\cdot\dot{3}\dot{6}$ and extract the square root of the quotient to four figures.

8. A creditor receives upon a debt of R3270 a dividend of 9 annas 2 pies in the rupee, and afterwards he receives a further dividend upon the deficiency of 3 annas 4 pies in the rupee; how much does he receive on the whole?

9. Extract the cube root of $\frac{5}{27}$ to three places of decimals.

10. Find the true present value of two sums of R100 payable at the end of one year and two years respectively, money making $7\frac{1}{2}$ per cent. per annum.

11. If mangoes be bought at the rate of seven for an anna, how must they be sold to gain 33 per cent?

12. Four French feet are equal to $1\cdot3$ metres, and 15 French feet are equal to 16 English feet; how many metres are 27 English feet equivalent to?

1863.

1. Explain the principle of the Decimal System of Numeration. Write down in words the number 4010010. What number expressed in the Decimal System, is identical with the number 4321, in which the base of the system of numeration is 12?

2. Divide R6148. 5*a*. 4*p*. by 136.

3. *A* barters some sugar, with *B* for flour which is worth 2*s*. 3*d*. per stone, but uses a false stone-weight of $13\frac{1}{2}$ lbs.; what value should *B* set upon his flour, that the exchange may be fair?

4. An annual tax of R2255 is laid upon a district containing four villages—A, B, C, D,—and the rate to be paid by each of the villages A, B, and C, is to the rate to be paid by D, as 3 to 2: what are the annual payments due from the villages?

5. Explain the following terms—*an improper fraction*, *a compound fraction*, *a mixed number*. Add together $\frac{1}{4}$ of $\frac{3}{7}$ of a year, $\frac{5}{8}$ of $\frac{6}{8}$ of a day and $\frac{7}{8}$ of $\frac{3}{8}$ of $19\frac{1}{2}$ hours.

6. Divide ·00333822 by ·1357. Reduce 18*s*. $9\frac{3}{4}$*d*. to the decimal of one pound.

7. The area of the entire surface of a pond is 9 acres 2 roods 15 poles; find to 3 places of decimals, the number of yards in the side of a square piece of ground of equal area.

8. A man sells a horse for R246 and loses $26\frac{1}{6}$ per cent. on what the horse cost him: what was the original cost?

9. Explain the difference between *interest* and *discount*; and find the discount on £397. 6*s*. 8*d*. due 9 months hence, at 4 per cent. per annum.

10. If the carriage of 150 feet of wood, that weighs 3 stones per foot, cost R30 for 40 miles, how much will the carriage of 54 feet of wood, that weighs 8 stones per foot, cost for 25 miles?

1864.

1. Express in figures the following distances in miles of some of the planets from the sun:—

Thirty-seven millions (for Mercury).

Sixty-nine millions (for Venus).

Four hundred and ninety-four millions (for Jupiter).

Write down in words the numbers signified by the following figures:—

900300804, 60660608008.

2. Find the greatest common measure of the numbers 12129 and 30081. Investigate whether the numbers 3794 and 1815 have a common measure or not.

3. Express in the scale of 8, the number seven hundred and eighty-four millions three thousand and forty-two.

4. To the sum, difference, and product of $\frac{7}{8}$ and $\frac{9}{11}$ find a fourth proportional.

5. Find the sum, difference, product and ratio of the decimal numbers 4075·32, and 186·4215. Demonstrate the rule for pointing the quotient in the division of decimal fractions.

6. The proportions used in making English gunpowder are saltpetre 75 parts, sulphur 10 parts, and charcoal 15 parts. How many pounds weight of each material are there in 10 cwt. of gunpowder

7. Extract the square root of 115·297356.

8. *A*, *B* and *C* form a Joint stock of R75000 of which R36000 are contributed by *A*, R30000 by *B*, and the remainder by *C*. At the end of the year, the profit is found to be R16791. Required the shares of this which each is to receive, R800 a month being allowed as salary to *C* as acting partner.

9. Calculate the interest on 4 lakhs of rupees from the 23rd November 1864 to the 25th May 1865 at 8 per cent. per annum.

10. If 12 iron bars, each 4 feet long, 3 inches broad and 2 inches thick, weigh 576 lbs., how much will 11 weigh, each 6 feet long, 4 inches broad and 3 inches thick?

1865.

1. Point and write in words, both according to the English and Indian numerations, the two numbers :—

1234567654321.
5020040003060.

2. Subtract R45867. 12*a*. 6*p*. from R86325. 8*a*. 3*p*. How are the numbers placed in subtraction?

3. If a room is 28 feet long, 20 feet wide, 13 feet high, and the windows and doors take up half the walls, find the cost of papering at 12*a*. a square yard.

4. How many square feet are there in 578 pieces of Grey Domestics 39 inches wide and 72 yards long? And what is the price at R20. 14*a*. per piece?

5. Multiply $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ by $1\frac{1}{2}\times 1\frac{1}{3}\times 1\frac{1}{4}\times 1\frac{1}{5}\times 1\frac{1}{6}$.

6. Reduce $\frac{1}{30}$, $\frac{1}{31}$, $\frac{1}{32}$ to decimals.

7. If I sell R500, 4 per cents. at 93, and buy $5\frac{1}{2}$ per cents. at 109, what is the change in my income?

8. Divide a lakh of rupees between *A*, *B* and *C*, in the proportion of 2, 3, 4, and the same amount between *D*, *E* and *F* in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

9. If I sell 40 shares of R250 each in the Oriental Bank at 121 per cent. premium, how many shares of R1000 each in the Madras Bank at 72 per cent. premium can I buy? and how much will be left?

10. A person travelled 120 miles by railway at 15 miles an hour, 120 by road at 8 miles an hour and 60 by bullock-cart at 2 miles an hour: how long did he take?

11. Find the square root of 173388·96 and the cube root of 1860·867.

1866.

1. Represent in figures :—

Ninety-nine millions, ninety-nine thousand and ninety-nine. And by the old English method of numeration, eight billions, two hundred and seven thousand and five.

Point and write in words 319680209078 and 20090060002, the first according to the Indian method and the second according to the English method of Numeration.

2. Add together $\frac{3}{4}$ of $\frac{5}{8}$ and $\frac{7}{4}+\frac{11}{12}-\frac{19}{88}$, and explain why fractions must be reduced to a common denominator for the purpose of Addition and Subtraction.

(*a*) What fraction must be divided by $\frac{3}{5}$ to give a quotient $\frac{11}{12}$?

3. A person who has $\frac{5}{8}$ of a mine sells $\frac{3}{4}$ of his share for ₨1,500; what is the value of his share and of the whole mine?

4. Explain why in reducing a fraction to a terminating decimal, the number of decimal places depends on the form of the denominator of the fraction and not on that of the numerator.

5. Reduce 1 cwt. 3 qrs. 5 lbs. to the decimal of $\frac{3}{5}$ of a ton.

6. Perform the operations indicated below:—

(i) $47{\cdot}03 - 2{\cdot}876843$.

(ii) $5{\cdot}776 \times 2{\cdot}003$.

(iii) $62{\cdot}5 \div 125{\cdot}125$.

(iv) $6{\cdot}25 \div {\cdot}000125$.

(v) $\sqrt{(2119{\cdot}6816)}$.

7. Define the terms, Stocks, shares, consols. State some of the circumstances which affect their value in the market.

How much stock can be purchased by the transfer of ₨2000000 from the 4 per cents. at 90, to the $5\frac{1}{2}$ per cents. at 110; and what change would be effected in the income derived from the two investments?

8. Find, by Practice, the price of 549 yards at 18*s*. $9\frac{1}{2}$*d*. a yd.

9. I bought cloth at 15*a*. a yard and lost 5 per cent. in selling; what was it sold for?

10. If a person owe ₨100 payable in 2 months, and ₨750 payable in 7 months, what is the just time for the payment of the two debts?

1867.

1. Give a demonstrative example, illustrative of the following axiom:— If the divisor be increased a certain number of times, the quotient is diminished in the same degree; but if the divisor be diminished, the quotient is increased.

2. Define *prime* and *composite* numbers. Resolve 54180 into prime factors.

3. Reduce $\dfrac{26\frac{3}{7}-1\frac{1}{21}}{\frac{4}{5}+\frac{6}{5}-\frac{1}{3}}$ to its simplest form.

4. Reduce $\frac{6}{7}$ to a circulating decimal; and find the fraction equivalent to $1.70\dot{1}\dot{6}$.

5. Find the product by contracted multiplication of 72·49 and 10·87632 to three places of decimals.

6. If $\frac{3}{8}$ of a maund of sugar cost R10, what will $\frac{3}{8}$ of a seer cost at the same rate? Give the answer in annas as well as in rupees.

7. Explain *direct* and *inverse* proportions.

8. 250 men are employed to work on a Railway embankment, a mile and a half long, which they are expected to finish in four weeks. But at the end of one week it is found that they have only finished 520 yards. How many more men must be engaged to finish it in the required time?

9. What time must elapse between the time of placing R250 in the Government Savings' Bank and taking out the amount just as it goes over R300, supposing interest at 5 per cent. per annum, compound interest?

10. In a school of 250 children, 44 per cent. are learning Geography, 36 per cent. are learning Grammar, 12 per cent. cannot read, and 4 per cent. have advanced as far as Algebra. What are the actual numbers of each?

11. Extract the square roots of 6085, ·00025 and $\frac{7 \cdot 98}{52 \cdot 4}$.

12. What is the cost of a marble slab, 6 ft. 3 in. long, 2 ft. 8 in. broad, and 4 in. thick at R7. 8*a.* per cubic foot?

What is the weight of the slab, one cubic foot weighing 170 lbs.?

1868.

1. How many yards of matting 2 feet 3 inches wide will be required for a square room whose side is 18 feet 9 inches?

2. What will be the cost of a Bill of Exchange on London for £1364. 14*s.* 6*d.* at 1*s.* 10½*d.* per rupee?

3. Reduce $\frac{\frac{9}{10} \times 1\frac{1}{3}}{3 - \frac{1}{3}} \times (\frac{1}{3} \times \frac{1}{6})$ to its simplest form.

4. What is the difference between $\cdot\dot{0}\dot{7}$ and $\cdot 0\dot{7}$?

5. If an ounce of gold be worth £$4 \cdot 1895\dot{3}$, what is the value of $\cdot\dot{0}3753$ lbs.?

6. If *A* owns ·24 of a ship, and *B* the rest, and the difference in the value of their shares is £28·76, what is the value of the whole ship?

7. What sum must be invested in 5½ per cent. Promissory Notes to produce a monthly income of R350?

8. At what rate per cent. would R17,200 amount to R18,650 in 5 years?

9. There are two schools, one containing 650 boys and the other 340 boys; 5 per cent. of the former are generally absent and 7·5 of the latter; what is the average attendance in each?

10 If 8 per cent. be gained by selling 218 yards of cloth for £92. 13*s.*, at what price per yard must it be sold so as to gain 17 per cent.?

11. If 400 men could do a piece of work in 3·4 days, how many men would do $\frac{1}{4}$ of the same work in 15 days?

12. What is the value of a beam of timber whose length is 30 feet, breadth $3\frac{1}{2}$ feet, and thickness $2\frac{1}{4}$ feet, at 3*s.* 9*d.* per cubic foot?

13. Find the cube root of $\cdot\dot{4}$.

1869.

1. Find the G. C. M. of 2231 and 4656; and the L. C. M. of 4, 9. 16, 28, 42.

2. Add together $\frac{3}{8}$, $\frac{1}{12}$, $\frac{1}{21}$, $\frac{11}{24}$, $\frac{9}{128}$.

3. Find the value of :—

$$\frac{1}{3\frac{1}{8}} + \frac{2\frac{1}{4}}{9} + \frac{3\frac{5}{8}}{2} + \frac{\frac{4}{7}}{4\frac{4}{7}}.$$

4. Convert into vulgar fractions the decimals ·015625 and $\cdot 01\dot{1}9047\dot{6}$ and reduce the results to their lowest terms.

5. Reduce R6. $7\frac{5}{8}a.$ to the decimal of R10.

6. Divide the sum of R3281. $12\frac{1}{3}a.$ among 4 persons in the proportion of 3, 5, 8, 9.

7. If £442 amount to £530. 8*s.* in 5 years, what is the rate per cent. of simple interest?

8. Find the amount of £1000 in 6 years, at 5 per cent. compound interest.

9. If 27 men take 15 days to mow 225 acres of grass, how long will 33 men take to mow 165 acres?

10. A person has R100000 stock in Government 4 per cents.; he sells out all his stock at $92\frac{1}{4}$; he then reinvests the purchase money in Bank of Bombay Shares of R500 each, at R625, which pay 6 per cent. per annum; find the alteration in his income.

11. Find the square roots of 3129361 and $434\cdot 02\dot{7}$.

12. Show that the cube root of $\cdot\dot{0}3\dot{7}$ is $\cdot\dot{3}$.

1870.

1. Write down in figures the following :—

Six hundred and fifty-four thousand three hundred and twenty-three billions, four thousand and twenty-one millions, fifty thousand three hundred and one.

Express in words the number 1327875430029 according to the English and Hindu systems of numeration.

2. Find the value of $3\frac{2}{5}+4\frac{1}{8}+1\frac{11}{16}+3\frac{11}{125}$ both by vulgar fractions and decimals, and show that the results coincide.

3. Divide the difference of $7\frac{1}{3}$ and $9\frac{2}{3}$ by their sum, and multiply the quotient by $\frac{5}{8}$ of $7\frac{1}{2}$.

4. If an ounce of gold be worth £$4\cdot\dot{0}09\dot{9}$, what is the value of a bar of gold weighing $1\cdot\dot{6}83$ lbs. ?

5. If a family of 9 persons spend R4800 in 8 months, how much will serve a family (living upon the same scale) of 24 persons for 16 months ?

6. Three equal glasses are filled with a mixture of spirit and water ; the proportion of spirit to water in each glass is as follows : in the first glass as 2 : 3, in the second glass as 3 : 4, and in the third as 4 : 5. The contents of the three glasses are emptied into a single vessel ; what is the proportion of spirit and water in it ?

7. What are the weights of a sovereign and a shilling, the pound Troy of standard gold being coined into £46. 14*s*. 6*d*., and the pound of silver into 66 shillings ?

8. Find the interest on £215. 12*s*. for 3 years 8 months and 10 days at $4\frac{1}{2}$ per cent. per annum.

9. A ship worth R9000 being entirely lost, of which one-fourth belonged to *A*, one-sixth to *B*, and the remainder to *C*, what loss will each sustain, supposing R5400 of the ship were insured ?

10. Extract the square roots to 6 places of decimals of ·099 and of 3·3.

11. How much stock in the 3 per cents. must I sell to pay off a debt of £550, the price of the stock being $94\frac{1}{8}$, and commission of $\frac{1}{8}$ on £100 of stock being also taken into consideration ?

1871.

1. The distance of the sun from the earth is ninety-one millions seven hundred and seventy-six thousand miles, and light travels from the former to the latter in seven minutes and fifty-eight seconds ; find the velocity of light per second.

2. Find the G. C. M. of 441441 and 844372 and the L. C. M. of 7, 11, 21, 63, 91, 99, 117, 143.

3. Define a fraction, and prove that the value of a fraction is not altered if we multiply both its numerator and denominator by the same whole number.

Bring $\left\{\left(\frac{5\frac{1}{4}-\frac{1}{3}\text{ of }2\frac{5}{6}}{\frac{3}{8}\times 4\frac{1}{6}+\frac{1}{12}}+\frac{2\frac{3}{7}}{4\frac{3}{7}}\right)\div 21\frac{3}{16}\times 3\frac{19}{20}\right\}$ cwt. to the fraction of $4\frac{1}{7}$ tons.

4. State and prove the rules for reducing terminating and circulating decimals into their equivalent vulgar fractions.

Ex. $\cdot 03125$ and $\cdot\dot{7}2\dot{9}$.

Find the value of $\cdot 03125$ of R2 + $\cdot\dot{7}2\dot{9}$ of R$3\frac{1}{9}$ + $\cdot\dot{7}2\dot{9}$ of R$4\frac{3}{8}$.

5. If 10 horses and 98 sheep can be kept 9 days for £37. 17*s.* 6*d.*, what sum will keep 45 horses and 216 sheep for 40 days supposing 5 horses to eat as much as 76 sheep ?

6. If the par of exchange be two English shillings for the Indian rupee, but if an Indian bill of exchange for R540. 12*a.* be negotiated in London for £51. 10*s.*, how much per cent. below par is the rate of exchange ?

7. Distinguish between interest and discount. The interest on a certain sum of money for three years is R825, and the discount for the same time is R645, simple interest being reckoned in both cases. Find the rate per cent. per annum and the sum.

8. A person desires to paper his room with postage stamps ; the room is 14 feet 9 inches long, 9 feet 3 inches broad and 10 feet 6 inches high ; it contains two windows, each $5\frac{1}{2}$ feet by 4 feet and 3 doors each 6 feet by 3 feet ; a postage stamp is $\frac{13}{16}$ inch long and $\frac{3}{4}$ inch broad. Find the number of postage stamps required to cover the room.

9. A person invests 1,250 gold mohurs in the Government five per cent. rupee stock at 105. The stock is converted subsequently to $4\frac{1}{2}$ per cents. at 95. Find the difference in his income, each gold mohur being considered equivalent to R17.

10. A certain number of persons agree to subscribe as many pies each as there are subscribers ; the whole subscription being R5,797. 0*a.* 1*p.* How many subscribers were there ?

1873.

. Simplify :—

$$\frac{\frac{310}{166488} - \frac{91}{1303}}{1 + \frac{1}{1303} \times \frac{810}{166488}}$$

2. Find the value of $\cdot 375$ of a guinea + $\cdot\dot{5}\dot{4}$ of 8*s.* 3*d.* + $\cdot\dot{0}2\dot{7}$ of £2. 15*s.*, and reduce the result to the fraction of a guinea and a half.

3. A mans owns $\frac{7}{8}$ of a ship and sells $\cdot\dot{3}57142\dot{8}$ of his share ; what fraction of the ship does he still own ?

4. If the income-tax be 6 pies in the rupee for the first half of the year and 3 per cent. in the second, what is the gross income of a gentleman whose net annual receipts amount to R1,454. 1*a.* ?

5. Five men do $\cdot 60\dot{0}6$ of a piece of work in 2·12 hours, how long will 6 boys take to finish it, it being known that 3 men and 7 boys have done the whole piece of work in 3 hours ?

6. If the difference between the simple and compound interest of a sum of money for 2 years at 5 per cent. be £5. 18*s*. 9¾*d*., find the sum.

7. When the three per cents. were at 90, I found that by selling out and investing in the 4 per cents. at 95 I could improve my income by ₨243. What was the amount of my stock in the three per cents. ?

8. A gardener plants an orchard with 5776 trees and arranges them so that the number of rows of trees equals the number of trees in each row. How many rows were there ?

9. How many seconds will a train 184 feet in length, travelling at the rate of 21 miles an hour, take in passing another train 223 feet long, proceeding in the same direction at the rate of 16 miles an hour ?

10. Find the cube root of 1879080904.

1874.

1. Simplify the fraction :—

$$\frac{\frac{1}{3}+\frac{2}{11}+\frac{7}{24}-\frac{1}{3}\text{ of }\frac{2}{11}\text{ of }\frac{7}{24}}{1-\frac{1}{3}\text{ of }\frac{2}{11}-\frac{2}{11}\text{ of }\frac{7}{24}-\frac{7}{24}\text{ of }\frac{1}{3}}$$

2. Divide $8{\cdot}064$ by $\{ \cdot 84\dot{6}+\frac{30}{7}\text{ of }\cdot 291\dot{6} \}$.

3. A man owns $\frac{8}{14}$ of a house, and sells $\cdot 1\dot{3}5\dot{1}$ of his share ; what fraction of the house does he still own ?

4. In a subscription list one-half of the subscription are a guinea each, one-third a half-guinea each, and the 5 shilling subscriptions which complete the list amount to £12 ; find the whole amount subscribed.

5. If the work done by a man, a woman and a child be in the ratio of 3, 2, 1, and there be in a factory 24 men, 20 women and 16 children, whose weekly wages amount to ₨204 ; what will be the yearly wages of 27 men, 40 women, and 15 children ?

6. The debts of a bankrupt amount to £2134. 10*s*. 6*d*., and his assets consist of property worth £916. 19*s*. 4*d*., and an undiscounted bill of £513 due 4 months hence, simple interest being reckoned at 4 per cent. How much in the pound can he pay his creditors ?

7. A merchant buys 4,000 maunds of rice, one-fifth of which he sells at a gain of five per cent., one-fourth at a gain of ten per cent., one-half at a gain of twelve per cent., and the remainder at a gain of sixteen per cent. If he had sold the whole at a gain of eleven per cent., he would have made ₨728 more. What was the cost of the rice per maund ?

8. The shares in a banking concern are ₨1000 each, ₨426. 10⅔*a*. are only paid up, and the shares are quoted in the market at ₨460. The dividend is ₨7½ per share quarterly. A gentleman holds 100 original shares. Find what interest he makes per cent ; and what he would make and how much per cent., if he sold out and invested in 4 per cent. Government stock at par.

9. *A* and *B* are the termini of a Railway 144 miles long. *A* fast train starts from *B* at 9 h. 0 m.; another fast train, travelling at the same rate, starts from *A* at 10 h. 0 m. A slow train starts from *B* at 10 h. 20 m.; the fast train from *A* meets the other fast train at 11 h. 30 m., and the slow train at 12 h. 32 m.; find the rates at which the trains travelled.

10. Arrange in order of magnitude :—

$$\sqrt{(50)}, \quad \sqrt[3]{(344)}, \quad \sqrt[4]{(2402)}.$$

1875.

1. Write out in words the following expressions :—

(*a*) 8271096.

(*b*) 9032804.

(*c*) 319080259417.

(*d*) 8004640.

2. What is the rule for the addition of concrete numbers? Add together 17 miles, 3 furlongs, 19 poles, 28 yards, 2 feet, 10 inches; 4 miles, 3 furlongs, 8 poles, 7 yards, 2 feet and 9 inches.

3. Explain what is meant by the following words and give examples :—

Measure, Multiple, Greatest Common Measure, and Least Common Multiple.

4. How many acres are contained in three countries, of which the first comprises 723100 square miles, the second 12342, and the third 89704 square miles?

5. Divide $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{7}$ of 42 by the sum of $2\frac{1}{6}$ and $4\frac{5}{6}$.

6. What are *continued fractions*, and when do you make use of them?

Find three fractions approximating to $\frac{186}{262}$.

7. Find the product of 17·302 and ·579 to three places of decimals, by the rule of contracted Multiplication.

8. What sum will discharge a debt of R7,200 due a year and a hal hence at 4 per cent. per annum?

9. Find the square root of 745·29 and the cube root of 32768.

10. Divide a guinea between *A*, *B*, *C*, *D*, so that *B*'s share is $\frac{1}{3}$ more than *A*'s, *C*'s $\frac{1}{3}$ more than *B*'s and *D*'s $\frac{1}{3}$ more than *C*'s.

11. How much stock can be purchased by the transfer of R20000 stock from the 3 per cents. at 90 to the $3\frac{1}{2}$ per cents. at 96; and what change will be effected in income by it?

12. Required the number of square feet there are in a piece of slate $2\frac{1}{3}$ feet $\frac{1}{4}$ in. in length, and $1\frac{1}{4}$ feet $\frac{1}{2}$ in. in width.

1877.

1. Define the arithmetical terms :—*notation*, *numeration*, *unit*, *integer*, *fraction*, *abstract*, *concrete*. Can you (1) multiply concrete numbers together ? (2) divide a concrete number by a concrete number ? Give examples to illustrate the nature of such operations.

2. Two men *A* and *B* start together, and when *A* has gone a mile, *B* has gone $\frac{8}{20}$ of $1\frac{2}{3}$ of $\frac{1\frac{1}{5}}{5\frac{2}{3}}$ of $\frac{\frac{2}{5}+\frac{1}{2}}{\frac{3}{2}-\frac{1}{3}}$ of $71\frac{3}{7}$ of $\dfrac{\frac{5}{1-\frac{1}{27}}+\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{7}\text{ of }\left\{\frac{\frac{2}{5}}{1-\frac{2}{5}}+\frac{1}{3}\right\}}$ of a mile : which is in advance of the other ?

3. Express the difference between $\cdot\dot{3}7\dot{8}$ of 13*s.* $10\frac{1}{2}$*d.* and $\cdot\dot{3}7\dot{8}$ of 16*s.* 6*d.* as a fraction of

$$\cdot 42\dot{6} \text{ of } \frac{3\cdot\dot{3}}{\cdot 08} \text{ of } \frac{\cdot 3}{\cdot 73\dot{5}} \text{ of } \frac{\cdot 147\times 4\cdot\dot{4}}{11\cdot 1} \text{ of } £1.\ 17s.\ 6d.$$

4. A lb. of tea and 3 lbs. of sugar cost R3, but if sugar rose 50 per cent. and tea 10 per cent., they would cost R3. 8*a.* ; find the prices per lb. of tea and sugar.

5. The circumferences of the wheels of a carriage are $6\frac{9}{14}$ feet and $8\frac{1}{18}$ feet : what is the *least* distance in which both wheels will *simultaneously* complete an integral number of revolutions ? How often will the lowest points of the two wheels at starting touch the ground together in 10 miles ?

6. *A*, *B* and *C* rent a field for R2,878. *A* puts in 12 horses for 5 months and 45 sheep for 3 months ; *B* puts in 15 oxen for 6 months and 54 sheep for two months ; *C* puts in 6 horses and 48 oxen for 3 months. Now, 4 horses and 3 sheep together eat as much as 5 oxen and 1 horse, and 2 oxen eat as much as 7 sheep ; how much of the rent should *A*, *B*, *C*, pay respectively ?

7. What sum of money will amount to 699*l.* 13*s.* 2·4*d.* in 2 years, reckoning compound interest for the first year at 4 per cent. and for the second $3\frac{1}{2}$ per cent. per annum ?

8. A person finds that if he invest a certain sum in railway shares paying £6 per share when the £100 share is at £132, he will obtain £10. 16*s.* a year more for his money than if he invest in 3 per cent. consols at 93. What sum has he to invest ?

9. Find the value of $\sqrt{(.00139876)} - \sqrt[3]{(\cdot 000030664297)}$.

10. A man near the sea-shore sees the flash of a gun fired from a vessel steaming directly towards him, and hears the report in 15.″ He then walks towards the ship at the rate of 3 miles an hour, and sees a second flash 5 minutes after the first, and immediately stops ; the report follows in 10″·5. Find the rate of the ship, the velocity of sound being 1,200 feet per second.

1878.

1. Seven men find a lump of gold weighing 13 lbs. $7\frac{1}{2}$ oz. Troy. What will be each man's share, supposing gold to be worth £3. 17*s*, $10\frac{1}{2}$*d*. per ounce?

2. Simplify :—

$$1\frac{1}{11} - \frac{1-\frac{7}{22}}{2-\frac{1}{3}} + \frac{1\frac{7}{8}}{3\frac{1}{2}} - \frac{5\frac{2}{3}}{6\frac{1}{4}} \text{ of } \left\{\frac{1}{3} - \frac{\frac{1}{2}-\frac{1}{3}}{4\frac{1}{4}-3\frac{3}{5}}\right\}$$

3. Find the value of :—

$$\cdot 3\dot{8}\dot{7} \text{ of } £8.\ 16s\ 3d. + 6\tfrac{1}{2} \text{ of } \tfrac{10}{63} \text{ of } 7s.\ 8\tfrac{3}{4}d. + \tfrac{7}{11} \text{ of } 1d.$$

4. What is the length of the edge of a cubical cistern which contains as much as a rectangular one whose edges are 154 ft. 11 in., 70 ft. 7 in., and 53 ft. 1 in.?

5. In 1861 three towns had populations of 17650, 19600, 18760 respectively. In 1871 the population of the first had decreased 18 per cent., that of the second had increased 21 per cent., while the population of the third had increased by 4690; find the change per cent. in the population of the third town.

6. A bankrupt has goods worth R9750; and had they realised their full value, his creditors would have received 13*a*. in the rupee; but $\frac{2}{5}$ths were sold at 17·5 per cent., and the remainder at 23·75 per cent., below their value. What sum did the goods fetch, and what dividend was paid?

7. What sum will amount to £1,591. 13*s*. 2·16*d*. in 3 years at compound interest; the interest for the first, second and third years being 3, 2 and 1 per cent. respectively?

8. Find the true discount on £2,750 due two years hence at $4\frac{1}{2}$ per cent.

9. If 4 men earn as much in a day as 7 women, and one woman as much as 2 boys, and if 6 men, 10 women and 14 boys working together for 8 days earn £22, what will be the earnings of 8 men and 6 women working together for 10 days?

10. A person having a certain sum of money to invest, finds that an investment in a railway stock bearing five per cent. interest at $117\frac{1}{2}$ will yield him £29 more annually than an investment in the 3 per cents. at $92\frac{1}{4}$. How much money has he to invest?

1879-80.

1. Add the following numbers :—Eighty-four thousand three hundred and one; nine hundred and thirty-three thousand; forty-seven millions six thousand three hundred; and subtract from the result two millions eighty-one thousand and eighty.

2. Explain the terms *measure*, *common measure* and *greatest common measure*, and prove that every common measure of dividend and divisor is a measure of the remainder.

3. Find the value of ·45 of £1. 3*s*. 9*d*. + $\cdot 2\dot{5}\dot{7}$ of £11. 5*s*. 6*d*. + ·3125 of £5.

4. Find the value of $\frac{\frac{7}{10}-\frac{3}{8}}{\frac{4}{15}+\frac{1}{8}} \div \frac{\frac{1}{8}}{9\frac{1}{2}}$ and also of $\frac{1}{3}+\frac{2}{5}-\frac{4}{15}+\frac{4}{15}$.

5. If by selling wine at R6 per gallon I lose 25 per cent., at what price must I sell it to gain 25 per cent. ?

6. A person borrows £130 on the 5th of March, and pays back £132. 10*s*. 6*d*. on the 18th October; find the rate of interest charged.

1880-81.

Simplify the following expressions:—

$$2+\cfrac{1}{5+\cfrac{1}{1}}\ ;\quad \frac{4\cdot 5\times 2\cdot 3}{5\cdot 341\dot{3}}\times 2\frac{253}{875}$$: and add together the results.

2. Three boys agree to start together and run, until all come together again, round a circular court 15 yards in circumference. One runs at the rate of six, the second seven, and the third eight, miles an hour. In how many seconds will the race end ?

3. If three soldiers or 10 coolies can dig 155 cubic feet of earth in 5 days, how many coolies must be employed to assist 7 soldiers in removing 600 cubic feet of earth so as to get it done in 4 days ?

4. In what time will R2,250 amount to R2,565 at 7 per cent. per annum ?

5. A merchant sells a lakh of rupees out of the four per cents. at 16 discount, and invests the proceeds while exchange is at 2*s*. 1*d*. in the three per cent. consols at 96. What income does he derive therefrom ?

1881-82.

1 If the income-tax be 7*d*. in the pound in the first half of the year, and $3\frac{1}{2}$*d*. in the second, what is the net income of a gentleman whose gross annual receipts are £1,542. 10*s*. 6*d*. ?

2. A passenger train going 41 miles an hour, and 431 feet long, overtakes a goods train on a parallel line of rails. The goods train is going 28 miles an hour, and is 713 feet long. How long does the passenger train take in passing the other ?

3. Find the cost of painting the outside of a cubical box whose edge is 3·5 feet, at 1·3 shillings per square yard.

4. A person invests R48,000 in the 4 per cents, at 80, and at the end of each year invests the dividend, which becomes due, in the same stock ; supposing the funds to remain at 80 for 3 years, find his dividend at the end of the third year.

5. Define *Discount*. If the discount on R2,261-5-4 due at the end of a year and a half be R128, what is the rate of interest ?

6. Find the square root of $\frac{\cdot 00125}{\cdot 18}$ and the cube root of 423564·751.

1882-83.

1. Find the value of £·596875, and reduce 11 poles 4 yards $4\frac{1}{2}$ inches to the decimal of one mile.

2. A railway passenger counts the telegraph posts on the line as he passes them. If they are 58 yards apart and the train is going 48 miles per hour, how many will he pass per minute ?

3. Three men can do as much work as five boys ; the wages of three boys are equal to those of two men. A work on which 40 boys and 15 men are employed takes 8 weeks and costs £350 ; how long would it take if 20 boys and 20 men were employed, and how much would it cost ?

4. What sum will amount to £5431. 15s. $11\frac{1}{4}$d. in 6 years at $4\frac{1}{4}$ per cent. simple interest ?

5. The sides of two squares contain 77 yards 1 foot 9 inches and 7 yards 2 feet 4 inches respectively ; find the side of a square whose area is equal to the sum of the areas of the two squares.

1883-84.

1. (*a*) Express in figures:—Sixteen billions, seventy-five millions, forty thousand and two.

(*b*) Simplify the expression—

$$\left(\frac{1\frac{6}{17}-\frac{6}{7}}{1\frac{3}{8}+\frac{3}{10}}\right)\div\left(\frac{\frac{9}{11}-\frac{2}{13}}{1\frac{1}{3}+\frac{9}{23}}\right).$$

(*c*) Find the value of:—3·75 of 5*s.* 6*d.* + 5·05 of £3. 1*s.* 8*d.* + 5·07 of ·7*s.* 6*d.* + 3·135 of £2. 1*s.* 3*d.*

2. At the examination of a school $\frac{1}{16}$ of the children were presented in the 6th standard, $\frac{1}{8}$ in the 5th standard, $\frac{1}{3}$ in the 4th, $\frac{1}{6}$ in the 3rd, $\frac{1}{4}$ in the 2nd, and the remainder 107 in the 1st standard ; how many were presented altogether, and how many in each of the other standards ?

3. In a bicycle race of two miles over a circular course of 1 furlong, the winner in his last round overtook the second at a point in his fifteenth round. Their paces were as 159 to 149. At what distance was this point from the winning post ?

4. Find the expenses of an excursion, which includes 5782 miles of railway at $\frac{3}{4}$*d.* per mile, 517 miles of carriage at $10\frac{1}{2}$*d.* per mile, 57 days of hotel keep at 14*s.* 3*d.* per day, allowing 5 guineas for extras.

5. Divide $1 \cdot 04$ by $\cdot 000078125$ and prove your result by vulgar fractions. Find the square root of $8658 \cdot 3025$ and the cube root of $753 \cdot 571$.

1884-85.

1. Reduce to a vulgar fraction $\cdot \dot{4}2857\dot{1}$. Divide $301 \cdot 6$ by 416. Find the value of $\cdot 475$ of £1 + $\cdot 42$ of £2. 17*s.* 9*d.*

2. A merchant buys 1260 maunds of corn, one-fifth of which he sells at a gain of 5 per cent., one-third at a gain of 8 per cent., and the remainder at a gain of 12 per cent. If he had sold the whole at a gain of 10 per cent., he would have obtained £22. 13*s.* more. What was the cost price per maund?

3. A room, 10 ft. 6 in. high, 22 ft. long and 14 ft. broad, is painted up to one-third of the height and the remaining two-thirds papered. The painting is charged at $7\frac{1}{2}$*d.* per square yard, the paper costs 5*s.* 2*d.* per square yard, and the work of papering is charged at 2*d.* per square yard. How much will the whole cost amount to?

4. A person sells out £3850 four per cent. stock at 104 and invests the proceeds in another stock at 143. If the dividend on this be $5\frac{3}{4}$ per cent., what will be the change in his income?

5. What must be the rate of interest in order that the discount on £387. 7*s.* $7\frac{1}{5}$*d.* payable at the end of 3 years may be £41. 10*s.* $1\frac{1}{3}$*d.*?

1885-86.

Reduce $\dfrac{2\frac{1}{2}-\frac{5}{6}}{2\frac{1}{2}+\frac{5}{6}}$ of 2 guineas + $\frac{7}{12}$ of $\dfrac{9 \div \cdot 1}{14 \times 3}$ of 4 crowns − $\cdot 8\dot{3}$ of $1\frac{6}{7}-\frac{1}{7}$ of £1 to the decimal of 5 half-guineas and prove that $\dfrac{6+5}{11+7}$ is greater than $\frac{6}{11}$ and less than $\frac{5}{7}$.

2. A man contracts to perform a piece of work in 30 days and immediately employs 15 men on it; at the end of 24 days the work is only half done. How many boys should be given to assist them that the contract may be fulfilled, each boy working two-fifths as much as each man?

3. A person buys 80 tons of coal, and after selling them again at 1*s.* 6*d.* per sack finds that he has gained £4; had he sold them for 1*s.* 4*d.* per sack he would have lost £6. Find the weight of each sack and the cost price per ton.

4. A field of 7 acres is sown with wheat, barley and maize, the areas of the crops being respectively as $2\frac{1}{2}$: $3\frac{1}{2}$: $4\frac{1}{2}$. If the values of an acre of each be also respectively in the same ratios, and an acre of wheat be worth £7, what is the worth of all the crops in the field?

5. If the three per cents. are at $92\frac{3}{8}$ and the four per cents. at $123\frac{1}{4}$, in which should one invest? And how much is one investing when the difference in income is a shilling?

1886-87.

1. Explain carefully the meaning of *prime number, factor, divisor, measure, multiple.*

Resolve 5005 into its prime factors.

Add together as decimals $8{\cdot}1\dot{3}\dot{8}$, $14{\cdot}6\dot{5}65\dot{1}$, ${\cdot}205\dot{0}896\dot{3}$.

2. The circumference of the forewheel of a carriage is $6\frac{7}{8}$ feet and that of the hindwheel is $12\frac{5}{6}$ feet. How many feet must the carriage pass over before the wheels shall have made a complete number of revolutions?

3. A vessel is filled with a liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

4. (i) The surface of a cube is $308{\cdot}1\dot{6}$ square feet. Find the length of its edge.

(ii) Extract the cube root of $45{\cdot}\dot{6}9\dot{8}$ to four places of decimals.

5. If the price of gold be £3. 10*s.* $10\frac{1}{2}$*d.* an ounce and a cubic inch of gold weigh 10 ounces, what is the price of the gold that would be required to gild a dome whose surface is 5000 square feet, the thickness of the gold gilding being ·0002 of an inch?

6. A person invests in 4 per cent. Government paper so as to receive 4 per cent. clear when the income-tax is 5 pies in the rupee. What percentage will be received if the tax be increased to 7 pies the rupee.

1887-88.

1. Simplify $\dfrac{{\cdot}\dot{1}4285\dot{7} \times {\cdot}\dot{0}7692\dot{3}}{{\cdot}\dot{0}1098\dot{9}} + \dfrac{2{\cdot}7\dot{5} \times 11{\cdot}25}{6{\cdot}2}$.

2. If 9 lbs. of rice cost as much as 4 lbs. of sugar, and 14 lbs. of sugar are worth as much as $1\frac{1}{2}$ lbs. of tea, and 2 lbs. of tea are worth 5 lbs. of coffee, find the cost of 11 lbs. of coffee if $2\frac{1}{2}$ lbs. of rice cost $6\frac{1}{4}$*d.*

3. If R165. 14*a.* and $1\frac{7}{17}$*p.* be the discount of a debt of R2820, simple interest being at the rate of $3\frac{3}{4}$ per cent., how many months before due was the debt paid?

4. The price of gold is £3. 17*s.* $10\frac{1}{2}$*d.* per oz.; a composition of gold and silver weighing 18 lbs. is worth £637. 7*s.*, but if the proportions of gold and silver were interchanged, it would be worth only £259. 1*s.* Find the proportion of gold and silver in the composition, and the price of silver per oz.

5. By selling 4 dozen mangoes for 13 rupees, it was found that $\frac{3}{10}$ths of the outlay was gained; what ought the retail price per mango to have been in order to have gained 60 per cent.?

1889-90.

1. Simplify :—
$$\frac{5\frac{1}{2} \text{ of } \dot{.2} \text{ of } 2.\dot{5}7142\dot{8} - 1 \div (\frac{1}{8} + .\dot{5})}{1 - \frac{3}{14} \text{ of } \left\{ .5 + \frac{1}{2} \text{ of } \frac{.05}{.\dot{1}4285\dot{7} \text{ of } 1\frac{1}{20}} \right\}}$$

2. A rectangular cistern, whose length is equal to its breadth, is $5\frac{5}{7}$ feet deep and contains 5 tons of water. If a cubic foot of water weighs 1000 ounces, find the dimensions of the cistern.

3. *A*, *B*, and *C* can walk at the rate of 3, 4, 5 miles an hour; they start from Poona at 1, 2, 3 o'clock respectively; when *B* catches *A*, *B* sends him back with a message to *C*; when will *C* get the message?

4. If I borrow money at 3 per cent. per annum, interest payable yearly, and lend it immediately at 5 per cent. per annum, interest payable half-yearly (receiving compound interest for the second half year), and gain thereby at the end of the year R660; what was the sum of money which I borrowed?

5. A person buys tea at 6 annas per seer and also some at 4 annas per seer. In what proportions must he mix them so that by selling the mixture at $5\frac{1}{4}$ annas per seer he may gain 20 per cent. on each seer sold?

1891-92.

1. Simplify :—

(i) $$\frac{\frac{7}{13} \text{ of } \frac{9}{128} + \frac{5}{9} \text{ of } \frac{13}{33}}{5\frac{2}{77} \text{ of } \frac{133}{183} - \frac{399}{323} \text{ of } \frac{19}{33}}.$$

(ii) $$\frac{3.6\dot{4}2857\dot{1} - (.009923 + .0102 - .000123) \frac{.145}{.0056}}{\sqrt{34.5744} - \sqrt[3]{9.663597}}.$$

2. Two passengers have together 5 cwt. of luggage and are charged for the excess above the weights allowed 5*s.* 2*d.* and 9*s.* 10*d.* respectively; but if the luggage had all belonged to one of them he would have been charged 19*s.* 2*d.* How much luggage is each passenger allowed to carry free of charge, and how much luggage had each passenger?

3. Two clocks *A* and *B*, whose rates are uniform, at noon yesterday indicated 11 hrs. 55 min. A. M. and 0 h. 2 m. P. M. respectively. *A* indicated the correct time at 9 P. M. yesterday and *B* at 6 A. M. this morning. When did *A* and *B* last agree and what time did they then indicate?

4. A person borrows two equal sums of money at the same time at 5 per cent. and $3\frac{3}{4}$ per cent. simple interest respectively, and finds that if he repays the former sum with interest on a certain date a year before the latter, he will have to pay in each case the same amount, *viz.* R 736. Find the amounts borrowed.

1892-93.

1. What decimal of a rupee is ·954 pie? Find the value of ·97625 rupee.

Simplify :—

$$\frac{\frac{5}{11}-\frac{3}{7}\text{ of }\frac{1}{2}}{\frac{5}{16}+\frac{7}{12}\text{ of }3\frac{1}{4}-\left(\frac{1}{8}\text{ of }2\frac{2}{7}-\frac{1}{8}\right)}\div\frac{\frac{1}{8}\text{ of }\frac{1}{2}+\frac{2}{3}\text{ of }5}{9\frac{1}{3}-1\frac{2}{3}}.$$

2. How long will two examiners, working 8 hours a day, take to look over the answers to this paper, if four examiners, working 5 hours a day, can do it in 8 days?

3. On a river, B is intermediate to and equidistant from A and C; a boat can go from A to B, and back, in 5 hours 15 minutes, and from A to C in 7 hours; how long would it take to go from C to A?

4. What income will a retired officer obtain in England, from one lakh of rupees, Indian Government $4\frac{1}{2}$ per cent. bonds, when for drawing and remitting it, his agents in India charge him 3 per cent., and exchange is at 1*s.* $2\frac{1}{4}$*d.* for the rupee?

5. Three equal glasses are filled with a mixture of spirits and water, the proportion of spirits to water in each glass being as follows: In the first glass as 2 : 3, in the second 3 : 4, and in the third 4 : 5. The contents of the three glasses are poured into a single vessel; what is the proportion of spirits to water in it?

1893-94.

(Set in the mofussil).

1. Divide each of the numbers 2,572,125 and 4,061,250 by 125; and express as a decimal the first quotient divided by the second.

2. Find, by Practice, the value of 5 yds. $22\frac{1}{2}$ in., at £2. 1*s.* 2*d.* a yard.

3. If the carriage of 2 cwt. 1 qr. and 18 lbs. of goods, for 56 miles, be £1. 1*s*, what weight can be carried at the same rate, 200 miles for £4. 3*s.* 4*d.*?

4. A man invests £3,000 in the 5 per cents. If after deducting an income-tax of 8*d.* in the pound, the man's clear income is £174, what is the price of the 5 per cents.?

5. A cistern is filled by two taps A and B in 4 hours and 6 hours respectively, and is emptied by a waste pipe C in 3 hours. When the cistern is half full, A and B are closed, and C is opened; after one hour, B is turned on; and after half an hour more, A is turned on. In what time after C is first opened, does the cistern become full?

6. A person buys two kinds of tea, at 5*s.* a lb. and 6*s*, a lb. respectively; and after mixing them he sells the mixture at 6*s.* 6*d.* a lb., thereby gaining 17 per cent. In what proportion does he mix them?

1893-94.

(Set at Bombay).

1. Reduce to their simplest forms :—

$$(i)\quad \frac{\frac{1}{2}+\frac{1}{3}-\frac{1}{4}}{\frac{1}{2}\text{ of }\frac{1}{3}\text{ of }\frac{1}{4}}\;; \qquad (ii)\quad \frac{2}{3+\dfrac{4}{5-\frac{6}{7}}}.$$

2. Find, by Practice, the value of 9 cwt. 3 qrs. 24 lbs. at £3. 5*s*. 8*d*. per cwt.

3. If 40 men, 60 women or 80 children can do a work in 6 months, in what time will 10 men, 10 women, and 10 children do one-third of the work?

4. A person invested £1,000 in the 3 per cents. at $90\frac{5}{8}$; but the price rising to $91\frac{1}{4}$, he sold out, and invested the proceeds in the $3\frac{1}{2}$ per cents. at $97\frac{1}{8}$; find the increase in his income.

5. A cistern can be filled by two pipes, *A* and *B*, in 12 minutes and 14 minutes, respectively, and can be emptied by a third, *C*, in 8 minutes. If all the taps be turned on at the same moment, what part of the cistern will remain unfilled at the end of 7 minutes?

6. Two clocks point to 2 o'clock at the same instant on the afternoon of 25th April; one loses 7 seconds, and the other gains 8 seconds, in 24 hours; when will one be half an hour before the other, and what time will each clock then shew?

1894-95.

1. When the number representing the year is a multiple of four, it is a leap year, consisting of 366 days, except when this number is a multiple of 100, in which case it is an ordinary year, consisting of 365 days, but when the number is a multiple of 400, it is again a leap year; on this supposition, calculate the number of days from the 1st January 1495 to 31st December 1894, both days inclusive.

2. A school of boys and girls consists of 453 children; the number representing the boys is ·52 of the number of the girls. How many boys were there?

3. Two-thirds of a certain number of poor persons received 1*s*. 6*d*. each, and the rest 2*s*. 6*d*. each; the whole sum spent being £2. 15*s*., how many poor persons were there?

4. If 3 men and 5 women do a piece of work in 8 days, which 2 men and 7 children can do in 12 days, find how long 13 men, 14 children and 15 women will take to do it.

5. *A* sells a house to *B* for R4860, thereby losing 19 per cent.; *B* sells it to *C* at a price which would have given *A* 17 per cent. profit. Find *B*'s gain.

6. The compound interest on one rupee is one quarter of a rupee at the end of three years; find the rate per cent. per annum, correct to two places of decimals; and calculate exactly the compound interest at the end of 9 years.

ENTRANCE EXAMINATION PAPERS.

MADRAS.

1857.

1. Simplify the expression $(\frac{3}{8} - \frac{1}{7}) \times (\frac{3}{5} + \frac{2}{7}) \times 2\frac{8}{31} + \frac{1}{8}$.

2. Divide ·000247 by ·013.

3. What is the equivalent in Indian coin, of 39*l.* 5*s.* $9\frac{3}{4}$*d.* when a rupee is worth 2*s.* $0\frac{3}{4}$*d.* ?

4. Extract the square root of 187·9641.

5. A tank is 300 yards long and 150 yards broad ; with what velocity per second must water flow into it through an aperture 2 feet broad and $1\frac{1}{2}$ feet deep, that the level may be raised 1 foot in 9 hours ?

6. Find the interest of £250 for $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. simple interest.

1858.

1. A company of 87 men have subscribed each a month's pay amounting to R13. 11*a.* 7*p.* for the benefit of the widows of their deceased comrades. There are 24 applicants ; what is each widow's share ?

2. A cubical tank, 24 feet long, 18 ft. 6 in. wide and 12 ft. 4 in. deep, is filled with water. Find the weight of water supposing that a cubic foot weighs 1000 oz. How long will it take to discharge itself at the rate of 15 gallons a minute assuming that a pint of water weighs 1 lb. ?

3. A rectangular field is $\frac{3}{16}$ths of a mile long and $\frac{2}{16}$ths of a mile wide ; find the length of a line joining two opposite angles.

4. Find the number of degrees, minutes and seconds in an arc of a circle which is equal in length to its radius, the ratio of the diameter to the circumference being 1 : 3·14159.

5. What must I pay for a bill of exchange on London for £73. 15*s.* 6*d.*, the exchange being at the rate of 1*s.* $10\frac{1}{8}$*d.* for the rupee ?

6. A person having R8,500 in 4 per cent. Government bonds sells out when they are at $8\frac{1}{4}$ per cent. discount, and with the amount thus realised purchases 5 per cent. bonds, which are at $6\frac{5}{8}$ per cent. premium : what does he gain or lose in annual income by the change ?

1859.

1. In Long Multiplication the general product is the sum of the several partial products. Illustrate this in the example, 2359×576 ; and write down, separately the several partial products with their factors.

2. The receipts on the Madras Railway for a certain week in January 1859, when there were 96 miles open, were R9,554. 3*a.* 10*p.* ; for the

corresponding week in 1858, when there were 81 miles open, they were R8,554. 6*a*. Compare the average receipts *per mile* for the two years.

3. State the Rule for division of Vulgar Fractions, and prove it, taking an example.

4. First multiply, and then divide ·2 by ·03, and verify your results. Finally find the sum of their square roots to three places of decimals.

5. A piece of land measures 10 cawnies, 11 grounds, 1075 square feet; find how many acres, roods and perches it contains, the cawny being 1·3223 acres.

6. The French unit of linear measure is a *metre*, equal to 39·371 English inches; the square formed on a line of 10 metres (called an *are*) is the French unit of surface. Find the equivalent in English square measure, of a hectare (100 ares).

7. The number of pupils in a school is 287, composed as follows: Hindoos 196, Mahomedans 63, Christians 28. The average daily number of absentees is 58; *i. e.*, Hindoos 37, Mahomedans 16, Christians 5. Find, to the first place of decimals, the percentage of *attendance*, both on the aggregate, and of each class of pupils.

1860.

1. Multiply 76489 by 743, and explain fully the various steps of the process.

In what cases does multiplication increase, leave unaltered, diminish, the multiplicand?

2. Find the least common multiple of 2191, 1252 and 1878. Illustrate the proof of your rule by this example.

3. What decimal of 9*s*. 3*d*. will be equivalent to a rupee, when the exchange is at 1*s*. 10$\frac{1}{4}$*d*.?

4. Find the vulgar fraction which will represent in the simplest manner

$$\frac{5·75}{4·25} \text{ of } \frac{1}{4} \div \frac{5}{7} + \frac{4}{8} \times \frac{3}{8} - \frac{1}{2}.$$

5. Extract the square root of 1156·272016.

6. If a cloth, 4 yards long and 15 inches wide, cost R3. 5*a*., how much should you give for a cloth 19 yards long, 12 in. wide, and every square inch of which is worth $\frac{4}{5}$ths of the value of a square foot of the former?

1861.

1. A bankrupt is indebted to *A*, *B*, *C* and *D*;—*A*'s debt is twice *B*'s; *B*'s three times *C*'s; *C*'s half *D*'s. How much should each receive of assets to the amount of R45,680?

2. Add together $\frac{4}{5}$, $\frac{5}{12}$, $\frac{3}{8}$ and $\frac{1}{6}$ and fully explain the process.

3. Reduce to their simplest forms

$$\frac{\frac{7}{9}-\frac{1}{6}}{3\frac{3}{4}+4\frac{1}{6}} \text{ and } \left(\frac{9-4}{5}\div\frac{1}{7}\right)^3.$$

4. Multiply ·892 of R16. 5*a*. 4*p*. by 4·678.

5. How much should you pay for a bill on London for £647, when the exchange is at 1*s*. 11½*d*.?

6. Divide 764·0468 by ·0007. Give the rule for the position of the decimal point in your quotient, and shew that the rule is correct.

7. What is the square root of ·004225?

8. If the daily wages of a labourer rise from four and three-quarters to six annas, what percentage of the increase in the price of food and other commodities will cause his position to be unaltered?

9. A gentleman buys a house for R24650 and spends 23 per cent. in additions and improvements. At what monthly rental will he secure 8 per cent. per annum on his whole investment?

1862.

1. Explain the decimal system of numeration. Write in words 14006, 3179040601, and 17·0461.

2. "Multiplication is a shortened form of addition"; of all additions, or of some, and if only of some, of what kind?

Do the two statements, *twice two are four* and *four times five are twenty*, rest upon the same ground? Could you shew, without reference to the multiplication table, that five times five must exceed four times six by one?

3. State and prove the rule for the division of vulgar fractions; divide

$$\frac{1\frac{4}{7}}{6\frac{2}{3}} \text{ by } \frac{19\frac{1}{2}}{\frac{7}{8}}.$$

4. Find the greatest common measure of 323 and 391.

5. If, when the exchange is at 1*s*. 1½*d*. per rupee, you wish to remit R4,891. 4*a*. 3*p*. to London, what should be the amount of your bills in English money?

6. Reduce $\frac{4}{7}$ to decimals. Prove the correctness of your method.

7. Find the square root of 64·064.

8. A steam-ship whose speed averages 14 miles an hour, reaches a certain port in 12 days; how many days afterwards will a sailing vessel arrive, which started at the same time and sailed on an average 8 miles an hour?

6. A train has been travelling 20 miles an hour; the steam-power is doubled, whilst from various causes the resistance of the train is increased by one-half. (The original steam-power is three times the resistance.) At what rate will the train now travel?

1863.

1. Divide 480813 by 245 in two ways,

(1) By long division, (2) by factors.

2. Add together £7. 16*s.* 9*d.* ; £19. 4*d.* ; £3. 2*s.* $6\frac{1}{2}$*d.* ; R142. 3*a.* 10*p.* ; R354. 4*a.* 8*p.* ; R1269. 14*a.* 2*p.* ; (1) in English money (2) in Indian money, one rupee being equal to two shillings.

3. Simplify the following fractions :—

$$(1)\ \frac{7887}{7953},\qquad (2)\ \frac{3}{5}\text{ of }\frac{4}{9}+8\frac{1}{2}+\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}},$$

(3) $\frac{3}{8}$ of R9 + $\frac{3}{5}$ of 10*a.* − $\frac{2}{3}$ of 6*p.*

4. Multiply 4·37 by 1·01 and divide 7·4 by ·018.

5. Reduce £56.125 to the ordinary notation.

6. Find the square root of (1) 127449 ; (2) of 12.7449 ; (3) of 2 to three places of decimals.

7. Shew that no number can be a perfect square which has an odd number of decimals after the point.

8. How must R1075 be divided betwixt two persons, so that one may have twice as much as the other ?

9. A sailing vessel reaches Madras from Calcutta in 6 days ; a steamer whose speed is to that of the sailing vessel as 3 : 2 starts at the same time but meets with detentions that average 6 hours daily. Which will reach Madras first ? And by how much ?

1864

1. A man rides at the rate of 11 miles an hour, but stops 5 minutes to change horses at the end of every 7th mile ; how long will he take to go a distance of 94 miles ?

2. How are vulgar fractions compared in regard to magnitude ? Of the fractions $\frac{7}{21}$, $\frac{113}{356}$, which is the greater, and what is the difference ?

3. A cubic foot of air weighs 1·29 oz. avoirdupois. What will be the weight of air in a room 18 feet broad, 30 feet long, and 16 feet high ?

4. Simplify the expression

$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}+\frac{1}{5}}+\frac{1}{3}\text{ of }\cdot 4.$$

5. What will be the cost of a beam of wood 14 feet long, 16 inches broad, and 9 inches thick, as R1 . 0 . 8 per cubic foot ?

6. Extract the square roots of $17\frac{8}{49}$ and ·015129.

7. Find (by Practice) the value of 371 articles at 6*s.* 3*d.* each.

8. Express in a decimal form

$$3+\frac{4}{10}+\frac{6}{1000}+\frac{11}{10000}.$$

9. A train starts from A at 12 o'clock, and runs towards C, which is 100 miles distant, at the rate of 30 miles an hour; at the same time the mail cart starts for C, from B, which is half-way between A and C, and runs at 10 miles an hour; at what distance from C will it be overtaken by the train?

1865, A.

1. Find (by Practice) the value of $237\frac{1}{4}$ yds. of cloth at 1*s.* $10\frac{1}{2}$*d.* per yard.

2. A person goes into a bookseller's shop with a certain sum of money, and after buying 20 books at R2 . 4 . 0 each, finds that $\frac{5}{8}$ of his money remains. How much had he when he entered the shop?

3. Reduce to their simplest forms each of the following expressions, and show that the second is double of the first—

$$(1)\quad \frac{8\frac{11}{88}}{\frac{7}{8}\text{ of }2{\cdot}25}\text{ of }\frac{{\cdot}1625}{\frac{1}{16}\text{ of }5\frac{4}{7}}\div\left(\frac{2}{21}+\frac{7}{81}\right).$$

$$(2)\quad \left\{37+\frac{3{\cdot}\dot{7}03\dot{7}}{100}\right\}\times{\cdot}54.$$

4. A room is 16 ft. 5 in. long and 19 ft. 7 in. broad, and the cost of painting the walls at 7*a.* 6*p.* per square yard is R43 . 3 . 0. Required the height of the room.

5. Extract the square roots of $5\frac{3}{8}$ and ·0045, each to 4 places of decimals.

6. A merchant buys goods for £568. 4*s.*, and sells half of them at a gain of 1*d.* in the shilling of the cost price, one-third of them at a gain of 2*d.* in the shilling and the remainder at a gain of £15. 15*s.* 8*d.* How much per cent. does he gain on the whole transaction?

7. Express $\frac{5}{8}$ of 12*s.* 6*d.* + ·625 of 7*s.* 6*d.* − $\cdot\dot{5}0\dot{5}$ of 16*s.* 6*d.* as a decimal of £1.

8. A person after paying an income-tax of 1 anna in the rupee, devotes $\frac{1}{16}$ of the remainder of his income to purposes of charity, and finds that he has Rs 5,175 left; what is his income?

1865, B.

1. A person paid a tax of 10 per cent. on his income, and had R15,000 per annum remaining. What was his income?

2. Find, by Practice, the time of building a wall 27 yards long by 6 feet high, of which one square yard is built in 3 hr. 18 min. 45 sec.

3. How much will 3,630 square yards of land cost when an estate of 144 acres is worth R46275?

4. Simplify the expression $\dfrac{\frac{5}{8}\text{ of }\frac{18}{25}+\frac{1}{7}\times 2\cdot\dot{3}}{3-(\frac{3}{5}+\frac{7}{10})\div 2\cdot\dot{3}\dot{6}}$.

5. If 10 compositors who can set 3 letters in 5 seconds, finish 27 pages in an hour and a half, how many compositors who can set 5 letters in 6 seconds, will complete 50 pages in an hour?

6. What is the value of

(*a*) $\frac{3}{8}$ of 1*s*. + $\frac{7}{10}$ of 2*s*. 6*d*. + $\frac{3}{8}$ of £1 expressed in the fraction of a guinea?

(*b*) ·0625 of ₹10½?

7. Find the square root of (*a*) 53·4361, (*b*) $\cdot 18\dot{7}$.

8. *A* can do a piece of work in 3 days, *B* can do 3 times as much in 8 days, and *C* 5 times as much in 12 days. In what time will they do it together, supposing them to work at the rate of 9 hours a day?

1866.

1. One inch of rain falls on an acre of ground. How much will it weigh reckoning the weight of one cubic foot as 1000 ounces?

2. A person bought a horse for ₹750 and kept it 15 months. It cost during that time, in gram, ₹190. 10*a*. 6*p*.; in servant's wages ₹135; and in other expenses, ₹35. 14*a*. 6*p*. He sold it for ₹625; what was the average monthly cost of the horse?

3. Reduce to its simplest form

$\dfrac{\frac{5}{8}-\frac{2}{5}}{\frac{1}{3}-\frac{1}{7}}$ and convert the result into a decimal.

4. A body of 3249 men is formed into a solid square. How many men will there be in each side?

5. What fraction of a rupee and a quarter is $\frac{1}{7}$ of $\frac{5}{6}$ of ₹5. 4*a*.?

6. Extract the square root of 195·1 to three places of decimals.

7. A merchant clears 20 per cent. on a gross income of ₹50,000. How much per cent. must he clear if he receives the same amount from a gross income of ₹40,000?

8. A ship-captain owns $\frac{3}{8}$ of his vessel. In virtue of his command he receives $\frac{1}{15}$ of the profits, and of the remainder his share as proprietor. What proportion of the whole does he receive?

9. A person with a monthly income of ₹264 spends as much in 4 months as he earns in three. After twelve years he divides his savings amongst his three children in such a manner that the eldest has twice as much as the second, and thrice as much as the youngest. How much did each receive?

1867.

1. If 16 men can do a piece of work in 255 days, how many men can do the same in 17 days?

2. How much carpet $2\frac{1}{4}$ feet wide would be required for covering the floor of a room 28 ft. long and $16\frac{1}{3}$ feet wide?

3. Reduce to its simplest form:

$$\frac{1\frac{1}{3} \text{ of } 2\frac{1}{4} \text{ of } 3\frac{1}{5} \text{ of } 4\frac{1}{6}}{3\frac{3}{5} \text{ of } 2\frac{5}{11}}.$$

4. Find the square root of ·00826462810.

5. A man buys 16 lbs. of tea at R2. 2*a.* per lb., also 12 lbs. at R2. 5*a.* 4*p.* per lb.; and 24 lbs at R2. 6*a.* 10*p.* per lb. At what price per lb. must he sell the mixture so as to gain R35. 12*a.* on the whole?

6. If it is high water at noon on a certain day, find after how many days it will again be high water at noon, supposing the time of high water to be three quarters of an hour later every day.

7. A crow wishing to quench its thirst came to a vessel which contained 28 cubic inches of water. The crow being unable to reach the water picked up several small stones, each three quarters of a cubic inch in size, and let them drop into the vessel until the water came to the top of the vessel. If the size of the vessel was such that it would exactly hold 73 cubic inches of water, find the number of stones dropped in by the crow.

8. A book containing between 900 and 1,000 pages is divided into four parts, each part being divided into chapters. The whole number of pages in each of the four parts is the same. Each chapter in the first part contains 20 pages, each chapter in the second 40, each chapter in the third 60, and each chapter in the fourth 80. Find the whole number of chapters in the book.

1868.

1. Simplify $\left(\frac{1}{30} \times 3\frac{1}{4}\right) + \left(\frac{3}{5} \div \frac{27}{35}\right) - \left(\frac{1}{1\frac{1}{7}} - \frac{7\frac{3}{8}}{9}\right)$.

2. The difference in the values of the two shares into which a certain property is divided is £48·575 and one share is ·51 of the whole. Find the value of the property and of each share.

3. What is the income corresponding to an Income-Tax of 25 guineas at the rate of 7*d.* in the pound?

4. Find, to within a foot, the length of the fence enclosing a square field whose area is $3\frac{1}{2}$ acres.

5. A barrack for 30 men is $73\frac{1}{3}$ feet long and $24\frac{3}{4}$ feet broad; how high should it be to allow each man 1000 cubic feet of air or space?

6. A person sets out to walk 26 miles; for a quarter of the distance he goes at the rate of 5 miles an hour, for half the remaining distance at

4 miles an hour, and 3 miles an hour for the other half. State the exact time occupied in the journey.

7. The Fort-Barracks are lighted with gas from 100 burners. Find the cost of lighting them per night of 10 hours, at the rate of R$5\frac{1}{3}$ for 1000 cubic feet of gas, assuming that for the first 3 hours each burner consumes 1 cubic inch per second, and during the remainder of the night the light is so reduced that the oonsumtion of gas by each burner is only $\frac{3}{4}$ths of that quantity per second.

8. If two Malabar miles are equal to 1 kros, and 7 Malabar miles are equal to 10 English miles, how many kros are there in 25 English miles?

9. A contractor bought 2,250 pharas of unslaked lime at Madras at the rate of 45 rupees for 100 pharas. On slaking it, every phara gave 3 cubic feet of lime, but of this $\frac{1}{15}$ was unserviceable: the carriage of the remainder to the place where it was required (distant 18 miles) cost 4 annas per 100 cubic feet per mile. At what rate per cubic foot must he sell it there, in order to gain 90 rupees on his outlay?

1869.

1. Which is greater

$$\tfrac{2}{3} \text{ of } 1\tfrac{3}{5} - \frac{1\frac{3}{5}}{6\frac{2}{3}} \text{ of } \tfrac{18}{20} + \tfrac{3}{7} \text{ of } \frac{6\frac{5}{12}}{3\frac{2}{3}} \text{ or}$$

$$\tfrac{5}{9} \text{ of } 1\tfrac{3}{5} + \frac{6\frac{2}{3}}{1\frac{3}{5}} \text{ of } \tfrac{18}{20} - \tfrac{1}{4} \text{ of } \frac{6\frac{5}{12}}{3\frac{2}{3}} \text{ ?}$$

and express the difference as a decimal.

2. Express as a fraction the difference between $\frac{355}{113}$ and 3·1416: and reduce 2 tons 3 cwt. 2 qrs. 26·996954 lbs. to the decimal of atbis of 3·086 lbs.

3. A rectangular field, whose diagonal measures 825 feet, has one of its sides ·75 of the length of the other. Find the length of each side in yards, and the area in acres.

4. A person had a legacy left to him, which he thus divided amongst 3 charities. To one he gave $\frac{3}{10}$, to the second $\frac{2}{7}$ of the remainder, and to the third $\frac{2}{3}$ of what now remained; and he then had 1,500 rupees left. Find the amount of the legacy, and how much was given to each charity.

5. A creditor received on a debt of 3,600 rupees a dividend of 9*a.* 10*p.* in the rupee; and a further dividend of 6*a.* 8*p.* upon the remainder. What did he receive altogether, and what fraction was it of the entire debt?

6. *A* and *B* each lends £250 for three years, *A* lends at $4\frac{1}{2}$ per cent. simple interest, and *B* at $4\frac{1}{4}$ per cent. per annum, compound interest. Find the difference in the amount of interest they receive.

7. A contractor agrees to supply $10\frac{1}{2}$ lacs of bricks for a particular work. His bricks cost him $3\frac{1}{2}$ rupees per 1,000 to make, and of these

$12\frac{1}{2}$ per cent. are rejected. How many bricks must he make in order to fulfil his contract, and what price per 1,000 must he put on those supplied in order to gain 25 per cent. on his outlay ?

8. The distance by Railway from Madras to Salem is $206\frac{1}{4}$ miles. A passenger train travelling 20 miles an hour leaves Madras at 7 A. M. ; and a special train at 10 A. M. the same day. At what rate must the latter travel, so as just to overtake the former at Jollarpett Junction (132 miles from Madras), and at what hour must a goods train leave Salem for Madras travelling 15 miles an hour, so as to reach Jollarpett at the same time as the other trains ?

9. Extract the square root of

$$\frac{1\cdot\dot{7} \times 29\frac{4}{25}}{\cdot000729}.$$

10. A work can be completed in 36 days by 30 men working 6 hours a day ; in what time would 18 men and 60 women working 9 hours a day complete it ; supposing that 3 men can do as much as 5 women, and that in the longer days a man does only $\frac{4}{5}$ per hour of what he does per hour in the shorter days ?

1871.

1. A person mixes together 10 lbs. of tea at R1. 4*a.* a lb., 12 lbs. at R1. 6*a.* and 14 lbs. at R1. 8*a.* a lb. He reserves 6 lbs. of the mixture for himself and sells the remainder at R1. 13*a.* 4*p.* a lb. How much does he gain ?

2. (*a*) Simplify $\frac{1}{1-\frac{2}{\cdots}} - \left\{\frac{1}{3}-\left(1+\frac{1}{12}\right)\right\} \div 3\left(1-\frac{5}{8} \text{ of } 2\frac{1}{3}\right)$.

(*b*) Express $\frac{3}{5}$ of $\frac{7}{15}$ of £1. 10*s.* + $\frac{2}{3}$ of $\frac{5}{8}$ of 5*s.* 4*d.* − $8\frac{1}{3}$ of $\frac{1}{4\frac{1}{2}}$ of 5*s.* $3\frac{3}{4}$*d.* as the fraction of 2*s.* $1\frac{1}{2}$*d.*

3. *A* has shares in an estate to the amount of $\cdot15 \div \cdot3\dot{6}$ of it. *B* has shares in the same estate to the amount of $\cdot\dot{4}7\dot{2}$ of it ; find the difference in value between the properties of *A* and *B*, when $\cdot0\dot{5}6$ of the estate is worth £373·3.

4. A reduction in the income-tax diminishes a tax which is R15 when the tax is 8 pies in the rupee by R3-12-0 ; what is the diminished rate of the tax in the rupee ?

5. 23 cwt. 3 qrs. 7 lbs. are bought at £2. 10*s.* 8*d.* per cwt. and 72 cwt. 2 qrs. 8 lbs. at £2-7-10 per cwt. Find, by Practice, the amount expended and give the average price per lb.

6. A person borrows £500 at 5 per cent. per annum, and subsequently £400 at $3\frac{1}{2}$ per cent. ; if the amount of both sums 6 months after the latter

was borrowed is £957, find the time for which interest is paid on the former sum.

7. A cask of $144\frac{2}{3}$ gallons is bought for £50 and kept 10 years, during which $\frac{7}{15}$ of a gallon evaporated yearly; at what rate per gallon must the contents be sold so as to clear 20 per cent. on the amount of the original outlay at 4 per cent. per annum simple interest?

8. Water flows into a rectangular cistern whose dimensions are 12 ft. 1 in. long, 11 ft. 8 in. wide, and 5 ft. 4 in. deep, through a pipe of 10 sq. inches aperture at the rate of $2\frac{1}{2}$ ft. per second, and flows out through an orifice at the rate of 2 ft. 5·8 in. per second; if the cistern is filled in two hours, find the size of the orifice.

9. A lump composed of gold and silver measures 6 cubic inches and weighs 100 oz.; if a cubic inch of gold weighs 20 oz. and an equal bulk of silver 12 oz., find the weight of gold in the mixture.

10. A train which travels at the uniform rate of 30·8 ft. a second leaves Madras at 7 A.M.; at what distance from Madras will it meet a train which leaves Arconum for Madras at 7·20 A.M., and travels one-third faster than it does, the distance from Madras to Arconum being 42 miles?

1873.

1. A person buys a piece of land at £25 an acre, and by selling it in allotments finds that the value is increased by one-half, so that, after reserving 20 acres for himself, he clears £200 on his purchase money by the sale of the remainder. How many acres were there?

2. Simplify

(*a*) $\dfrac{7}{5-\frac{4}{5}} \div \dfrac{3-\dfrac{2}{3-\frac{3}{2}}}{4-\frac{3}{2}} - \frac{5}{7}$ of $\left\{\dfrac{1}{1\frac{3}{7}} + \text{[illegible]} \text{ of } \dfrac{3\frac{1}{3}-2\frac{1}{2}}{\text{[illegible]}}\right\}$.

(*b*) Reduce $(\dot{5}7\dot{5} + 16 \times \dot{1}\dot{5})$ of 4 viss to the decimal of $1\frac{1}{8}$ cwt., a viss being equal to 3 lbs. 2 oz. avoirdupois.

3. A vessel's cargo, $\frac{2}{3}$ of which is worth £$6666\dot{6}$, gets damaged, and the owner in consequence sells $\dfrac{\cdot 8\dot{3} + \cdot 041\dot{6}}{1\cdot 05}$ of it for half the original value of the whole cargo. What is the value of the remainder at the same rate and what is the loss on the whole cargo?

4. Find how much rice a family requires monthly, when a reduction in the price from 7 to 10 measures for the rupee reduces the total monthly expenses from R$31\frac{1}{4}$ to R30.

5. A person going from Pondichery to Ootacamond travels 90 miles by steamer, 330 miles by rail, and 30 miles by horse-transit. The journey occupies 30 hours 50 minutes, and the rate of the train is three times that of the horse-transit and $1\frac{1}{2}$ times that of the steamer. Find the rate of the train.

6. A person bought 10 Bank of Madras shares at R1540 each, and for 5 years got interest on his investment at the rate of $5\frac{1}{2}$ per cent. He then sold his shares at a loss of $22\frac{1}{2}$ per cent. How much did he make by the transaction, and what rate per cent. per annum had he for his money ?

7. A person borrows two equal sums at the same time at 5 and 4 per cent. respectively, and finds that if he repays the former sum with interest on a certain date, 6 months before the latter, he will have to pay in each case the same amount, *viz.*, £1100. Find the amount borrowed and the time for which interest is paid.

8. A dealer buys 10 horses at R400 each, 8 horses at R500 each, and 4 horses at R600 each. He keeps the horses for 6 months, during which each costs R15 a month, and then sells them, clearing $12\frac{1}{2}$ per cent. on his original outlay, after paying all his expenses. Find the selling price.

9. A stream of water, 8 yds. broad at the surface and 6 yds. at the bottom, and 2 yds. deep, flows at the rate of $1\frac{1}{8}$ miles an hour, into a tank, 220 yds. long and 56 yds. broad, which holds 74,250 tons of water. Find the depth of the tank and the time in which it will be filled, a cubic foot of water weighing 1,000 oz.

10. Two trains, running at the rates of 25 and 20 miles an hour respectively, on parallel rails in opposite directions, are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rates as before, a person sitting in the faster train observes that he passes the other in $31\frac{1}{2}$ seconds ; find the lengths of the trains.

1874.

1. Find the greatest number which will divide 201 and 671, leaving remainders 6 and 8 respectively ; and the least number which when divided by 5, 7 and 9 gives in each case a remainder 4.

2. A wine merchant mixes together one pipe (126 gallons) of wine at £80, one at £90, and one at £100, and sells one-third of the mixture at 13*s.* 4*d.* a gallon ; at what price per gallon must he sell the remainder so as to gain $£34\frac{1}{2}$ by the transaction ?

3. Simplify

(*a*) $$\left\{\frac{2}{3-\frac{1}{1-\frac{1}{2}}}-\frac{1}{3}\text{ of }\left(5-\frac{2}{\frac{3}{4}-\frac{1}{3}}\right)\right\}\div\frac{\frac{1}{2}+\frac{2}{3}}{1\frac{1}{2}}.$$

(*b*) $$\frac{\cdot1\times\cdot1\times\cdot1+\cdot01\times\cdot01\times\cdot01}{\cdot2\times\cdot2\times\cdot2+\cdot02\times\cdot02\times\cdot02}.$$

4. Find the value of $\cdot\dot{4}2857\dot{1}$ of $£1\cdot05+\cdot3\dot{8}$ of $1\cdot5s.$, and express the result as the decimal of £43. 2*s.* 6*d.*

5. *A* barters sugar with *B*, for rice which is worth $1\frac{7}{8}$ annas a measure, but in weighing his sugar uses a false maund weight. *B* discovers this, and to make the exchange fair raises the price of his rice to $2\frac{1}{8}$ annas a measure. Find the real weight of the false maund which *A* uses.

6. A certain sum put out at compound interest amounts in two years to £270·4, and in three years to £281·216. Find the sum and the rate per cent.

7. A person pays an income-tax of 4*d.* in the £ during the first half of the year, and of 3*d.* in the £ during the second half, and finds that owing to an increase in his income he pays the same amount of tax for the second as for the first half of the year. If his gross income for the year is £700, find his net income.

8. A cistern measuring 13 feet in length, 5 feet in breadth, and 4 feet in depth, has a tap which, not being properly opened discharges 54 gallons an hour less than it would otherwise do and empties the cistern in 7½ instead of 6 hours. How many cubic inches are there in a gallon?

9. Gold costs £3. 17*s.* 10½*d.* per oz., and silver 5*s.* 6*d.*; in what proportion must these metals be mixed that a lb. of the mixture may be worth £32. 5*s.*?

10. A peon walks from *A* to *B* at the rate of 3 miles an hour, and after transacting some business which occupies him an hour, returns to *A* by the tramway at the rate of 5 miles an hour. He then finds he has been absent 2 hours 20 minutes. Find the distance from *A* to *B*.

1875.

1. A merchant purchases 231 gallons of spirits at R10. 12*a.* 4*p.* per gallon; 126 gallons at R12. 11*a.* 7*p.*; and 70 gallons at R14. 8*a.* 9*p.*; if he sell the mixture at R13 per gallon, how much will he gain by the transaction?

2. Define a decimal; and show how its value is affected by affixing and prefixing cyphers.

Reduce $\dfrac{1+\dfrac{2}{3\frac{1}{8}}}{1-\dfrac{2}{3\frac{1}{8}}} \div \left\{1+\dfrac{4}{9-\dfrac{3}{1-\frac{7}{8}}}\right\}$ to a decimal,

and find the value of $\dfrac{\cdot044\times2\cdot1}{\cdot000035}\div\dfrac{3\cdot\dot{0}7692\dot{3}}{2\cdot\dot{3}\times5\cdot\dot{6}}$.

3. Express the sum of $\cdot\dot{5}7142\dot{8}$ of a viss, ⅞ of $\dfrac{1}{3\frac{1}{4}}$ of $\dfrac{317}{384}$ of a maund and $\dfrac{4491}{[illegible]}$ of a hundredweight as a decimal of one ton. (One viss = 3 lbs. 2 oz.; one maund = 82$\frac{2}{7}$ lbs.)

4. If 210 coolies in 7 days of 10 hours each, dig a channel 1 mile long, 6 feet broad, and 2 feet deep; in how many days of 7 hours each should 35 coolies dig a channel 660 feet long, 7½ feet broad, and 2½ feet deep? And how many cubic feet does each cooly dig in an hour?

5. The expenses of a family when rice is 12 seers for a rupee are 50 rupees a month: when rice is 14 seers for a rupee the expenses are 48 rupees a month (other expenses remaining unalterable); what will they be when rice is at 16 seers per rupee?

6. What are the prime factors in 45090045, and what is the smallest whole number by which it must be multiplied in order to become a perfect square?

7. The cost of carpeting a room is £7. 4*s.*, and of papering the same room, with paper at 2½*d.* per square foot, £10. 12*s.* 6*d.* The length of the room is 18 feet, and if the width had been 4 feet less, the cost of the carpet would have been £1. 16*s.* 0*d.* less. Find the height of the room.

8. Find the sum for which the difference between the simple and compound interest at 5 per cent. per annum for 3 years, is £12. 4*s.* 0*d.*

9. What length of wire will go round the edges of a cube, the surface of which contains 187 yards 54 inches?

What is the least number of such cubes which will contain an exact number of cubes whose edges are 1 foot 3 inches?

10. A merchant's average rate of profit for five years was 5 per cent. on his capital, and for the first four years his average profit was 4 per cent. What was his rate of profit in the fifth year?

1877.

1. A merchant buys 264 gallons of spirit at R12. 8*a.* 4½*p.* per gallon, 378 gallons at R9. 10*a.* 7*p.* per gallon and 420 gallons at R12. 15*a.* 6½*p.* per gallon. If he sell the whole quantity at R12. 4*a.* 0*p.* per gallon, what profit will he make by the transaction?

2. If 2 men and 5 women can do a piece of work in 8 days of 9 hours each; how long will it take 3 men and 6 women to do a piece of work twice as great working 8 hours a day; the work of a man being double that of a woman?

3. Extract the square root of ·0002890; and find in yards to four places of decimals the side of a square field containing ·254 of an acre.

4. Find the value of $\cdot01\dot{6}$ of R260. 2*a.* 6*p.* + $\cdot\dot{3}5\dot{1}$ of R13. 14*a.* 0*p.* + $1\cdot0\dot{0}03\dot{3}$ of R7. 14*a.* 3*p.*

5. A merchant buys cloth at such a price that by selling it at R2. 3*a.* per yard he will gain 5 per cent. on his outlay. What percentage will he lose if the cloth be sold at R1. 13*a.* per yard?

6. Find the interest (simple) at 4 per cent. per annum on R595. 9*a.* for 4 years and 17 weeks, reckoning 52 weeks equal to a year.

7. A sum of R18,240 is remitted to England at the rate of exchange of one shilling and 8¼ pence per rupee, and is invested in the 3 per cent. consols at 95. Find the yearly income in pounds sterling.

8. A man bequeathed $\frac{4}{15}$ of his estate to one son, $\frac{7}{15}$ of the remainder

to another son, and the balance to his widow. The children's shares differ by R1320; find the widow's share.

9. A merchant buys in Madras 210 bags of rice at R10. 12*a*. 0*p*. per bag of 164 pounds. He sends them by rail 320 miles at $6\frac{1}{4}$ pies per ton per mile, but during the journey $7\frac{3}{4}$ pounds are stolen from each bag. Find at how many measures per rupee he must sell the remainder in oredr to clear R95. 15*a*. 0*p*. by the transaction. (One measure $= 3\frac{1}{8}$ pounds)

1878.

1. Find by *Practice* the value of

(*a*) 6 tons. 17 cwt. 2 qrs. 24 lbs. at R125. 6*a*. 8*p*. per ton.

(b) 29,764 articles at R1. 11*a*. $9\frac{3}{4}$*p*. each.

2. The materials of an old building were sold for R1,500 upon condition that they should be removed within 30 days under a penalty of R10 per day for every day beyond 30 days. The purchaser employed 40 men at $3\frac{1}{2}$ annas per day to do the work, and after selling the materials for R2365, he cleared R190 by his bargain. Find the number of days the men were at work.

3. (*a*) Divide ·0576 by 180, and by ·018.

(*b*) Find the value of

$$\frac{2\cdot8 \text{ of } 2\cdot\dot{2}\dot{7}}{1\cdot\dot{3}\dot{6}} + \left(\frac{4\cdot\dot{4} - 2\cdot8\dot{3}}{1\cdot\dot{3} + 2\cdot62\dot{9}} \text{ of } 8\cdot2\right).$$

4. *A* and *B* enter into partnership; *A* supplies the whole of the capital, amounting to R45000 upon condition that the profits are to be equally divided, and that *B* pays *A* interest on half the capital at 10 per cent. per annum but receives R120 per mensem for carrying on the concern. Find their total yearly profits when *B*'s share is equal to $\frac{1}{3}$ of *A*'s share.

5. Find the difference between the *true* discount on R259·2 due two years hence and the interest on the same sum for two years, allowing in both cases simple interest at 4 per cent. per annum.

6. A room, 21 feet long by $13\frac{1}{2}$ feet wide is surrounded by walls $1\frac{1}{2}$ feet thick and 14 feet high. There are two doors each $4\frac{1}{2}$ feet by 6 feet, and one window 3 feet by $4\frac{1}{2}$ feet. Find (1) the cost of building the walls at the rate of R5. 1*a*. 0*p*. per cubic yard, and (2) the number of bricks, each measuring 9 in. × 4 in. × $2\frac{1}{4}$ in., required for the work.

7. If 38 men working 6 hours a day can do a piece of work in 12 days, find in what time 57 men working 8 hours a day can do a piece of work twice as great, supposing 2 men of the first set to do as much work in 1 hour, as three men of the second set can do in $1\frac{1}{2}$ hours.

8. Extract the square root of ·002 and of 764·9, each to four places of decimals.

9. A person's net income from 5 per cent. Government paper is R1225 after paying income-tax at the rate of 2 per cent. Find the number of shares of R1000 each owned by him.

1879.

1. A person purchases 18,426 articles at Rupees 2-8-9¾ each, and 2,204 articles at Rupees 4-11-7½ each. He sells the whole number at R2-13-0 each. Find how much he gains by the transaction.

2. In 12 days 20 men could finish a piece of work. Being assisted by 6 women for 5 days, and by 10 women for the rest of the time, the work is done in 9 days. How long would 28 women take to do the work?

3. *A* sold a horse to *B*, who sold it to *C* at a loss of 10 per cent. *C* sold it for R891, and cleared 20 per cent. on his bargain. What did *B* give for the horse, and what gain per cent. was the last price on the first price?

4. Find the value of $\cdot\dot{9}\dot{6}$ of R9-1-9 + $\cdot5\dot{9}2\dot{5}$ of R7-5-0 + ·0027 of R 13-0-4.

5. Extract the square root of 4·376 and of $\cdot\dot{3}$ each to 4 places of decimals.

6. An oblong piece of ground measures 57 feet 3 inches × 36 ft. 7½ inches. From the centre of each side a path 5 ft. 4 inches wide goes across to the centre of the opposite side. Find the cost of paving these paths at the rate of R1-5-0 per square yard.

7. On what sum will the difference between the simple and compound interest for 3 years at 5 per cent. per annum amount to R13-11-7⅛?

8. A person in England has a certain sum invested in Indian 4½ per cent. Government bonds, which after deducting 2 per cent. as agent's charges for drawing and remitting the money, and when the rate of exchange is 1*s.* 7½*d.* per rupee, brings him an income of £429-19-6 per annum. Find the amount of the investment in rupees.

9. A train 132 yards in length, travelling at a uniform speed, overtook a man walking along the line at the rate of 6 miles an hour, and passed him in 12 seconds. Twenty minutes later the train overtook a second man and passed him in 11 seconds. How many hours after the train overtook the second man would the first man also overtake him?

1880.

1 The circumference of a circle being equal to 3⅐ times its diameter, find the diameter of an engine-wheel which makes three revolutions a second when the engine is moving at 40 miles an hour.

2. If 24 men build a wall 2½ miles long, 2 feet broad, and 6 feet

high, in 146 days of 10 hours each, what length of wall $2\frac{1}{4}$ feet broad, and 5 feet high, will 15 men build in 365 days, working 8 hours a day ?

3. Express $\cdot\dot{3}4\dot{5}$ of R16 . 0 . 8 – $\cdot07\dot{3}$ of R6 . 4 . 0 as the decimal of R8 . 9 . 3.

4. A person sold 86 measures of rice for R13 . 7 . 0 thus gaining 25 per cent. ; and 154 measures at a profit of 10 per cent. Supposing he had sold the whole at a profit of 16 per cent., how much more would he have gained ?

5. The length of a room is $32\frac{1}{2}$ feet. The cost of painting the walls at R1. 14. 0 per sq. yd. is R308 . 2 . 0 ; and the cost of carpeting the room at R2 . 4 . 0 per sq. yd. is R150 . 5 . 0. Find the height and width of the room.

6. Extract the square root of 6095961. Also of $\cdot006\dot{2}$ to four places of decimals.

7. Five men start to walk round a race course, which is $1\frac{1}{4}$ miles round. They walk at the rates of 3, $3\frac{3}{4}$, 4, $4\frac{1}{2}$ and 5 miles per hour. How long will it be before they all meet again at the starting point ?

8. If R32,000, put out at compound interest, amount in 2 years to R34,279$\frac{1}{5}$, what is the rate per cent. ?

9. A person leaves R6780 to be divided among his 5 children and 4 brothers, so that after the legacy duty has been paid, each child's share shall be twice as great as each brother's share. The duty on a child's share is one per cent., and on a brother's share 4 per cent. Find what amounts they respectively receive.

1881.

1 Find by *Practice* the cost of :—

8 cwt. 3 qrs. 12 lbs. at R27. 4*a*. 4*p*. per cwt.

7 mi. 5 fur. 165 yds. at R682. 7*a*. 4*p*. per mile.

2. A room measuring 42 feet 6 in. by 22 feet 9 in. inside, with walls 2 feet 3 in. thick, is surrounded by a verandah 10 feet 6 in. wide. Find the cost of paving this verandah with tiles measuring $4\frac{1}{2}$ in. by 3 in., and costing R3. 2*a*. 0*p*. per hundred.

3. A bankrupt has book-debts equal in amount to his liabilities, but on R8640 of such debts he can recover only $8\frac{1}{2}a$. in the rupee, and on R6300, only $5\frac{1}{4}a$. in the rupee. After allowing R1054. 11*a*. 0*p*. for the expenses of bankruptcy, he finds he can pay his creditors 12*a*. in the rupee. Find the total amount of his debts.

4. Extract the square root of $2329\frac{19}{81}$.

Also of $\frac{1}{10}$ to four places of decimals.

5. Reduce $\cdot0\dot{3}\dot{6}$; $\cdot001875$; $\cdot3\dot{9}0\dot{9}$ and $\cdot\dot{9}2\dot{5}$ to equivalent vulgar fractions in their lowest terms.

6. A sum of money in 10 years at $3\frac{1}{2}$ per cent. simple interest amounts to R727. 0a. 6p. In how many years would it amount to R840. 2a. 0p. at 4 per cent. ?

7. Find the cost in rupees of one mile of railway, which consists of two rails each weighing 40 lbs. per yard on wooden sleepers weighing 70 lbs. each placed 2 ft. 8 in. apart. The rails cost in England £6. 13s. 0d. per ton, and the sleepers 2s. $4\frac{1}{2}d.$ each The rate of freight is £1. 5s. 0d. per ton, and landing charges amount to R2. 8a. 0p. per ton. Rate of exchange 1s. 8d. per rupee.

8. For what sum should a cargo worth R26315 be insured at $7\frac{3}{8}$ per cent., so that the owner may recover in case of loss the value both of cargo and the sum paid for insurance ?

9. Two trains measuring 330 feet and 264 feet respectively, run on parallel lines of rail. When travelling in opposite directions they are observed to pass each other in 9 seconds. but when they are running in the same direction at the same rates as before, the faster train passes the other in $27\frac{1}{2}$ seconds. Find the speeds of the two trains in miles per hour.

1882.

1. What decimal fraction of a mile is 681 yds. 0 ft. $4\frac{8}{25}$ in. ?

2. Simplify $\left\{\sqrt[3]{\frac{\frac{1}{3}+\frac{1}{6}}{\frac{1}{6}-\frac{1}{7}}+2\left(\frac{\frac{1}{8}+\frac{1}{6}}{\frac{1}{6}-\frac{1}{8}}\right)}\right\}^2$.

3. The wheels of a cart are 13 ft. 6 in. in circumference. One breaks down, and is replaced by a new one, which is rather small To test it, the owner makes a chalk mark on each wheel where it touches the ground, and tells his man to drive over a piece of level road, and to count the turns made by each wheel until the chalk marks next touch the ground at the same time. The man obeys ; but, when he returns to his master, can only recollect that one wheel made one more turn than the other. His master, however, measures the distance traversed by the cart, 360 yds., and thence finds the circumference of the new wheel. What is it ?

4. Find the value of $\frac{15+\sqrt{(\cdot 009)}}{1-\sqrt{(\cdot 9)}}$ correct to three places of decimals.

5. (*a*) What is the smallest whole number which is divisible by $3\frac{1}{2}$, 15, and $17\frac{1}{2}$ without remainder ?

(*b*) What is the greatest number which will divide 3051 and 2331, leaving remainders of 8 and 4 respectively ?

6. The table below shows the marks gained at an examination in seven different subjects by a class of six boys *A*, *B*, *C*, *D*, *E*, *F*. Complete the table so as to show, correct to one place of decimal :—

(*a*) What percentage of the total marks is gained by each boy ;

(*b*) What percentage of the marks awardable in each subject is gained by the class ;

(*c*) What percentage of the total marks is gained by the class.

	Arithmetic 85	Algebra 80.	Euclid 70.	English 120.	History 110.	Geography 100	Hand writing 35.
A	33	27	12	95	79	63	3
B	76	49	52	73	67	82	15
C	48	69	43	61	58	85	21
D	53	41	27	91	61	47	23
E	71	62	39	85	73	68	14
F	47	18	21	78	92	27	12

7. Divide 5·89651 by 13·75854, expressing the quotient as a decimal.

8. A Bank advances R1,500 to a person on agreement that interest at the rate of 9 per cent. per annum shall be paid half-yearly for its use. The person fails to make any interest payment, and at the end of eighteen months, the Bank obtains judgment against him for the principal and compound interest at the rate and on the terms agreed to. Find to the nearest pie the amount he has to pay.

9. The roof of a verandah is supported by 16 teak beams, each 9 ft. long, 3 in. broad, and 5 in. deep. If the weight of a cubic inch of teak is $\frac{1}{2}\frac{5}{8}$ of that of a cubic inch of water, and if a cubic foot of water weighs 1000 oz., find the weight in lbs. of the timber in the verandah.

1883.

1. A cistern, whose capacity is 43092 gallons, is to be filled with water by a pipe which conveys 23 gallons 1 qt. per minute. On account of a leakage, the cistern is only just filled in 31½ hours. What is the average amount of leakage per hour?

2. I sold some goods, weighing 13 cwt. 2 qrs. 9 lbs. for £72-17-7½, gaining thereby 3½*d.* per lb. How much should I have gained per lb., if I had sold them at £5-12-0 per cwt.?

3. If 40 men and 50 boys can do a piece of work in 6 days, working 6 hours a day, in how many days will 8 men and 20 boys, do a piece of work half as large again, working 7 hours a day, assuming that a man does as much work in 3 hours as a boy in 5 hours?

4. Three equal circular wheels revolve round a common horizontal axis with different velocities. The first makes a revolution in 5⅓ minutes, the second in 2⅘ minutes, the third in 3⅗ minutes. Three marks, one in each wheel, are in a horizontal line at a certain moment. What is the shortest interval after which they will be in a horizontal line again?

5. Find, by Practice, the cost of 475 tons of coal at £2-16-8 per ton....If this is sold again for £1453. 10*s*. 0*d*., what is the whole gain, and what the gain per cent. ?

6. *A* and *B* start on a journey at the same time. *B* travels at $\frac{4}{5}$ths of *A*'s rate, and arrives 3 hours 15 minutes after him. In what time did each complete the whole journey ?

7. If an investment of £75 becomes £78. 15*s*. 0*d*. in eight months, what sum, invested at the same rate of interest, will become £201. 17*s*. 6*d*. in ten months ?

8. Simplify the expression :—

$$\frac{\sqrt{(75\frac{1}{9})} - 9\frac{7}{8} \text{ of } \cdot\dot{7}\dot{2}}{5\frac{5}{6} - \sqrt{(2\frac{7}{9})}} - \frac{10\frac{2}{11}}{3\cdot\dot{3}\dot{9} \times 3}.$$

9. *A* and *B* started on a race and ran a certain distance exactly together. Then *B* began to fail and gave up the race when he had run 56 yards further, *A* having gone during the same time 320 yards. The average of the entire distances run by the two men was 1188 yards. What distance had they run together ?

1884.

1. Simplify $\frac{5\frac{3}{4} + 4\frac{5}{8}}{8\frac{3}{8} - 5\frac{4}{7}} \div \left\{\frac{\frac{7}{8} - \frac{4}{7} + \frac{5}{6}}{\frac{4}{7} - \frac{7}{8} + \frac{2}{3}} \text{ of } (1\frac{3}{11} \times 3\frac{1}{7})\right\}$.

2. Find, by Practice, the cost of 15 tons 11 cwt. 3 qrs. 10 lbs. 8 oz. at R93. 5*a*. 4*p*. per ton.

3. Extract the square root of $1\frac{1}{15}$ to five places of decimals ; and divide $1\cdot43\dot{8}$ by $\cdot01\dot{3}$, giving the result in decimals.

4. When the rupee is worth 1*s*. 7½*d*., what is the nearest sum of Indian money equivalent to £79. 3*s*. 7½*d*. ?

5. A tea-merchant has a rectangular space for storing tea. It is 15¾ ft. long, 10½ ft. broad and 9⅓ ft. high. He wishes to fill this space with packets of a cubical shape all of the same size. What is the largest size of such cubical packets that can be made to fill it exactly, and what would be the number of such packets ?

6. *A* starts in business at the beginning of the year with R3000. On March 1st, he takes a partner *B*, with R4000. And on June 1st, he receives another partner *C*, with R5000. The profits at the end of the year amount to R1480. What share of the profits should each partner receive ? And what is the rate per cent. per month of the profits on the capital invested ?

7. What sum of money must I invest at 4 per cent. compound interest, so that I may gain R390. 3*a*. 2⅔*p*. in three years ?

8. A tradesman has been accustomed to give his customers — months' credit, but wishes to introduce the ready money system into his business. For how much ready cash should he sell an article that he has

hitherto sold for £8. 2*s.* 0*d.*, the rate of interest charged being 5 per cent. per annum?

9. What rate per cent. will be received for money invested in $3\frac{1}{2}$ per cent. stock, at 84?

10. Find the cost of building the walls of a rectangular room, 20 ft. long, 16 ft. broad, and 10 ft. high, with a door 7 ft. by 4 ft. and a window 5 ft. by 3 ft., at $2\frac{1}{2}$*a.* per cubic foot, the walls being 2 ft. thick.

1885.

1. Explain how the value of a fraction is not altered when its numerator and denominator are multiplied by the same number.

Simplify $\left(\dfrac{\frac{4}{11} \text{ of } 1\frac{4}{7} - \frac{5}{6} + \frac{9}{7}}{\frac{8}{7} \text{ of } 1\frac{5}{9} + 1\frac{3}{4}}\right) \times 4\frac{3}{10} - \frac{5}{6} \text{ of } \frac{4}{5}$.

2. If the rupee is worth 1*s.* $6\frac{3}{4}$*d.*, express R6 – 5 – 4 as a fraction of £1; and find the least number of rupees equal in value to an integral number of pounds.

3. State the rule for converting recurring decimals into vulgar fractions; and find the value of $0\cdot\dot{0}\dot{3}$ of $2\cdot7\dot{5}$ of £3. 2*s.* 6*d.* + $0\cdot\dot{2}8571\dot{4}$ of $1\cdot3$ of £7. 5*s.* 10*d.* – $0\cdot\dot{5}9\dot{2}5$ of £2. 16*s.* 3*d.*

4. Find by any method the value of 5 cwts. 2qrs. 21lbs. of goods at £3. 7*s.* 6*d.* per cwt.

5. The carriage of $17\frac{1}{2}$ cwt. for 52 miles on a certain railway is 8*s.* 4*d.*; find what will be the cost of carrying $4\frac{3}{4}$ cwt. for 300 miles on a railway on which the rate per mile is 9 per cent. lower.

6. A landlord pays 1 per cent. for collecting his rents and a tax of 7 pies in the rupee on what he receives after paying the collector. He has a clear rental of R1,831 – 8 – 0. Find his gross rental.

7. A grocer mixes four kinds of tea which cost him 5*s.*, 4*s.*, 3*s.*, 2*s.* per lb. respectively in the proportions of 2, 3, 4, 7 respectively. Find at what rate he must sell the mixture so as to gain 25 per cent. on the whole.

8. Define the terms *interest*, *discount*, and find in what time £533. 6*s.* 8*d.* will amount to £672 at $6\frac{1}{4}$ per cent. per annum simple interest.

9. A person invests £4800 in 4 per cent. stock at 96, and after a year sells out at $92\frac{1}{8}$ and invests the proceeds together with the interest for the year in stock at $96\frac{1}{4}$. How much stock does he then purchase?

10. Find to four places of decimals the square root of [illegible]; and calculate the cost of surrounding with a fence a square field of $22\frac{1}{2}$ acres at 3*d.* per yard.

11. The population of a country increases at the rate of 7 per cent. every 10 years. What was the population 20 years ago of a country whose present population is 4,007,150?

1886.

(*N. B.*—Answers in money must be stated in £. *s. d.* or in R *a. p.* as the case may be, and not as fractions of £1 or of R1.).

1. State and explain the rule for the multiplication of vulgar fractions.

Simplify $\dfrac{\frac{2}{3}(1\frac{2}{3} - \frac{5}{8}\text{ of }1\frac{3}{5}) + \frac{4}{17}}{\frac{3}{4} \times 1\frac{4}{7} \div 1\frac{1}{2} - \frac{11}{28}} \times \dfrac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}} - 20.$

2. Express £66-14-5¼ as the decimal of R1000, the rupee being worth 1*s.* 4½*d.*

3. Distinguish between pure and mixed recurring decimals.

Find the value of $0\cdot\dot{9}4\dot{5}$ of £2-3-6½ + $0\cdot37\dot{2}5\dot{9}$ of £1-8-1½.

4. Find by any method the rent of 156 ac. 3 r. 24 p. 11 sq. yds. at R25-3-4 per acre.

5. A clock which gains 3 m. 56 s. in 24 hrs. was set correctly at noon on the 1st of January 1884. Find to the nearest minute the next date at which it indicated correct time.

6. Twenty men are employed to make a tank 40 ft. long, 20 ft. broad, and 6 ft. deep. They work for 30 days and have just completed one-third of the work, when it is resolved to increase the length of the tank by 10 ft., the breadth by 4 ft. and the depth by 2 ft. How many additional men must be employed in order that the work may be completed in 30 days more?

7. The difference between the simple and compound interest on a sum of money for 3 years at 5 per cent. is £7-12-6. Find the sum.

8. The capital of a certain railway is £1000000 in 20000 shares of £50 each, fully paid up. The gross annual receipts are 105000 of which 48 per cent. is absorbed in working expenses, £4600 goes to the reserve fund, and the remainder to pay dividend. Find what annual income a person will obtain from the investment of £4500 in the undertaking, the shares being at £62-10-0.

9. Ice is manufactured for 6 pies a pound. Two-thirds of the quantity made is kept for sale at the factory and the remainder is sent to branch shops. If the average loss from melting of the former be 12½ per cent. and that of the latter be 25 per cent., find the gain on every ton made.

10. The average width and depth of a river at its mouth are 240 yds. and 6 feet respectively, the average rate of flow is 3 miles per hour, and the amount of sediment per cubic foot of water discharged is 1½ cubic inches. Find the amount of sediment deposited annually; and the depth of the deposit, supposing it spread uniformly (*i.e.* to the same depth throughout) over an area of 146 square miles.

[illegible]

(*N. B.* (1) Answers in [illegible] must be stated in £. *s. d.* or in R. *a. p.* as the case may be and not as [illegible] of £1 or R1. (2) Except in the

Question 1, the process by which each result has been obtained must be given in full).

1. Add together (without copying out) the following sums, and write down the results :—

(1)	£	s.	d.	(2)	R	a.	p.
	2065	19	$6\frac{1}{2}$		20581	15	4
	149	0	7		3690	4	11
	6695	4	$3\frac{3}{4}$		28	11	7
	12964	13	$11\frac{1}{2}$		308	8	8
	24	8	$9\frac{1}{4}$		19075	0	5
	1549	17	$5\frac{3}{4}$		9	14	6
	707	3	10		207	9	10
	19208	10	$0\frac{1}{2}$		53968	13	1
	6	15	$9\frac{3}{4}$		6072	8	7
	358	1	$10\frac{1}{3}$		90	1	9
	58877	4	$11\frac{1}{4}$		7782	12	3
	4059	17	$5\frac{3}{4}$		30259	15	10

2. Simplify $\dfrac{6\frac{3}{8}-4\frac{1}{8}}{5\frac{1}{2}-4\frac{7}{8}} - \dfrac{2\frac{2}{15} \div 1\frac{11}{18} + 1\frac{1}{4} - \frac{9}{13} \text{ of } 3\frac{1}{2}}{\frac{3}{5} \times 3\frac{2}{7} - 5\frac{3}{5} \div 3\frac{4}{15}} \times 13\frac{1}{2}.$

3. Find the value of $1\frac{3}{17}$ of $\cdot 012\dot{3}\dot{6}$ of R5-11-8 ; and taking the rupee as worth 1*s.* $4\frac{1}{2}$*d.*, express the result as the decimal of one shilling.

4. Find by any method the value of 9 tons. 17 cwt. 3 qrs. 25 lbs. of coffee at £72-18-4 per ton.

5. When iron is at £3-7-6 a ton, the cost of laying a railway 10 miles 2 fur. 20 po. in length with rails weighing 270 lbs. each is R67500. Find the cost of laying a railway 25 miles 220 yds. long with rails of the same length weighing 500 lbs. each, when iron is at £3-14-3 a ton.

6. Find the present value of £482-6-$10\frac{1}{2}$ due 3 years hence at 5 per cent. compound interest.

7. When exchange is at the rate of 1*s.* $4\frac{1}{2}$*d.* per rupee, a person in Madras orders from a bookseller in England a parcel of books, the published price of which is £5. The bookseller allows discount at the rate of 25 per cent. on the published price, but includes in his bill a charge of 13*s.* for packing, freight, &c. When the books arrive in India, a further sum of R2-8 has to be paid on account of landing charges and cost of delivery. If the books can be obtained from a bookseller in Madras at the rate of $9\frac{1}{2}$ annas per shilling of the published price, find how much the person loses by ordering from England.

8. A person holds forty Rs. 500 shares in a concern which pays dividend at the rate of 6 per cent. per annum. When the shares are at R675, he sells out and invests half the proceeds in 4 per cent. stock at 90. With the other half he buys a house, for which he receives an annual rental of R1,440, subject to a deduction of 3*a.* 9*p.* per rupee for repairs and taxes. Find the alteration in his annual income.

9. In a certain year a country produces 50,000,000 bushels of wheat. Of this quantity 40 per cent. is available for export at R3-2 per bushel.

In the following year the acreage under wheat has increased 20 per cent. but the yield per acre is only seven-eighths of what it was in the previous year, while the quantity required in the country has increased 5 per cent. If at the same time the export price has fallen to R3. per bushel, find the increase in the value of the wheat available for export.

10. The population of a country is 33,264,000, and there are 99 males to 101 females. 2 out of every 11 boys and 1 out of every 33 girls of school-age are under instruction. If the boys of school-age form one-seventh of the male population, and the girls of school-age form one-seventh of the female population, find the total number of pupils under instruction.

1889.

[*N. B.*—(1) Answers in money must be stated in £. *s.* *d.* or in R. *a.* *p.* as the case may be, and not as fractions of £1 or of R1. (2) Except in the case of question 1., the process by which each result has been obtained must be given in full.)

1. Add together (without copying out) the following sums and write down the results:—

£.	*s.*	*d.*	R	*a.*	*p.*
172	19	$7\frac{3}{4}$	12851	3	4
4372	13	$6\frac{1}{2}$	208	13	10
267	11	9	3796	10	7
29	3	$0\frac{3}{4}$	82	1	9
7901	9	$11\frac{1}{4}$	53028	9	5
99	7	$8\frac{1}{2}$	203	15	11
5	3	$10\frac{1}{4}$	8888	9	1
149	0	7	535	15	8
6	15	$9\frac{1}{2}$	26	0	10
1000	6	$5\frac{3}{4}$	24370	12	2

Simplify $\dfrac{\frac{1}{8}+\frac{1}{7}\div(\frac{1}{7}-\frac{1}{6})}{(\frac{1}{8}+\frac{1}{7})\div\frac{1}{7}-\frac{1}{6}}\div\dfrac{1\frac{1}{5}+\frac{3}{28}}{1\frac{7}{8}-\frac{1}{4}}-\dfrac{\frac{1}{2}\text{ of }\frac{1}{8}}{\frac{1}{2}\div\frac{1}{8}}$.

3. Multiply $41\cdot36\dot{5}1\dot{4}$ by $\cdot001\dot{9}$, expressing the result as a decimal; and find the value of $\cdot3472$ of £1. 4*s.* − $\cdot032\dot{8}8$ of £2. 6*s.* 3*d.*

4. Find by any method the cost of 79 ca. 17 m. 5 v. 25 pal. of salt at R21. 10*a.* 8*p.* per candy.

5. The cost of rice for a family of 2 adults and 3 children from January 1st, 1889, to December 11th, 1889, both days inclusive, during which time rice was selling at 15·4 seers per rupee, was R70. 7*a.* What will be the cost of rice for a family of 3 adults and 5 children from December 19th, 1889, to May 11th 1890, both days inclusive, assuming that the price of rice will be 14·7 seers per rupee, and assuming also that the quantity required per day by each adult is the same in both cases, and that in both cases the quantity required by a child is two-fifths of the quantity required by an adult?

6. On what sum due 1 year 4 months hence does the true discount amount to £100. 18*s.* 9*d.*, simple interest being reckoned at $4\frac{3}{4}$ per cent. per annum?

7. How much 3 per cent. stock must a person sell when the selling price is 91, in order that by investing the proceeds in the $4\frac{1}{2}$ per cents. at $113\frac{3}{4}$ he may derive from the investment an annual income of R9817. 8*a.*, after paying income-tax at the rate of 5 pies per rupee?

8. *A* and *B* can do a piece of work in 10 days, *B* and *C* in 15 days, and *C* and *A* in 20 days. They all work at it for 6 days; then *A* leaves, and *B* and *C* go on together for 4 days more. If *B* then leaves, how long will *C* take to complete the work?

9. In a certain year the total amount received by a railway company for the carriage of passengers was R2751000. Of this sum 6 per cent. was contributed by first class passengers, 5 per cent. by second class, and the remainder by third class. The fares were 18, 6, and $1\frac{1}{4}$ pies per mile for first, second, and third class passengers respectively. Assuming that the average distance travelled by each third class passenger was 36 miles, and the average distance travelled by each passenger of the other classes was 160 miles, find the total number of passengers carried during the year.

10. The length of a rectangular field is twice its breadth. If the rent of the field at £3. 7*s.* 6*d.* an acre is £151. 17*s.* 6*d.*, find the cost of surrounding it with a fence at $4\frac{1}{2}$*d.* per yard.

11. Extract the cube root of 9 to five decimal places.

1890.

1. Reduce 2149908480 sq. inches to acres, etc. If this is the area of a rectangle the length of which is 5 m. 7 fur. 5p. 1 ft. 6 in., find its breadth.

2. Simplify $\frac{1835}{2202}+\frac{5468}{12303}+\frac{147}{441}-3\frac{1}{7}$ of $\frac{6{\cdot}25}{5{\cdot}5}$ of $\frac{{\cdot}04}{1{\cdot}\dot{2}8571\dot{4}}$.

3 Find the value of 237 candies 17 maunds 6 viss at R4100. 1*a.* 4*p.* per candy.

4. 300 coolies are set to build a tank-bund. In 14 weeks they have done $\frac{7}{10}$ of the work when rain stops the work for 4 weeks and washes away $\frac{3}{8}$ of what they have done. At the end of that time the work is resumed with only 250 coolies. In what time from the commencement will the work be finished?

5. Find the amount of R5859375 for 3 years at $4\frac{1}{4}$ per cent. per annum, reckoning compound interest.

6. Explain the difference between discount and interest. If the discount on £2830. 15*s.* $7\frac{1}{2}$*d.* be equal to the simple interest on £2784. 7*s.* 6*d.* for the same time, find the time, the rate of interest being 5 per cent. per annum.

7. A person invests £34539 in the 3 per cents, at 87. After receiving one year's dividend he sells out at 89. He then invests the whole in Rail-

-way stock, paying 5 per cent. at 115. What will the difference in his income be?

8. A cistern 10 ft. 6 in. long by 7 ft. 6 in. wide, by 3 ft. 4 in. high is lined inside with lead, 7 lbs. of which cover a square foot. Find the weight of the lead and its cost at 53*s.* 4*d.* per cwt.

9. A cask contains 16 gallons of spirit. Two gallons are drawn off and the cask filled up with water. Two gallons are again drawn off and the cask filled up as before. This is done a third time. Compare the quantities of spirit and water remaining in the cask.

10. Find the square root of 379749833·583241.

1891.

Add together

£	s.	d.
104	14	$6\frac{3}{4}$
39	11	$4\frac{1}{2}$
166	15	$0\frac{1}{4}$
27	0	$5\frac{1}{2}$
1103	19	$3\frac{1}{2}$
1002	15	$4\frac{3}{4}$
6	3	$11\frac{1}{2}$
32234	15	$7\frac{1}{4}$
8192	12	6
8	4	10
13	7	$0\frac{1}{2}$

2. Subtract 13 times ₨17. 6*a.* 11*p.* from 17 times ₨13. 6*a.* 11*p.*

3. ₨330. 3*a.* 7*p.* are to be divided among 193 persons, two of whom receive ₨2 each, and ten ₨3 each. The others receive equal shares. Find the value of each share.

4. Find the value of $\dfrac{\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}}{\frac{1}{3}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}} \times 3\frac{1}{2} \div \dfrac{\frac{1}{15}+\frac{1}{20}+\frac{1}{24}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}-\frac{1}{6}}$, and simplify (without reduction to vulgar fractions if you can)

$$2\cdot03 + 1\cdot\dot{3}\dot{4}5 + 27\cdot34 + 16\cdot\dot{2}3\dot{1}7.$$

5. How long will it take to walk round a square field 14 acres 1 rood 24 poles in extent at the rate of 3 miles an hour?

6. Find the cost of white-washing a room $22\frac{1}{2}$ ft. × 12 ft. and 11 ft. high, at one anna per square yard, making allowance for four windows each 4 ft. × $2\frac{1}{2}$ ft. and two doors each $8\frac{1}{2}$ ft. × 4 ft. Find also the cost of a carpet for the same room with 3 ft. border all round the carpet, costing ₨4 per square yard and the border ₨6 per square yard.

7. Find the compound interest on £3143. 6*s.* 8*d.* for 3 years at 3 per cent. per annum.

8. A cistern can be filled by three pipes in 30, 40, and 60 minutes respectively, and emptied by an escape-pipe in half an hour. The three

taps are turned on at noon, but the escape pipe is at the same time accidentally left open and not closed for a quarter of an hour. At what time will the cistern be full?

9. I purchase 16 lbs. of tea at *1s. 7d.* per lb., 14 at *2s. 2d.* and 17 at *1s. 8d.* Seven pounds of the mixture becoming spoiled have to be sold at a low price, but by selling half the remainder at *2s. 4d.* per lb. and the other half at *2s. 7½d.*, I eventually make a profit of 25 per cent. on the original outlay. At what price per pound was the spoiled tea sold?

10. A person invests a sum of money in the 4 per cents. at 102. When they have risen to 104, he transfers R5000 stock to another investment paying 5 per cent. of which the shares are at 120. When the 4 per cents. fall to par, he transfers the remainder to the 5 per cents., which are still at the same price and now finds his income R25 more per annum than it was at first. What was the sum originally invested?

1892.

(*N. B.* (1) Answers in money must be stated in £. *s. d* or in R. *a. p.* as the case may be, and not as fractions of £ 1 or R. 1; (2) Except in the case of question 1., the process by which each result has been obtained must be given in full.)

1. Add together (without copying out) the following sums, and write down the results:

	tons.	cwts.	qrs.	lbs.
(1)	3124	17	2	27
	65097	3	1	19
	385	14	3	6
	20039	10	0	13
	1470	11	2	19
	38965	0	1	7
	13	7	3	21
	5082	8	3	14
	97654	19	1	0
	38046	16	0	25
	909	3	2	11
	41203	15	3	26

	ca.	md.	v.	pal.
(2)	19937	19	7	39
	2016	13	3	25
	26	10	2	19
	4309	17	6	35
	18197	9	4	12
	98006	14	0	26
	1779	0	5	17
	909	7	3	31
	1472	18	7	9
	99	15	2	37
	3201	7	0	21
	69547	12	3	16

2. Simplify $\dfrac{\frac{4}{5} \text{ of } 1\frac{3}{8} - \frac{3}{8} \text{ of } \frac{4}{5}}{1 - \frac{1}{7} \times (\frac{6}{11} + \frac{1}{3})} \times \dfrac{\frac{1}{4} + \frac{1}{7} \div (\frac{1}{7} - \frac{1}{8})}{(\frac{1}{8} + \frac{1}{7}) \div \frac{1}{7} - \frac{1}{8}}$.

3. Find the value of $\cdot041\dot{6}$ of £33-7-6 − $\cdot03\dot{4}\dot{5}$ of £32-13-1½; and express R371-2-6 as the decimal of a lakh of rupees.

4. Find by any method the cost of making a road 37m. 6f. 31p. 3yds. long at R1785-3-4 per mile.

5. Find the present value of £482-6-10½ due three years hence at 5 per cent. per annum compound interest.

6. Extract the square root of 13·697142031225 to six places of decimals.

7. The annual rainfall of a district is 49·7 inches. Assuming that the fall is distributed uniformly over the district, and that a cubic foot of water weighs 62·5 lbs., find the weight in tons of the rain that falls throughout the year on a square mile.

8. When exchange is 1*s.* 2⅜*d.* per rupee, a Madras bookseller sends to a London publisher a bill for £104 in payment of books ordered. Freight and landing charges amount to R37-8. The publisher allows the bookseller discount at the rate of 35 per cent. on the published price, and the latter sells the books at the rate of 10½ annas per shilling of the published price. Find how much he gains on the transaction.

9. In the year 1891, the cost of rice for a family of 2 adults and 4 children was R86-7-9. In that year rice sold at 11·2 seers per rupee, and each child received two fifths of the amount given to an adult. Assuming that in 1893 the price of rice will be 13·5 seers per rupee, what will be the cost of rice for the same family from January 5 to August 11 both days inclusive, if the allowance of each adult be increased by one-fourth and the allowance of each child be three-sevenths of that of an adult?

10. The capital of a railway company amounts to R18,90,00,000 of which one-fourth is 5 per cent. preference stock and one-third 4½ per cent. preference stock. In a certain year the receipts are R1,81,50,000, and the working expenses amount to 55 per cent of the receipts. Of the net receipts R540000 are added to the reserve fund, and the remainder, after paying dividend on the preference stock, is divided among the ordinary shareholders. What rate of interest will they receive?

11. In the ten years from 1871 to 1881 the population of a country increased at the rate of 9·5 per cent., and in the ten years from 1881 to 1891 the rate of increase was 10·5 per cent. If the population in 1891 was 31,023,759, find what it was in 1871.

1894.

[*N. B.*—(1) Answers in money must be stated in £. *s.* *d.* or in R. *a.* *p.* as the case may be, and not as fractions of £1 or of R1.

(2) ... in the case ... 1. the process by which each result has be[en] ... [o]btained must be ...

1. Add together (without copying out) the following sums, and write down the results.

(1)

R.	a.	p.
166595	13	5
5598	6	11
201550	4	11
157726	4	9
24066	2	6
8339	15	10
190131	7	3
483	11	11
403663	11	8
972177	12	7

(2)

m.	fur.	po.	yd.	ft.	in.
26	7	25	4	2	9
3	0	6	3	2	5
209	4	37	2	1	6
43	6	0	0	2	10
95	0	29	1	1	7
1	5	19	4	0	8
10	3	9	3	2	11
179	6	39	3	0	4
83	1	21	2	1	5
101	7	13	4	2	9

2. Simplify $\dfrac{\frac{3}{4}-\frac{1}{6} \text{ of } \frac{7}{8}+\frac{5}{6}}{\frac{4}{9}\div\frac{6}{7}-\frac{1}{8}+\frac{2}{3}} - \dfrac{\frac{6}{11}-\frac{4}{9}}{1-\frac{5}{6}\div\frac{3}{8}}$.

3. Find the value of $2\cdot0475\dot{2}$ of £2. 2*s.* 1*d.*—1·734375 of £2. 6*s.* 8*d.*

4. Find by any method the value of 59 ca. 14 m. 7 v. 27 pal. of salt at R26. 10*a.* 8*p.* per candy.

5. In a certain year the produce of a tea-estate was sold in London at an average rate of $9\frac{1}{2}$*d.* per lb., and the amount realised was remitted at an average rate of exchange of 1*s.* $2\frac{1}{4}$*d.* per rupee. In the following year the average price realised was only $8\frac{3}{4}$*d.* per lb., but the quantity sold exceeded by $12\frac{1}{2}$ per cent. the quantity sold in the previous year and the average rate of exchange at which remittances were made fell to 1*s.* $1\frac{1}{8}$*d.* If in this year the total amount realised from sales in London was R105000, find how much was realised in the previous year.

6. A sum of money was invested for four years, interest payable annually. The rate of interest was 5 per cent. per annum for the first two years and 4 per cent. per annum for the last two; and the amount at the end of four years was £1,164. 10*s.* $3\frac{3}{4}$*d.* What was the sum invested?

7. Ice is manufactured for $2\frac{1}{2}$ pies per lb. and sold at 6 pies per lb. Of the total quantity made one half is kept for sale at the factory, and the remainder sent to branch shops. The loss from melting is $12\frac{1}{2}$ per cent. in the case of the former and 25 per cent. in the latter; and the agents at the branch shops receive commission at the rate of 15 per cent. on the price of every pound sold by them. Find the profit on every ton of ice manufactured.

8. Two persons, *A* and *B*, set out together on a journey. They walked at the rate of 3 miles an hour; and after they had proceeded for three quarters of a mile, *B* returned, walking at the same rate, to the place of starting. Here he was detained three quarters of an hour. Setting out again he overtook *A*, who had been walking all the time, at the end of $2\frac{1}{2}$ hours from the second time of starting. At what rate did he walk?

9. A person sold 25 Bank of Madras shares and invested the proceeds in the Government $3\frac{1}{2}$ per cents. when they were at $3\frac{1}{4}$ premium. If his net annual income from the investment, after paying income-tax at the

rate of 5*p*. in the rupee, be R876-9-0, find the price at which he sold each of his bank shares.

10. In the year 1891 the population of a country was 35640000 and there were 1025 females to every 1000 males. Of the total population 7·5 per cent. could read and write, but of the females only 1 per cent. could do so. Find what percentage of males could read and write.

11. Extract the square root of 81·1318315970410I to seven places of decimals.

ENTRANCE EXAMINATION PAPERS.

PUNJAB.

1875.

1. Write in figures one million, ten thousand and one. Subtract 397 from 1163 and explain the process.

2. Shew that when any number is divided by nine, the remainder is the same as when the sum of the digits is divided by nine.

3. State the rules for the multiplication and division of vulgar fractions. What is a complex fraction? and simplify

$$(1)\quad \left\{\frac{8}{17}+\frac{3}{8}\text{ of }7\frac{1}{2}\right\}\div 1\frac{8}{9}\ \text{ and }\ (2)\ \frac{\frac{8}{9}}{8}\div\frac{\frac{8}{7}}{\frac{6}{8}+7\frac{1}{2}}.$$

4. What is the value of ·3375 of an acre?

Reduce £1-10-4 to the decimal of two guineas.

5. Find the square root of 998001 and that of 3·14159 to three places of decimals.

6. If five pumps each having a length of stroke of 3 feet, working 15 hours a day for 5 days, empty the water out of a mine; how many pumps with a length of stroke of 2½ feet, working 10 hours a day for 12 days, will be required to empty the same mine; the strokes of the former pumps being performed four times as fast as those of the other?

1876.

1. How many revolutions will a cart wheel of three feet six inches diameter make in going a distance of 6 miles, the ratio of the diameter of a circle to its circumference being given as 1 : 3·14159.

2. A piece of land measuring 48 ghumas 3 kanals and 17 marlas of which 29 ghumas 4 kanals and 17 marlas are cultivated and the rest uncultivated is sold at the rate of R75/- a ghuma for cultivated and R33/- a ghuma for uncultivated land. What is the price of the whole?

3. The revenue of a village containing 15756 acres of cultivated land is assessed at 13 annas an acre. What will the local rate of $6\frac{1}{4}$ per cent. on the land revenue payable by the village amount to?

4. A bania purchases 1525 maunds of grain at 36 seers for a rupee. He sells one half at 26 seers the rupee; at what rate must he sell the remainder so as to clear 50 per cent. on the transaction?

5. Find the interest on 24485 rupees for 1 year and 131 days at 12 per cent. per annum.

6. A man hires a workman on this condition that for every day he worked he should get one rupee, but that for every day he was absent he should be fined 12 annas. When 356 days were past, the workman was to receive R118. How many days had he worked?

1877.

1. If a pound of pure silver be worth 62 shillings, the shilling containing 222 parts of pure silver in 240, what will be the value in shillings of a rupee weighing 180 grains, the rupee containing 979 parts of pure silver in 1,000?

2. (*a*) How much is ·0125 of a day?

(*b*) Find the value of $3\frac{2}{5}+4\frac{1}{8}+1\frac{11}{40}+3\frac{13}{625}$.

Express the result both as a vulgar and a decimal fraction.

3. Divide ·10724 by ·003125 and extract the square root of the result to 3 places.

4. (*a*) What sum at simple interest will amount to R6,000 in six years at 4 per cent. per annum?

(*b*) How much Government paper of the six per cent. can be bought for R500 when the funds are at 94 and what dividend will be got on it yearly?

1878.

1. If 135 rupees 4 annas be divided equally amongst 24 persons, what will each receive?

2. Define a vulgar fraction. By how much does the difference of $1\frac{1}{26}$ and $\frac{8}{15}$ fall short of their sum? Express the defect as a decimal of 7.

3. (*a*) Subtract ·03 from $\cdot\dot{0}\dot{3}$ and divide the result by $\cdot 10\dot{2}$.

(*b*) Shew that $\dfrac{1}{7+\frac{1}{16}} = \cdot 14159$ nearly.

4. A room whose height is 11 feet and length twice its breadth, takes 143 yards of paper 2 feet wide for its four walls; how much carpet will it require?

5. At what rate (simple interest) will 1,300 rupees amount to 1,381 rupees 4 annas in 15 months ?

6. Find the square root of ·1 to 3 places of decimals. What number has ·01 for its square root ?

1879.

1. (*a*) Show by an example that if the numerator and denominator of a fraction be divided by the same number, the value of the fraction is not altered. (*b*) Reduce to their lowest terms [illegible] and [illegible] and express their difference in decimal form.

2. Simplify $\dfrac{\frac{1}{4}+\frac{11}{14}}{\frac{2}{7}+\frac{[illegible]}{11}} \div \dfrac{\frac{1}{5}+\frac{2}{3}}{\frac{1}{3}+\frac{2}{7}}$.

3. One cubic inch of water weighs 253·17 grains, while one cubic inch of air ·31 grain ; find the number of cubic inches of water (to three places of decimals) that would be equivalent to one cubic foot of air.

4. (*a*) What portion of R34 . 8*a*. is $\frac{3}{12}$ of $\frac{5}{9}$ of R50 − $\frac{7}{18}$ of R10½ ?

(*b*) Find (accurately to 4 places of decimals) the square root of ·001.

5. A rectangular field measures 6 acres and 960 yards ; its length is 3 times its breadth ; find the distance between the diagonal angles.

1881.

1. Distinguish between a vulgar fraction and a decimal fraction and show how to reduce one to the other.

2. Divide the continued product of ·021, ·0021, and 210 by that of ·14 and ·007 ; and extract the square root of 5·005 to four places of decimals.

3. Express $\dfrac{\frac{5}{8} \text{ of } 1\cdot\dot{5}}{2\cdot7+\frac{1}{3} \text{ of } \cdot\dot{3}}$ of a rupee to the decimal of a guinea (= R10½.)

4. A person withdrew R5000 from a bank, which paid him interest at 5½ per cent. and invested the money in the 6 per cent. Municipal Debenture at 103½. Find the change in his income.

1883.

1. (*a*) Divide the difference of ·4607 and ·00809 by the difference of 6[illegible] and 5[illegible].

(*b*) Prove that $\dfrac{3+4}{4+5}$ is greater than $\frac{3}{4}$ and less than $\frac{4}{5}$.

2. Divide $\frac{1}{2}[3+\frac{1}{2}\{3+\frac{1}{2}(3+1\frac{1}{2})\}]$ by ·125.

3. (*a*) Shew that the value of a decimal is not altered by adding ciphers to the right hand side.

(*b*) Find the value of $7\cdot\dot{5}\dot{7} \times \cdot 3\dot{6} - 2\cdot 3\dot{4}\dot{5}$ in vulgar fraction.

4. A railway train having travelled at $\frac{4}{5}$ of its proper speed reaches its journey's end $2\frac{1}{2}$ hours behind time; in what time should the journey have been done?

5. Five hundred boys are distributed in three houses; the smallest house contains $\frac{7}{25}$ of the whole number and the largest contains $\frac{11}{7}$ of the smallest: what is the number in each?

6. A person realises ₨185500 by selling his $3\frac{1}{2}$ per cent. stock at $92\frac{3}{4}$. He invests one-fifth of the realised money in the 4 per cents. at 96, and the remainder in 3 per cents. at 90. What is the difference in his income by this transaction?

1884.

1. Multiply and divide ₨625 by ₨25, if you think the operations possible. Give your reasons.

2. State and explain the rules for multiplying and dividing one decimal number by another; exemplify by multiplying ·0256 by 1·05 and ·105 successively, and dividing the results by ·00105.

3. Simplify $\dfrac{4}{7\cdot 5}\left\{\dfrac{\cdot\dot{3} + \dfrac{\cdot 75}{4\cdot 5}}{\dfrac{1 - \cdot 25}{2 - \cdot 5}} + \dfrac{7}{8}\right\}$.

4. Extract the square root of $\dfrac{1000\cdot 20001}{1000}$.

5. Find by Practice the value of 45 mds. 22 seers and 10 chts. of grain at ₨1. 6*a.* per maund.

6. The assets of a bankrupt consist of ₨9560. 4*a.*, a bankshare of ₨1200 quoted at $107\frac{5}{8}$, and an undiscounted bill of ₨3225, due 4 months hence at 4 per cent. per annum simple interest; his liabilities amount to ₨25014. How much in the rupee can he pay his creditors?

7. Compare the ratios $\dfrac{\sqrt{5}}{\sqrt{3}}$ and $\dfrac{31}{27}$.

1885.

1. Simplify $\dfrac{[illegible]}{[illegible]} + \dfrac{[illegible] + \frac{1}{10}}{[illegible] - [illegible] \text{ of } [illegible]}$ of [illegible] − [illegible], and find how many times ·027 can be taken from 3·33.

2. Convert $\frac{13}{20 \times 8}$ into a decimal : why is the result a terminating and not a recurring decimal ? Subtract ·03 from $\cdot\dot{0}\dot{3}$ and divide the result by $\cdot 00\dot{7}$.

3. Find, by Practice, the value of 12 maunds 8 seers 4 chataks of ghee at R72. 8a per maund.

4. A legacy of £1901. 5*s*. is to be distributed amongst a number of persons, in such a way that each shall receive as many shillings as there are persons ; what will be the portion of each ?

5. Find the Least Common Multiple of 35280 and 592704. What is the smallest number of square yards which can be measured either by roods or square chains ?

6. Four per cents. are offered at R98, five per cents. at R120$\frac{3}{8}$; which is the better investment ? How much is one investment when the difference of income is R30 ?

1886.

1. Simplify $\frac{4\cdot\dot{4} - 2\cdot\dot{0}\dot{3}}{1\cdot\dot{6} + 2\cdot\dot{6}2\dot{9}}$, and extract the the square root of the result to three places of decimals.

2. Reduec $\dfrac{}{7 - \dfrac{}{2 - \frac{1}{2}}}$ to a decimal fraction correct to four places.

Is there anything to suggest that the result will be a terminating or a recurring decimal ?

3. What fraction of £51120. 18*s*. is 17·975 of £71. 2*s*. ?

4. A clever housekeeper went out shopping and found that 2 cocoanuts were selling for the same price as 144 plums ; she hought half a dozen cocoanuts, exchanged one of them for 5 melons, and a couple of melons for 5 oranges ; she then gave 3 oranges for 42 limes, and finally secured a couple of plums for 5 limes. Has she gained or lost in buying the plums ?

5. Distinguish between Interest and Discount.

Find the Interest and Discount of R1450. 8*a*. for 3 years at 4½ per cent. per annum, simple interest.

1887.

1. (*a*) Write in figures—three billions, five millions four hundred and nine thousand and sixty-two.

(*b*) Write out measures of length and surface, both English and Indian.

(*c*) Express an acre as the decimal of a *bigha*, a cubit being equivalent to 18 inches.

2. Owning $\frac{4}{17}$ of an estate, I sold $\frac{8}{11}$ of $\frac{3}{8}$ of my share for £[illegible]; what is the value of $\frac{1\frac{3}{8}}{4\frac{1}{4}}$ of $\frac{3}{5}$ of the estate at the same rate?

3. A merchant having 100 maunds of grain sold 50 maunds at R9 per maund, and thereby gained $7\frac{1}{2}$ per cent. At what rate should he sell the remainder 50 that he may gain 10 per cent. on the whole?

4. A merchant in trade successively admits three partners at the end of 3 months, 5 months, and 6 months respectively from the opening of the business. The capitals embarked by them were R400, R450, R480 and R495 respectively. After 6 months more, the profit was found to be R1518. Divide this rateably between the partners.

5. What sum of money invested in the 4 per cents. at par would realise the same income as R10200 invested in the $4\frac{1}{2}$ per cents. at 102?

6. Extract the square root of—

$$\frac{\cdot 0025 \times 1\cdot\dot{6}}{3\cdot\dot{6} - 2\cdot 5} \text{ of } \frac{\cdot 42\dot{6} \times 2\cdot 625}{12\cdot 7 - 10\cdot 2}.$$

1888.

1. Simplify

$$\frac{1}{1-\frac{1}{12}} - \frac{1-\frac{7}{72}}{2-\frac{1}{3}} \cdot \frac{1\frac{2}{5}}{4-1\frac{1}{2}} - \frac{6\frac{3}{8}-\frac{3}{4}}{6\frac{1}{4}} \times \left\{\frac{1}{5} - \frac{\frac{1}{2}-\frac{1}{8}}{4\frac{3}{4}-3\frac{5}{8}}\right\}.$$

Express the difference between $\cdot\dot{3}7\dot{8}$ of 13*s.* $10\frac{1}{2}$*d.* and $\cdot\dot{3}7\dot{8}$ of 16*s.* 6*d.* as a decimal of $\cdot 42\dot{6} \times \frac{3\cdot\dot{3}}{\cdot 8} \times \frac{\cdot 3}{\cdot\dot{7}3\dot{5}} \times \frac{\cdot 147 \times 4\cdot\dot{4}}{11\cdot 1}$ of £1. 17*s.* 6*d.*

3. Four men working together all day, can finish a piece of work in 11 days, but one of them having other engagements can work only half time and another only quarter time. How long will it take the men to complete the work?

4. A merchant sells his goods worth R500 directly for R600 giving three months' credit. Find his profit per cent., interest being calculated at 12 per cent. per annum.

5. Find the value of $\frac{\cdot 12 + \sqrt{\cdot 009}}{1 - \sqrt{\cdot 4}}$ correct to 3 places of decimals.

1889.

1. Express 80080080·0975 in words and give the local value of the digits. What decimal of R75 is R24. 2*a.* 6*p.*?

What is the least number which when divided by 22, by 88, by 132 and by 198 gives in each case remainder 7?

2. Why is the fraction $\frac{0}{0}$ objectionable ?

After walking $4\frac{1}{2}$ miles, a man has accomplished $\frac{2\frac{1}{2} - 1\frac{1}{3} \text{ of } 2\frac{1}{4} + 1\frac{1}{7}}{(2\frac{1}{2} - 1\frac{1}{3}) \text{ of } (2\frac{1}{4} + 1\frac{1}{7})}$ of $\frac{\frac{9}{11} + \frac{1}{2}}{\frac{4}{7} - \frac{3}{11}}$ of his journey ; how far has he still to walk ?

3. Add together $\frac{5\cdot 7}{1\cdot 52}$ and $\frac{\cdot 0102}{\cdot 74}$.

Five bells which commence tolling together, toll at intervals of 1·2, 1·5, 1·75, 1·8, 2·1 seconds respectively ; after what interval will they again toll together ?

4. Define "present worth."

A farmer buys 57 sheep for R120, payable at the end of 12 months, and sells them directly at R1. 12*a*. a head ready money ; what does he lose by the transaction, supposing the interest of money to be 5 per cent. ?

5. Show which is greater $\sqrt{2}$ or $\sqrt[3]{3}$.

Which is the better investment, 3 per cents. at $83\frac{1}{3}$ or $3\frac{1}{2}$ per cents. at 3 per cent. discount ?

1890.

1. Simplify (*a*) $\dfrac{\dfrac{\frac{2}{5}}{1-\frac{1}{26}} + \frac{1}{3} + \frac{1}{7}}{1 - \frac{1}{7}\left(\dfrac{\frac{2}{5}}{1-\frac{1}{26}} + \frac{1}{3}\right)}$.

(*b*) $\dfrac{\cdot 47 - (\cdot 5 - \cdot 0\dot{3}0\dot{3})}{\cdot 0873 - (\cdot 0083 + \cdot 07)}$.

2. What part of $\frac{3}{8}$ of 5 cwt. is $\frac{1}{100}$ of a ton ?

Express $\cdot 3\dot{7}\dot{8}$ of 16*s*. 6*d*. as a decimal of $\cdot 42\dot{6}$ of £1. 17*s*. 6*d*.

3. A man bequeathed $\frac{4}{13}$ of his property to one son, 30 per cent. of the remainder to another, and the surplus to his widow. The difference of his sons' legacies was £754. How much did the widow receive ?

4. A ship with 1200 men on board had sufficient provisions to last 17 weeks. The survivors of a wreck having been taken aboard, the provisions were consumed in 15 weeks. How many men were taken aboard ?

5. At what price must a person invest in the 4 per cent. Government Promissory Note, so that after paying income-tax at the rate of 5 pies in the rupee, he may receive $4\frac{1}{2}$ per cent. on his investment ?

6. *A* and *B* travel together 120 miles by rail. *A* takes a return ticket for which he has to pay one fare and a half. Coming back they find that *A* has travelled cheaper than *B* by 4*a*. 2*p*. for every 100 miles. Find the fare per mile.

1891.

1. Simplify :—

(1) $$\frac{\frac{1}{7}+\frac{1}{8}+\dfrac{\frac{2}{5}}{1-\frac{1}{25}}}{-\frac{1}{7}\text{ of }\left(\frac{1}{8}+\dfrac{\frac{2}{5}}{1-\frac{2}{5}}\right)}.$$

(2) $$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}.$$

2. Express 7 · 7 oz. + ·075 cwt. as decimal of $2{\cdot}\dot{2}5$ of $\cdot\dot{2}7$ of a ton.

3. A sum of money invested at 5 per cent. per annum, simple interest amounts in 6 years to R1326 ; in what time will it amount to R1530 ?

4, What is discount ? Distinguish between true and commercial discount.

The interest on a certain sum at 5 per cent. per annum for a certain time is £50, and the discount at the same rate for the same time is £40. Find the sum and the time.

5. Nine gallons are drawn from a cask full of wine, it is then filled with water. Nine gallons of the mixture are drawn, and the cask is again filled with water. The quantity of wine now left in the cask is to that of the water in it as 16 : 9. How much does the cask hold ?

1892.

1. Find by how much the square root of $9+\dfrac{\cdot}{1+\dfrac{}{7+\frac{1}{5}}}$ differs from

[illegible]. Which of these comes nearest to $3+\frac{1}{10}\sqrt{2}$?

2. Find the value of

$$\left(\frac{\cdot 0019}{3{\cdot}1\dot{6}}\text{ of }\frac{4{\cdot}\dot{4}}{\cdot 0005}\right)\div\left(\frac{8{\cdot}\dot{8}}{7}\text{ of }\frac{4}{5{\cdot}625}\right).$$

3. A stream which flows at a uniform rate of 1·109 miles an hour, is 20 yards wide, the depth at a certain ferry being 6 feet : how many gallons pass the ferry in a minute ? (Each gallon contains about $277\frac{1}{4}$ cubic inches).

4. A person invests £14970 in the purchase of 3 per cents. at 90 and $3\frac{1}{2}$ per cents. at 97. His total income being £500, how much of each stock did he buy ?

5. A spirit merchant buys 80 gallons of whisky at 18s. per gallon, and 180 gallons more at 15s. per gallon, and mixes them. At what price must he sell the mixture to gain $8\frac{1}{3}$ per cent. upon his outlay ?

1893.

1. Add :—

R.	As.	P.
3436	12	2
5242	10	3
248	6	9
431	13	5
5302	11	$4\frac{1}{2}$
	8	$1\frac{1}{2}$
5001	15	$6\frac{3}{4}$
136854	7	2
298	9	$4\frac{1}{2}$
836993	1	$9\frac{3}{4}$

2. Multiply 319·9657 by ·04286.

3. Find the value of $\dfrac{\sqrt{(2-\sqrt{2})}}{\sqrt{(2+\sqrt{2})}}$ correct to 5 places of decimals.

4. Calculate the income-tax on R666. 10 annas 8 pies at 5 pies per rupee.

5. A local train which travels at the rate of twenty-four miles an hour, leaves Lahore at twenty minutes past eight and reaches Amritsar at five minutes past ten the same morning. It stops at Mianmir for ten minutes and at each of three other stations for five minutes. Find the distance between Lahore and Amritsar.

1894.

1. Convert $\frac{1}{7}$ and $\frac{4}{7}$ into circulating decimals and point out the relation between the figures in their periods.

2. The sides of a rectangle are as 3 : 4 and the area is 1452 square feet. Find its length and breadth.

3. Exchange R7080 for English money at 1*s.* $3\frac{3}{4}$*d.* per rupee.

4. What is discount ? How is it commonly calculated ? If a sum of R1000 becomes due three months hence, what is its present value as commonly calculated, and what as correctly calculated, interest being reckoned at 5 per cent. ?

5. Find the square root of 101 correct to five places of decimals.

1895.

1. Divide $\dfrac{48\frac{1}{4}}{1085\frac{1}{16}}$ by $\dfrac{7\frac{4}{11}}{174\frac{2}{17}}$, and reduce the quotient to a recurring decimal.

2. The Imperial gallon contains 277·27 cubic inches, and a cubic foot of water at its maximum density weighs 62·42 lbs. ; find the weight of a pint of water correctly to two places of decimals.

3. The capital of a firm consists of £713. 3s.; £964. 17s.; £2391. 3s. subscribed by three partners; divide £2231 among them in proportion to their several capitals.

4. Find the square root of 5 correctly to seven places of decimals.

5. The area of a rectangular field is $\frac{3}{8}$ of an acre; and its length is twice its breadth; determine the lengths of its sides approximately.

ENTRANCE EXAMINATION PAPERS.

ALLAHABAD.

1889.

1. Define a fraction and shew that $\frac{1}{3}=\frac{2}{6}$.

By how much does the difference of $1\frac{1}{26}$ and $\frac{9}{13}$ fall short of their sum? Express the defect as a decimal.

2. (*a*) Simplify $\dfrac{3\frac{3}{8}-1\frac{1}{2} \text{ of } 1\frac{1}{6}-1\frac{1}{7}}{(3\frac{3}{8}-1\frac{1}{2}) \text{ of } (1\frac{1}{6}-1\frac{1}{7})}$.

(*b*) Subtract ·03 from $\cdot\dot{0}\dot{3}$ and divide the result by $\cdot 10\dot{2}$.

3. Find the square root of ·001 to four places of decimals. What number has ·1 for its square root?

4. What sum of money will amount to R1,381 . 4 . 0 in 15 months at 5 per cent. per annum simple interest?

5. How long will it take to walk along the four sides of a square field which contains 16 acres 401 square yards, at 3 miles an hour?

6. *A* and *B* complete a piece of work in 8 days; *B* and *C* do the same in 12 days; and *A*, *B* and *C* finish it in 6 days. In how many days will *A* and *C* complete the work?

7. *A* who travels $3\frac{1}{2}$ miles an hour starts $2\frac{1}{2}$ hours before *B* who goes the same road at $4\frac{1}{2}$ miles an hour; where will *B* overtake *A*?

1890.

1. Multiply ·347695 by 2·0066, and divide the product by ·01905.

2. Simplify $1\frac{3}{8}+3\frac{1}{3}-5\frac{5}{6}+2\frac{3}{4}-1\frac{1}{12}$.

3. Find, by Practice or otherwise, the value of 2345 mds. 27 seers 10 ch. of wheat at R3. 10. 8 per md.

4. Extract the square root of $1-(\cdot 00135)^2$ to 5 places of decimals.

5. The weight of a cu. in. of water is 253·17 grains, that of a cu. in. of air is ·31 grains; find to 3 places of decimals how many cu. in. of water are equal in weight to one cu. ft. of air.

6. On measuring a distance of 32 yds. with a rod of a certain length, it was found that the rod was contained 41 times with $\frac{1}{2}$ an inch over. How many inches will there be over in measuring 44 yds. with the same rod?

1891.

1. Define "Notation," "Numeration"; and prove that "three times four" = "four times three."

2. Reduce to a single fraction :—

$$\frac{919\frac{2}{17}}{7\cdot954}+\frac{4\cdot\dot{1}0\dot{0}}{442\frac{20}{37}}+\frac{7}{11}\text{ of }\cdot07344.$$

3. The wine in a pipe when full is worth £19. 9*s*. 9*d*. How much has leaked away if what is left is worth £9. 16*s*. $7\frac{84}{221}$*d*.?

4. In discounting a bill, what do you mean by "The Banker's profit?" If the simple interest on £923. 18*s*. $1\frac{1}{2}$*d*. amounts to £17. 9*s*. $3\frac{3}{4}$*d*. exactly in 138 days, what is the rate of interest per cent. per annum?

5. Extract the square roots of 99,980001; and of $60\frac{61}{169}$.

1892.

1. How is a fraction affected by adding the same number to the numerator and the denominator?

Prove that $\frac{3+4}{4+5}$ is greater than $\frac{3}{4}$ and less than $\frac{4}{5}$.

2. (*a*) Divide $\frac{1}{8}[3+\frac{1}{8}\{3+\frac{1}{8}(3+1\frac{1}{2})\}]$ by ·125.

(*b*) Reduce [illegible] and [illegible] to their lowest terms and express their difference as a decimal.

3. Forty men finish a piece of work in 40 days; if 5 men leave the work after every tenth day, in what time will the whole work be completed?

4. Find the difference between the Simple interest and Discount of £330 in 4 years at $2\frac{1}{2}$ per cent. per annum.

5. Extract the square root of $\frac{1000\cdot2001}{1000}$.

1893.

1. Two recurring decimals are added together; prove that the number of digits in the period of the result, cannot exceed the product of the numbers of the digits in the original periods.

2. Find the value of $\dot{5}\dot{4}$ of $\dot{3}07\dot{2}$ of 1 mile 5 fur. 30 poles.

3. Multiply R2 anna 1. by $\dfrac{\frac{4}{5}+\frac{6}{7}+\frac{13}{15}+\frac{19}{21}}{\frac{1}{3}+\frac{5}{7}+\frac{7}{15}+\frac{13}{21}}$.

4. Find by Practice the cost of 10 cwt. 3qrs. 23 lbs. 8 oz. at £1-5-8 per cwt.

5. A sum of money was divided amongst 5 people; 4 of them received respectively ·15, $\frac{3}{20}$, ·1, $\frac{3}{8}$ of the whole, while the 5th received £105-3-6. What was the sum divided?

6. An oz. of standard gold, one-twelfth of which is alloy, is worth £3-17-10½; how many sovereigns would be coined from 36 lbs. 8 oz. of pure gold?

7. Find the square root of 6246·057024 and of $71\frac{184}{256}$.

1894.

1. (*a*) A multiplication sum having been worked is partially rubbed out; the figures that remain are the entire multiplicand 999 and the last three digits 193 in the product. Restore the complete work.

$$\text{Simplify}\quad \frac{1}{1\frac{1}{20}}\times\frac{1+\cdot0025\times\cdot05}{1\cdot0025-\cdot05}-\frac{45\times\cdot3\dot{5}}{8}.$$

2. (*a*) What decimal of R100 must be added to $\frac{141}{1205}$ of R5-10-8 that the sum may be 10 annas?

(*b*) Extract the square root of 25·6.

3. Two trains start at the same time from Mirzapur and Delhi and proceed towards each other at the rates of 16 and 21 miles per hour respectively. When they meet it is found that one train has travelled 60 miles more than the other. Find the distance between the two stations.

4. Two years and six months ago, I borrowed a sum which with simple interest at 6 per cent. per annum now amounts to R638-4-0. Find the sum.

ANSWERS

TO THE EXAMPLES IN THE APPENDIXES.

ANSWERS

TO THE EXAMPLES IN THE APPENDICES.

APPENDIX A.

Examples I. Pages 1, 2.

1\. (1). 12 ; 19 ; 17 ; 27. (2). 52 ; 39 ; 80 ; 94 ; 79.

(3). 512 ; 885 ; 805 ; 921 ; 901.

(4). 3000 ; 27504 ; 676050 ; 201324.

(5). 200000 ; 584911 ; 834508 ; 602079.

(6). 84525804 ; 98406606 ; 74335909.

(7). 7400000062407. (8). 4050000823517O5.

2\. 99999999 ; 1000000000. 3. Five thousand millions ; ten thousand crore crores.

4\. He should have written 80502004.

5\. Thirty thousand crore crores.

6\. He should have written 80800888.

9\. (1). Ten thousand two hundred and three ; forty thousand and twenty-five ; sixty-seven thousand eight hundred and nine ; eighty thousand and seven ; fifty-seven thousand ; ninety-two thousand and four.

(2). Three millions, six hundred and eighty-seven thousand five hundred and sixty-two ; two millions, one hundred thousand and seven ; five millions nine hundred and sixty thousand and three ; eight millions, three hundred and forty-five ; five millions, five hundred thousand five hundred and fifty-five.

10 Ninety-nine millions, nine hundred and ninety-nine thousand, nine hundred and ninety-nine ; one million.

11. They should have read it as fifty millions, seventy thousand and ten.

Examples II. Pages 2—5.

14. 22. 15. 32. 16. 20. 17. 16.

18. 48. 19. 40. 20. 27. 21. Rs. 40.

22. 35. 23. 80. 24. 25 yards.

25. (1). 99. (2). 239. (3). 280.
(4). 211. (5). 1000. (6). 1301.
(7). 10151. (8). 14878. (9). 486.
(10). 11111. (11). 123214. (12). 3611911.
(13). 1991810. (14). 9763066. (15). 2064937.
(16). 8765400. (17). 3427908. (18). 6357510.
(19). 5201952. (20). 4467789 (21). 1357477.
(22). 9509920. (23). 14507. (24). 15905.
(25). 110735. (26). 1659364. (27). 15741620.

26. (1). 38980. (2) 94186. (3). 11047541.

27. 2763, 20428, 180004. 28. 48831. 29. 5675.

30. 6913. 31. 38 ; 50. 32. Rs 160000. 33. 9637.

Examples III. Pages 5–7.

11. 5. 12. 19 years. 13 22. 14. 16 years.

15. (1). 23. (2). 103. (3). 117.
(4). 449. (5). 402. (6) 4334.
(7). 7045. (8). 3355. (9). 2222.
(10). 198. (11). 81113. (12). 500700.
(13). 9247058. (14). 1. (15). 91111102.
(16). 22968057. (17). 54732739. (18). 1357381269.

16. (1). 258774. (2). 9000. (3). 8790999.

(4). 6299999. (5). 1000000. (6). 1011112.

(7). 9199993. (8). 904688773.

17. Rs 355. 18. 3892. 19. 341238.

20. 72491. 21. 51 years. 22 Rs. 500056.

23. 52 years. 24. 1. 25. 853560.

27. 27000000. 28. Rs. 1580. 29. Rs. 2600.

30. 1803 ; 27 years. 31. Rs. 1416.

Examples IV. Pages 7—9.

5. 160. 6. 75 7. 156. 8. Rs 95. 9. 210.

10. 216. 11. 240. 12. 91. 13. 192. 14. 30.

15. 5. 16. 111.

17. (1). 692. (2). 2208. (3). 8019.

(4). 4024. (5). 21085. (6). 56938.

(7). 20084. (8). 30008. (9). 438750.

(10). 686104. (11). 6834888. (12). 2457414.

(13). 25700. (14). 8976000. (15). 26320000.

(16). 1708600. (17). 22851000. (18). 48660000.

(19) 42863800. (20). 64305000. (21). 682736000000.

18. 14151. 19. 17615 Rs. 20. 175 miles.

21. 8395200. 22. 4000. 23. 4145280.

24 Rs 1284. 25. Rs 2575.

26. (1). 834339635498O7. (2). 27828305912I252.

(3). 642596568711185. (4). 14446089217728.

(5). 758554535256. (6). 516760458000000.

(7). 683024925I3144.

27. 9 ; 36 ; 49 ; 64 ; 81 ; 25 ; 121 ; 144 : 169.

28. 4096 ; 5929 ; 33124 ; 541696 ; 88604569 ; 4598738596.

29. 216 ; 27 ; 8 ; 343 ; 512 ; 729.

30 250047 ; 438976 ; 658503 ; 2000376 ; 370146232 ; 929714176 ; 1879080904.

31. 276922881 ; 139314069504.

Examples V. Pages 9—11.

8. 3, 3. 9. Rs. 9. 10. 16. 11. 13 as. 12. Rs. 16.

13. (1). 52, rem. 24. (2). 56, rem. 32. (3). 78, rem. 18.
(4). 30, rem. 7. (5). 58, rem. 48. (6). 88, rem 51.
(7). 277, rem 95. (8). 66, rem. 574. (9). 63, rem. 524.
(10). 19, rem. 89. (11). 55, rem. 128. (12). 110, rem. 367.

14. (1). 16, rem. 51. (2). 18, rem. 2.
(3). 1201, rem. 107. (4). 158.
(5). 6083, rem. 3. (6). 68274, rem 580.
(7). 468333428. (8). 5696242, rem. 75.

15. 27. 16. Rs. 529. 17. Rs. 40. 18. 15.

19. Each son, Rs. 6000 ; each daughter, Rs. 3000. 20. 38.

21. Rs. 365. 22. Rs. 4. 23. Each cow Rs. 16, each sheep Rs. 4.

24. (1). 112, rem. 5. (2). 142, rem. 5. (3). 766, rem. 4.
(4). 246, rem. 7. (5). 579, rem. 4. (6). 1220, rem. 3.
(7). 154, rem. 9. (8). 418. (9). 229, rem. 7.
(10). 557, rem. 5. (11). 453, rem. 3. (12). 130, rem. 8.

25. 422. 26. Syam Rs 10, and each boy Rs 523.

27. (1). 591, rem. 7. (2). 20873, rem. 107.
473, rem. 3. 18554, rem. 75.
295, rem. 23. 19791, rem. 66.
145, rem. 38. 15903, rem. 147.
121, rem. 25.

(3). 13965, rem. 323. (4). 18119, rem. 272.
7481, rem. 533. 25480, rem. 212.
24938, rem. 197. 40265, rem. 137.

28. (1). 177, rem. 11.
133, rem. 1.
106, rem. 21.
88, rem. 41.
76, rem. 1.

(2). 678, rem. 91.
339, rem. 91.
226, rem. 91.
135, rem. 391.
96, rem. 691.

(3). 78834, rem. 925.
72770, rem. 725.
67572, rem. 925.
52556, rem. 925.

(4). 5640, rem. 172.
564, rem. 172.
650, rem. 1972.

(5). 79050, rem. 60.
7026, rem. 660.
702, rem. 6060.

(6). 100008, rem 460.
8764, rem. 7100.
137, rem. 464700.

Examples VI. Pages 11—12.

1. (1). 2^2 ; 2^3 ; $2^2 \times 3$; 3×5 ; 2×3^2 ; $2^2 \times 5$.

(2). $3^2 \times 3$; 5^2 ; $2^2 \times 7$; $2 \times 3 \times 5$; 5×7 ; 3×13.

(3). $2 \times 3 \times 7$; 7^2 ; $2^2 \times 13$; $2^3 \times 7$; $2^2 \times 3 \times 5$; 2^6.

(4). $2^3 \times 3^2$; 3×5^2 ; $2^2 \times 3 \times 7$; $2^3 \times 11$; 5×19 ; $2^5 \times 3$.

(5). $3 \times 5 \times 7$; $2^4 \times 7$; $2^2 \times 3 \times 11$; 4×47 ; $2^2 \times 7^2$; $2^2 \times 5^2$.

2. (1). $3^2 \times 23$; $2^2 \times 3^2 \times 7$; 5×43 ; $2^5 \times 3^2$; $2^4 \times 3^5$; $3^2 \times 5 \times 11$.

(2). $3 \times 5^2 \times 7$; 5^4 ; 3^6 ; $3^3 \times 37$; $2^2 \times 3^5$; $2^4 \times 3^4$.

(3). $2^4 \times 3^2 \times 13$; 11^3 ; $2^4 \times 7 \times 13$; $2^2 \times 3^2 \times 5 \times 11$; $2^4 \times 5^3$;
$3 \times 5^2 \times 7^2$.

(4). $3^2 \times 5 \times 107$; $3^3 \times 5^2 \times 11$; $2 \times 3 \times 5^3 \times 7$; $3^2 \times 7 \times 11^2$;
$2^3 \times 2437$.

(5). $3^2 \times 7 \times 11 \times 13$; $3 \times 5 \times 7 \times 11 \times 13$; $3^2 \times 5^2 \times 7^2$;
$3 \times 17 \times 19^2$; $2 \times 3^2 \times 5 \times 11 \times 7^2$.

(6). $3^2 \times 5^4 \times 7$; $2^3 \times 3^2 \times 7^3$; $7^4 \times 13^2$; $3 \times 2^9 \times 11^3 \times 47$;
$2 \times 3 \times 5^3 \times 7 \times 11 \times 13$.

3. 199 ; 197 ; 991 ; 2809.

Examples VII. Pages 12, 13.

1. (1). 2. (2). 5. (3). 6. (4). 3. (5). 2.
(6). 5. (7). 5. (8). 7. (9). 5. (10). 13
(11). 23. (12). 8.

2. (1). 4. (2). 17. (3). 31. (4). 53. (5). 32.
(6). 47. (7). 83. (8). 11. (9). 123. (10). 3.
(11). 121. (12). 851. (13). 7. (14). 29. (15). 23.
(16). 3. (17). 4. (18). 4. (19). 37. (20). 10.
(21). 23. (22). 2. (23). 5.

4. 12. 5. 91. 6. 17 and 102; or 34 and 85; or 51 and 68.
7. 50 and 350; or 150 and 250. 8. 35. 9. 625.
10. 23. 11. 27. 12. 8.

Examples VIII. Pages 13, 14.

1. (1). 8. (2). 18. (3). 24. (4). 45. (5). 48.
(6). 60. (7). 6. (8). 24. (9). 48. (10). 60.

2. (1). 252. (2). 720. (3). 432. (4). 60.
(5). 48. (6). 180. (7). 810. (8). 336.
(9). 80620. (10). 130350.

3. (1). 72. (2). 72. (3). 80.
(4). 120. (5). 600. (6). 720.
(7). 960. (8). 1728. (9). 26136.
(10). 643440. (11). 10080. (12). 2520.
(13). 98280. (14). 197064. (15). 323323.
(16). 286069792. (17). 416316833.

4. 840. 5. 149. 6. 292.
7. 308. 8. 7 hours. 9. 4 h. 40 m.
10. 48 miles. 11. 1 Rupee. 12. 10 yds. 1 ft.
13. 65 and 75. 14. 180 yds.

Examples IX. Pages 14—16.

1. 91091. 2. 542759. 3. 370.
4. 845. 5. 7693. 6. 19052.
7. Rs 12. 8. 5732, 94311. 9. Rs 28924.
10. 4473225. 11. Syam, Rs374542; Ram, Rs 375541
12. 0. 14. 3, 5, 7, 15, 21, 35.
15. 31450. 16. 13. 17. 24 years.
18. 300. 19. 3376. 20. 205.
21. 1111. 22. Horse Rs800; carriage Rs400.
23. 119. 24. 61750 mds. 25. 25.
26. 10 hours. 27. 15. 28. 22.
30. 9 hours after. 31. *A*, 32 years; *B*, 35 years.
32. Rs837. 33. 56077560. 34. Rs691.
35. 199. 36. 2352.
37. *A*, Rs185; *B*, Rs170; *C*, Rs160. 38. Rs438.
39. Rs1000. 40. 15 miles. 41. 100.

Examples X.—Pages 17, 18.

1. $\frac{65}{13}$, $\frac{91}{13}$, $\frac{104}{13}$, $\frac{117}{13}$, $\frac{156}{13}$. 2. $\frac{180}{15}$, $\frac{224}{16}$, $\frac{342}{18}$, $\frac{418}{19}$.

3. $\frac{920000}{200}$, $\frac{620000}{300}$, $\frac{1200000}{400}$.

4. (1). $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{8}$, $\frac{4}{5}$, $\frac{1}{3}$, $\frac{3}{5}$. (2). $\frac{1}{2}$, $\frac{1}{2}$, $\frac{7}{5}$, $\frac{3}{5}$, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{2}{3}$.
(3). $\frac{5}{6}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{3}{4}$, $\frac{4}{7}$, $\frac{7}{8}$, $\frac{5}{7}$, $\frac{1}{4}$. (4). $\frac{6}{11}$; $\frac{12}{13}$; $\frac{1}{2}$; $\frac{17}{26}$; $\frac{8}{27}$; $\frac{29}{40}$; $\frac{1}{4}$.
(5). $\frac{124}{437}$; $\frac{5}{7}$; $\frac{17}{31}$; $\frac{71}{121}$; $\frac{16}{25}$; $\frac{11}{15}$. (6). $\frac{2}{3}$; $\frac{2}{7}$; $\frac{1}{2}$; [illegible].

5. (1). $\frac{8}{3}$, $\frac{11}{4}$, $\frac{11}{3}$, $\frac{29}{3}$, $\frac{53}{6}$, $\frac{79}{8}$, $\frac{89}{9}$, $\frac{61}{12}$.
(2). $\frac{73}{6}$, $\frac{40}{9}$, $\frac{57}{5}$, $\frac{109}{10}$, $\frac{60}{8}$, $\frac{71}{4}$, $\frac{34}{3}$, $\frac{79}{6}$.
(3). $\frac{111}{4}$; $\frac{405}{11}$; $\frac{237}{5}$; $\frac{1189}{16}$; $\frac{873}{12}$; $\frac{2437}{35}$.
(4). [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; [illegible].

6. (1). $1\frac{1}{3}$, $1\frac{1}{8}$, $2\frac{3}{8}$, $2\frac{2}{5}$, $3\frac{1}{5}$, $5\frac{1}{4}$, $4\frac{1}{9}$, $3\frac{1}{6}$, 9.
(2). $4\frac{5}{12}$, $3\frac{1}{3}$, 7, $2\frac{1}{12}$, $2\frac{5}{15}$, $5\frac{15}{18}$, $17\frac{1}{25}$, $21\frac{1}{7}$.

(3). $31\frac{1}{18}$; 63 ; $51\frac{5}{18}$; $20\frac{8}{25}$; $45\frac{4}{7}$; $16\frac{9}{55}$; $11\frac{12}{13}$; $138\frac{33}{35}$.

(4). $18\frac{63}{100}$; $9\frac{403}{480}$; $75\frac{27}{43}$; $15\frac{77}{770}$; $32\frac{19}{771}$; $20\frac{27}{500}$; $102\frac{954}{1205}$.

7. (1). $\frac{9}{7}$; $4\frac{8}{9}$; 3 ; $10\frac{2}{3}$; 12 ; 1.

(2). 5 ; $36\frac{9}{10}$; 9 ; $\frac{1}{3}$. (3). $1\frac{13}{18}$; $49\frac{1}{15}$; $\frac{8}{75}$.

8. (1). $\frac{4}{7}$, $\frac{5}{8}$, $\frac{11}{21}$, $\frac{19}{42}$. (2). $\frac{13}{30}$, $\frac{5}{12}$, $\frac{25}{60}$, $\frac{11}{37}$, $\frac{7}{15}$.

(3). $1\frac{48}{60}$, $1\frac{20}{80}$, $\frac{48}{28}$. (4). $\frac{15}{4}$, $3\frac{1}{3}$, $\frac{2}{7}$ of $9\frac{3}{8}$.

(5). $\frac{33}{28}$ of $\frac{32}{80}$, $\frac{7}{8}$ of $\frac{15}{28}$, $\frac{9}{7}$ of $\frac{7}{26}$. (6). $\frac{33}{35}$ of $\frac{44}{88}$, $\frac{17}{51}$ of $\frac{90}{81}$, $\frac{9}{13}$ of $\frac{2}{13}$.

Examples XI. Pages 18, 19.

1. $\frac{5}{12}$. 2. $\frac{5}{7}$. 3. $1\frac{11}{12}$. 4. $1\frac{1}{6}$. 5. 4.
6. 6. 7. $13\frac{4}{5}$. 8. $7\frac{2}{3}$. 9. $1\frac{47}{144}$. 10. $2\frac{12}{35}$.
11. $1\frac{19}{300}$. 12. $2\frac{103}{184}$. 13. $40\frac{199}{288}$. 14. $71\frac{5}{8}$. 15 $20\frac{1}{3}$.
16. $73\frac{179}{504}$.

Examples XII. Page 19.

1. (1). $\frac{1}{3}$. (2). $\frac{1}{2}$. (3). $\frac{1}{2}$. (4). $\frac{1}{4}$.
(5). $\frac{1}{17}$. (6). $\frac{1}{7}$. (7). $\frac{1}{7}$. (8). $2\frac{1}{4}$.
(9). $1\frac{1}{8}$. (10). $1\frac{7}{8}$.
(11). $\frac{3}{35}$. (12). $\frac{35}{162}$. (13). $\frac{19}{120}$. (14). $\frac{1}{35}$.
(15). $11\frac{13}{16}$. (16). $1\frac{287}{345}$. (17). $\frac{849}{1001}$. (18). $10\frac{3}{5}$.

2. (1). $6\frac{1}{3}$. (2). $3\frac{13}{35}$. (3). $13\frac{99}{140}$. (4). $10\frac{19}{28}$.
(5). $15\frac{3}{4}$. (6). $20\frac{79}{100}$.

Examples XIII. Page 20.

1. $\frac{3}{4}$, $2\frac{2}{5}$, $\frac{4}{5}$, $\frac{10}{11}$, $1\frac{1}{6}$, $\frac{12}{17}$. 2. $4\frac{9}{10}$, $3\frac{3}{7}$, 6, $1\frac{11}{23}$, $2\frac{4}{5}$, $4\frac{4}{5}$.
3. $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{24}$, $\frac{1}{64}$. 4. $\frac{1}{56}$, $\frac{1}{48}$, $\frac{1}{45}$, $\frac{1}{36}$, $\frac{1}{105}$, $\frac{1}{120}$.
5. $19\frac{1}{3}$; $6\frac{4}{11}$; $37\frac{1}{2}$; $19\frac{1}{2}$; $46\frac{2}{7}$; 74.
6. 369 ; $3\frac{3}{8}$; $18\frac{1}{4}$; $9\frac{3}{8}$; $96\frac{3}{4}$; $285\frac{5}{7}$.
7. $292\frac{1}{2}$; $41\frac{1}{3}$; $176\frac{1}{2}$; $111\frac{3}{8}$; $2069\frac{3}{5}$.
8. $1\frac{7}{8}$; $5\frac{1}{7}$; 27 ; $32\frac{2}{5}$. 9. $\frac{1}{3}$; $\frac{4}{15}$; $\frac{1}{5}$.
10. 26. 11. 39. 12. 20.

Examples XIV. Page 21.

1. (1). $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{14}$; $\frac{1}{25}$; $\frac{1}{18}$; $\frac{1}{19}$. (2). $\frac{2}{35}$; $\frac{8}{15}$; $\frac{8}{75}$; $\frac{2}{11}$; $\frac{1}{8}$; $\frac{8}{11}$.

(3). $\frac{1}{15}$; $\frac{1}{57}$; $\frac{1}{34}$; $\frac{1}{21}$; $\frac{2}{25}$. (4). $\frac{1}{5}$; $\frac{3}{5}$; $\frac{7}{9}$; $\frac{7}{10}$; $\frac{8}{11}$; $\frac{3}{5}$.

(5). $\frac{15}{16}$; $\frac{1}{30}$; $\frac{14}{15}$; $\frac{8}{35}$; $\frac{9}{10}$; $\frac{1}{7}$.

(6). $\frac{8}{25}$; $\frac{1}{14}$; $\frac{5}{1036}$; $\frac{1}{14}$; $\frac{3}{78}$.

(7). $\frac{1}{32}$; $\frac{1}{20}$; $\frac{3}{7}$; $\frac{1}{56}$; $\frac{19}{21}$.

(8). $1\frac{1}{3}$; $1\frac{2}{3}$; $3\frac{1}{8}$; $1\frac{25}{832}$; 1.

(9). 156 ; $5\frac{17}{78}$; $1\frac{1}{11}$; 168.

(10). (11). $\frac{81}{125}$. (12). $\frac{1}{8}$. (13). $5\frac{6}{14}$.

2. $\frac{1}{35}$. 3. $5\frac{9}{16}$.

Examples XV. Pages 21—25.

1. (1). $1\frac{1}{3}$. (2). $\frac{1}{3}$. (3). $7\frac{1}{6}$. (4). 2.

(5). 3. (6). $\frac{1}{8}$. (7). $\frac{3}{4}$. (8). $\frac{1}{3}$.

(9). $3\frac{17}{27}$. (10). $18\frac{8}{13}$. (11). $12\frac{1}{24}$. (12). $4\frac{1}{4}$.

(13). $6\frac{281}{336}$. (14). $\frac{6}{67}$. (15). $\frac{8}{11}$. (16). $6\frac{4}{11}$.

(17). $13\frac{41}{60}$. (18). $1\frac{1}{12}$. (19). $\frac{3}{5}$. (20). $57\frac{13}{18}$.

(21). $7\frac{98}{125}$. (22). $\frac{31}{56}$. (23). 6. (24). $\frac{183}{224}$.

(25). $\frac{3}{5}$. (26). $3\frac{13}{18}$. (27). $\frac{1}{8}$. (28). $\frac{7}{65}$.

(29). $\frac{14}{15}$. (30). $3\frac{182}{385}$. (31). $73\frac{177}{222}$. (32). $2\frac{58}{495}$.

(33). $3\frac{91}{92}$. (34). $3\frac{187}{560}$. (35). 12. (36). $\frac{13}{27}$.

(37). $\frac{21}{625}$. (38). $1\frac{7}{30}$. (39). 1. (40). $\frac{1}{216}$.

(41). $1\frac{17}{72}$. (42). $22\frac{13}{14}$. (43). $\frac{4}{7}$. (44). $\frac{2333}{3332}$.

(45). $\frac{8}{9}$. (46). 5.

2. (1). $\frac{3}{13}$ and 4. (2). $\frac{8}{75}$ and $3\frac{1}{5}$. (3). $\frac{3}{14}$ and $13\frac{3}{8}$.

(4). $\frac{3}{560}$ and $26\frac{1}{4}$. (5). $\frac{1}{18}$ and $5\frac{1}{4}$. (6). $\frac{13}{600}$ and $409\frac{1}{3}$

(7). $\frac{88}{165}$ and $278\frac{1}{5}$.

3. $1\frac{7}{15}$. 4. 682. 5. 462 ft. 6. 70 minutes.

7. $14\frac{3}{10}$. 8. $8\frac{1}{7}$. 9. $13\frac{1}{6}$. 10. 7 and $\frac{5}{8}$.

11. $\frac{6}{13}$. 12. $\frac{1}{10}$. 13 7500. 14. $1\frac{10}{13}$.

15. 600. 16. 15 and 7. 17. $28\frac{4}{5}$ ft.

18. Rs. 16000 ; 1st son, Rs. 10000 ; 2nd son, Rs. 6000.

19. Rs. 42000. 20. $\frac{1}{3}$. 21. 1716. 22. 80 and 160.

23. Bengalis, 400 ; Hindustanis, 300 ; Englishmen, 200 ; Rajputs, 100
Sikhs, 200.

Examples XVI. Page 26.

1. (1). $\frac{21}{100}$; $\frac{7}{10}$; $\frac{31}{100}$; $\frac{1}{100}$; $\frac{11}{1000}$; $\frac{19}{10000}$; $\frac{39021}{10000}$.

   (2). $\frac{878319}{100000}$; $\frac{480}{1000}$; $\frac{390710}{100000}$; $\frac{8884317}{1000000}$; $\frac{600003}{1000000}$.

2. (1). $\frac{1}{5}$; $\frac{2}{5}$; $\frac{1}{2}$; $\frac{4}{5}$; $\frac{9}{20}$; $\frac{1}{20}$; $\frac{2}{25}$.

   (2). $\frac{1}{2000}$; $\frac{1}{4000}$; $\frac{21}{40}$; $\frac{35}{16}$; $\frac{38}{5}$; $1\frac{3}{16}$.

   (3). $\frac{303}{500}$; $\frac{105}{16}$; $\frac{2401}{400}$; $\frac{11}{1600000}$; $\frac{91}{32000000}$.

3. $5\frac{1}{2}$; $8\frac{3}{20}$; $12\frac{5}{8}$; $11\frac{1}{40}$; $140\frac{29}{400}$; $13\frac{27}{40}$.

4. (1). ·7 ; ·9 ; 1·21 ; ·57 ; ·91 ; 8·2353 ; ·5721.

   (2). ·061387 ; ·0002367 ; ·00071 ; ·05135 ; ·0000001.

Examples XVII. Page 26.

1. (1). 22·787. (2). 1236·11. (3). 8·86192.

   (4). 1541·777742. (5). 545·020196.

2. (1). 899·2512. (2) £1387·85. (3). Rs 732·975725.

   (4). 5683.622 miles.

Examples XVIII Pages 26, 27.

1. (1). ·998 ; 8·001 ; 12·482 ; ·0001.

   (2). 2·5098 ; ·09999 ; 18·99.

2. (1). ·0999 ; 639·880999 ; 9·86. (2). 699·93.

3. (1). Rs·664 ; £1.9979 ; £.0098248. (2). 239·996.

   (3). 26·754273. (4). 9·8309.

4. The first.

Examples XIX. Page 27.

(1). 7·36 ; 18·225 ; 14·91 ; 141·814.

(2). 639712·72 ; 24·22 ; 146·7738 ; ·0000 38665.

(3). 2005·236414 ; 148·977536 ; 8·227765.

(4). ·00000606 ; 7661·5009 ; 103·823.

(5). 70·75233 ; 19448100.

(6). 1·21 ; ·215784 ; 7·82688. (7). 346·624225.

Examples XX. Page 27.

1. (1). 941 ; ·811 ; 120. (2). 23000 ; 10·01 ; 32·67578125.
   (3). 259 ; 2401800 ; ·81.
   (4). ·0000479 ; ·00000067625 ; 3·59.

2. ·000038, ·0038, ·038, ·38, 3·8. 3. 6·25 ; ·14927 ; ·0232......

4. 32714·2857 ; ·000092307 ; 61·4175 ; 1·2588.

Examples XXI. Page 28.

1. (1). ·25 ; ·125 ; ·0625 ; ·375 ; ·75 ; ·52 ; ·875 ; ·8125.
   (2). ·7578125 ; ·376 ; ·68359375 ; ·888 ; ·0005859375 ; ·1136.

2. (1). ·5, ·25, ·125. (2). ·83383.., ·75, ·75.
   (3). ·583333..., ·2666..., ·1875. (4). ·55, ·4444..., ·04166...

3 (1). ·051. (2). ·000162. (3). 853·333...
  (4). ·9. (5). 85·1. (6). 326·4.

4. (1). ·538461 ; ·53 ; ·26 ; ·38 ; ·42 ; ·590 ; ·093 ; ·02439.
   (2). 1·230769 ; ·1764705882352941 ; 3·714285 ; 1·296 ; 5·02083 ;
   ·01 ; ·648.
   (3). 7·1893 ; ·4990774 ; 2·457002 ; 13·94230769 ; ·0945 ; 4·253 ;
   ·001.

5\. (1). $\frac{8}{11}$; $\frac{1}{33}$; $\frac{2}{909}$; $\frac{14}{33}$; $\frac{4}{11}$; $\frac{28}{37}$; $\frac{81}{100}$.

(2). $\frac{67}{21}$; $\frac{441}{44}$; $\frac{4021}{66}$; $\frac{17}{13750}$.

Examples XXII. Pages 28,

1\. (1). ·889. (2). ·6116. (3). 7·068.

(4). $1{\cdot}58\dot{4}66\dot{4}$. (5). $4{\cdot}13\dot{0}3999408949\dot{9}$.

(6). $18{\cdot}64\dot{0}8199\dot{1}$. (7). $84{\cdot}44\dot{0}3958\dot{5}$.

(8). $5073{\cdot}52\dot{4}9422\dot{1}$.

2\. (1). 40·01544. (2). 71·97417.

3\. (1). $3{\cdot}\dot{0}\dot{6}$. (2). $1{\cdot}7\dot{1}395224573307839289\dot{0}$.

(3). $30{\cdot}0\dot{7}7704\dot{9}$. (4). $3{\cdot}0\dot{8}8809962077\dot{2}$.

(5). $2{\cdot}9\dot{9}5\dot{7}$. (6). $.868\dot{2}\dot{4}$.

4\. (1). $4{\cdot}780\dot{1}6725\dot{8}$. (2). $1{\cdot}202\dot{6}6\dot{0}$.

5\. (1). $72{\cdot}199\dot{4}6\dot{3}$. (2). $4125{\cdot}7\dot{8}\dot{5}$.

(3). ·1127. (4). 2·98255...

6\. (1). 335·14620. (2). 12·29596.

7\. (1). $392{\cdot}2569\dot{6}28451393\dot{1}$. (2). $1{\cdot}4\dot{8}5714\dot{2}$.

(3). ·6481. (4). 2·5227.

8\. (1). ·4738. (2). ·0228.

Examples XXIII. Pages 29—31.

1\. (1). ·4. (2). 1. (3). ·2. (4). $1{\cdot}\dot{3}\dot{6}$.

(5). $\cdot6\dot{4}8\dot{1}$. (6). $\cdot9\dot{9}\dot{0}$. (7). ·125. (8). 3·2691...

2\. (1). 1·999. (2). 1·249. (3). 1·1111. (4). 1·41068.

3\. 8·267. 4. 25. 5. 2·1301. 6. $\frac{65}{148}$.

7. 2700 ; 2 in. 8. Rs. 56. 9. £10666. 13*s.* 4*d.*

10. 300 Hindus and 30 Christians. 11. 5 days.

12. 5725 mango-trees, 4203 jack-trees, 72 cocoanut-trees.

13. 2. 14. 10·25 miles. 15. 32.

16. 55 ; 1·92 gals. 17. ·006 in. 18. $\cdot\dot{3}8461\dot{5}$.

19. 247 20. Rs. 36.

21. (1). ·036. (2). ·002. (3). ·001. (4). 1·45.

22. (1). 37·8. (2). ·42. (3). 5·94. (4). 11·55.

Examples XXIV. Pages 32, 33.

1. (1). 1344 pice. (2). 2104 pice. (3). 7840 pice.
   (4). 8629 pice. (5). 15766 pice.

2. (1). 4779 pies. (2). 8761 pies. (3). 7422 pies.
   (4). 12211 pies.

3. (1). 3840 *gandas.* (2). 41689 *gandas.* (3). 38093 *gandas.*

4. (1). 16640 *karas.* (2). 1140938 *karas.* (3). 553600 *karas.*

5. (1). 19200 *q.* (2). 20256 *q.* (3). 118904 *q.*
   (4). 222496 *q.* (5). 55440 *q.* (6). 2400 *q.*
   (7). 8400 *q.* (8). 71280 *q.*

6. (1). 121440 *d.* (2). 77760 *d.* (3). 53928 *d.*
   (4). 179424 *d.*

7. 3435. 8. 171093.

9. (1). Rs. 10. 9 as. 6 p. (2). Rs. 40. 12 as. 8 p.
   (3). Rs. 111. 7 as. (4). Rs. 159. 6 as. 2 pice.
   (5). Rs. 457. 13 as. (6). Rs. 840. 2 as.
   (7). Rs. 589. (8). Rs. 18. 7 as. 5 gandas. 1 kara.

10. (1). £117. 6*s.* (2). £40. 11*s.* 11*d.*
    (3). £10 5*s.* 8*d.* 2*q.*

11. (1). 101 guineas. 15*s.* (2). 0 guinea. 2*s.*
    (3). 2 guineas. 17*s.* 1*d.* 1*q.*

12. (1). Rs. 1508. 8 as. 8 p. (2). Rs. 5323. 2 as.
(3). Rs. 12510. 15 as. 10 p.

13. (1). 226 crowns. 2*s*. (2). 3249 crowns.
(3). 3602 crowns.

14. (1). 26 guineas. (2). 490 guineas. 10*s*.
(3). 4800 guineas.

15. (1). 3*s*. 10*d*. (2). £2. 1*s*.
(3). £6. 8*s*. 7*d*.

16. (1). 14 mds. 9 srs. 14 cht. (2). 1 md. 1 cht. 3 kanchas.
(3). 8 mds. 5 srs. 13 chts. (4). 31 mds. 26 srs. 40 tolas. 11 mashas. 4 rattis (Bengal gold and silver weight); 151 mds. 39 seers. 20 tolas. 1 masha. 2 rattis (Bengal doctors' weight).

17. (1). 7 cwt. 10 lbs. 9 oz. (2). 40 cwt. 3 qrs. 11 lbs. 14 oz.
(3). 179200 oz. (4). 19 tons. 10 cwt. 25 lbs.

18. (1). 19 kths. 6 chts. 15 sq. cubits.
(2). 30 bghs. 1 kth. 1 cht.
(3). 245 bghs. 14 kths. 8 chts.

19. 41289588 sq. inches.

20. 1 ac. 2 ro. 5 sq. po. 15 sq. yds. 8 sq. ft. 106 sq. inches.

Examples XXV.—Pages 33, 34.

1. Rs. 1844. 12 as. 10 pies. 2. Rs. 710. 17*g*. 1*k*.
3. £50. 12*s*. 5*d*. 4. £106. 6*s*. 9*d*.
5. 1989 mds. 17 srs. 8 chts.
6. 195 tons. 7 cwt. 1 qr. 26 lbs. 12 oz.
7. 144 lbs. 4 oz. 8 dwts. 8 grs.
8. 2385 miles. 1 fur. 36 po. 2 yds. 1 ft. 6 in.
9. 74 d. 19 hrs. 5 min. 25 secs.
10. 962 ac. 2 ro. 35 po. 9 sq. yds. 1 sq. ft. 36 sq. in.

Examples XXVI. Pages 34, 35.

1. Rs. 493. 4 as. 2 pies.   2. Rs. 4674. 5 as. 6 pies.
3. £51. 15*s*. 4*d*.   4. £78. 11*d*. 1*q*.
5. 18 guineas. 19*s*. 1*d*.   6. 43 mds. 11 srs. 11 chts.
7. 36 tons. 1 cwt. 3qrs. 2 lbs. 1 oz.
8. 12 lbs. 6 oz. 6 dwts. 17 grs.   9. 113 mi. 3 fur. 16 po. $3\frac{1}{7}$ yds.
10. 190 ac. 1 ro. 38 po. 29 sq. yds. 1 sq. ft. 36 sq. in.

Examples XXVII. Page 35.

1. (1). Rs. 307 ; Rs. 358. 2 as. 8 pies ; Rs. 409. 5 as. 4 pies ; Rs. 460. 8 as

(2). £77. 4*s* ; £102. 18*s*. 8*d*. ; £64. 6*s*. 8*d*. ; £90. 1*s*. 4*d*.

(3). Rs. 454. 4 as. 11 *g*. ; Rs. 512. 4 as. 9 *g*. ; Rs. 570. 4 as. 7 *g*. ; Rs. 647. 9 as. 11 *g*.

(4). £213 ; £255. 12*s* ; £298. 4*s* ; £319. 10*s*.

(5). 120 mi. 3 fur. 6 po. 3 yds. ; 180 mi. 4 fur. 29 po. 4 yds. 1 ft. 6 in. ; 234 mi. 32 po. 4 yds. ; 260 mi. 6 fur. 34 po. 1 yd. ; 4334 mi. 1 fur. 35 po. 3 yds. 1 ft. 6 in.

(6). 390 bghs. 15 kths. ; 439 bghs. 11 kths. 14 chts. ; 1318 bghs. 15 kths. 10 chts. ; 1758 bghs. 7 kths. 8 chts.

(7). 2680 mds. 19 srs. 7 chts. ; 4212 mds. 7 srs. 11 chts. ; 4339 mds. 33 srs. 6 chts. ; 12253 mds. 26 srs.

(8). 1260 tons. 8 cwt. 2 qrs. 8 lbs. ; 9810 tons. 6 cwt. 2 qrs. 24 lbs. ; 6564 tons. 14 cwt. 2 qrs. 16 lbs.

(9). 3672 lbs. 7 oz. 4 dwts. 12 grs. ; 4254 lbs. 4 oz. 4 dwts. 2 grs ; 5240 lbs. 9 oz. 10 dwts. 8 grs.

(10). 646 sq. yds. 136 sq. in. ; 2180 sq. yds. 5 sq ft. 63 sq. in. ; 3634 sq. yds. 3 sq. ft. 9 sq. in. ; 5168 sq. yds. 7 sq. ft. 80 sq. in. ; 14698 sq. yds. 7 sq. ft. 142 sq. in.

2. Rs. 432. 13 as.   3. Rs. 52. 6 as.

Examples XXVIII. Pages 35, 36.

1\. (1). Rs. 413. 12 as. 3 pies ; Rs. 275. 13 as. 6 pies ; Rs. 190. 15 as. 6 pies.

(2). £176955. 12*s*. 6*d*. ; £75838. 2*s*. 6*d*. ; £66358 7*s*. 2*d*. 1*q*. ; £106173. 7*s*. 6*d*. ; £48260. 12*s*. 6*d*.

(3). 2887 mds. 10 srs. 13 chts. ; 2062 mds. 13 srs. 7 chts. ; 1604 mds. 1 sr. 9 chts. (4). 5 cwt. 1 qr. 16 lbs.

(5). 2 lbs. 9 oz. 13 dwts. 7 grs ; 1 lb. 9 oz. 8 dwts. 11 grs.

(6). 164 mi. 1 fur. 8 po. ; 140 mi. 5 fur. 24 po. ; 89 mi. 4 fur. 11 po. 3 yds 1 ft. 6 in. ; 46 mi. 7 fur. 8 po. ; 29 mi. 6 fur. 30 po. 3 yds.

(7). 36 bghs. 3 kths. 12 chts. ; 24 bghs. 2 kths. 8 chts. ; 13 bghs. 5 kths. 6 chts. ; 6 bghs. 10 chts.

(8). 5 mo. 20 d. 9h. 35$\frac{18}{33}$ min. ; 5 mo. 15 d. 9 h. 18$\frac{11}{17}$ min. ; 2 mo. 12 d. 2 h. 12$\frac{33}{38}$ min.

(9) 769 ac. 20 po. ; 461 ac. 1 ro. 36 po. ; 329 ac. 2 ro. 20 po. 153 ac. 3 ro. 12 po. ; 109 ac. 3 ro. 20 po. ; 65 ac. 3 ro 28 po.

2\. House, Rs. 300. 10 as. 5 *g*. ; furniture, Rs. 901. 14 as. 15 *g*.

3\. 30.

Examples XXIX. Pages 36—38.

1\. (1). Rs. 2. 3 as. 4 p. (2). Rs. 4. 3 as. 8 p.

(3). Rs. 34. 10 as. 2 *g*. (4). 8*s*. 4*d*. 3$\frac{1}{3}$ *q*.

(5). £13. 8*s*. 1*d*. 2*q*. (6). 3 yds. 10$\frac{1}{4}$ in.

(7). Rs. 2. 13 as. 2$\frac{4}{5}$ *g*. (8). 8 mds. 1 sr. $\frac{9}{11}$ chts.

(9) 2 ac. 2ro. 4$\frac{4}{5}$ po. (10). 3 d. 17 h. 59 m. $\frac{14}{15}$ secs.

(11). Rs. 19 1 a. 6 p. (12) Rs. 20. 6 as. 3 p.

(13). £2. 17*s*. 3·75*d*. (14). £78. 7*s*. 10*d*.

(15). £11. 17*s*. (16). 12*s*. 5*d*.

(17). 3 miles. 1680·684 yds. (18). 2 cwt. 2 qrs. 19·475 lbs.

2. (1). Rs. 40. 15 as. $11\frac{3}{8}$ p. (2). £33. 17*s*. $8\frac{3}{4}$*d*.

(3). 18 cwt. 3 qrs. $7\frac{1}{16}$ lbs.

3. (1). Rs. 2. 2 as. $1\frac{7}{30}$ pies. (2). £15. 9*s*. $3\frac{1}{4}$*d*.

(3). 10 cwt. 15 $\frac{13}{16}$ lbs.

4. (1). Rs. 52. 13 as. 7 $\frac{1}{2}$ pies ; Rs. 123. 5 as. $1\frac{1}{2}$ pies ;

Rs. 158. 8 as. $10\frac{1}{2}$ pies ; Rs. 140 15 as. ; Rs 211. 6 as. 6 pies.

(2). 131 mds. 26 srs. $5\frac{1}{3}$ chts 150 mds 18 srs. $10\frac{2}{3}$ chts. ;

225 mds. 28 srs. ; 206 mds. 35 srs. $10\frac{2}{3}$ chts.

(3). £113. 6*s*. $7\frac{1}{2}$*d*. ; £94. 8*s* 10*d*. ; £169. 19*s*. $10\frac{1}{2}$*d*. ;

£415. 10*s*. $10\frac{3}{4}$*d*.

(4). 33 tons. 2 cwt. 2 qrs. $6\frac{1}{2}$ lbs. 47 tons. 17 cwt. $3\frac{1}{8}$ lbs. ;

66 tons. 5 cwt. 13 lbs. ; 117 tons 15 cwt. 3 qrs. $1\frac{1}{3}$ lbs.

5. (1). Rs. 13. 14 as. $5\frac{7}{80}$ pies ; Rs. 10. 6 as. $9\frac{4}{11}$ pies ;

Rs. 4. 7 as. $5\frac{107}{117}$ pies ; Rs. 2. 9 as. $8\frac{81}{102}$ pies.

(2). 12 tons. 19 cwt. 3 qrs. $9\frac{17}{31}$ lbs. ; 10 tons. 2 cwt. $10\frac{20}{37}$ lbs. ;

5 tons. 6 cwt. 3 qrs. $27\frac{5}{61}$ lbs. ; 3 tons. 2 qrs. $14\frac{10}{41}$ lbs.

(3). 16 yds. 2 ft. $5\frac{11}{80}$ in. ; 12 yds. 1 ft. $9\frac{47}{48}$ in. ; 4 yds. $7\frac{47}{121}$ in. ;

2 yds. $\frac{181}{300}$ in.

(4). $32\frac{2935}{4782}$.

6. (1). Rs. 4. 12 as. $4\frac{18}{25}$ pies. (2). £7. 8*s*. 11*d*. $1\frac{1}{5}$*q*.

(3). 2 miles. 6 fur. 22 po. 3 in. (4). 31 cwt. 3 qrs. 12 lbs. 10 oz.

(5). 12 yds. 1 ft. $11\frac{1}{2}$ in. (6). Rs. 43. $4\frac{1}{2}$ pies.

(7). 14*s*. 9*d*. (8). Rs. 21. 15 as. $9\frac{3}{25}$ p.

(9). £4. 7*s*. 6*d*. (10). 19*s*. 1·5*d*.

7 (1). $\frac{5}{8}$. (2). $\frac{789}{997}$. (3). $\frac{97}{100}$. (4). $6\frac{1}{4}$.

(5). $\frac{91}{800}$. (6). $3\frac{5}{8}$. (7). $1\frac{1}{6}$. (8). $\frac{1}{12}$.

(9). $\frac{12}{25}$. (10). $\frac{281}{350}$. (11). $\frac{84}{475}$. (12). $\frac{2871}{10000}$.

(13). 32. (14). ·627083. (15). ·5203125. (16). 350·90.

(17). 4·78125. (18). ·75. (19). ·3125. (20). ·0625.

(21). ·4279... (22). ·206...

3·54375. 9. 9·50129. 10. ·0693.

Examples XXX. Pages 38—44.

1. 1 pie. 2. Rs. 75. 3. 13s. 6d. 4. 60 lbs.
5. 64. 6. 6250. 7. Rs. 4. 6 as. 9. 152 galls.
10. Rs. 4. 11. 162½ galls. 12. A, Rs. 281. 5 as. ; B, Rs. 231. 1 a. 9 p.
13. A, Rs. 5. 7 as. 8 p. ; B, Rs. 7. 13 as. 11 p. ; C, Rs. 11. 5 as
14. Ram, Rs. 7. 7 as. 6 p. ; each of the 9 others, Rs. 3. 1 a. 4 p.
15. A, Rs. 15. 6 as. 8 p. ; B, Rs. 19. 12 as. 8 p. ; C, Rs. 11. 8 p.
16. £2. 7s. 3d. ; £7. 1s. 9d.
17. 1st. Rs. 4. 11 as. ; 2nd. Rs. 14. 1 a. ; 3rd. Rs. 56. 4 as.
18. Each man, £12. 18s. ; each woman, £4. 6s. ; each boy, £2. 3s.
19. Rs. 28. 10 as. ; Rs. 80. 8 as.
20. Oranges, 200 ; pears, 600 ; plums, 800.
21. 320. 22. 20. 23. 120 of each coin.
24. Rs. 24 ; 72 eight-anna pieces ; 96 four-anna pieces ; 120 two-anna pieces.
25. Horse, Rs. 100. 8 as. ; cow, Rs. 20. 8 as. ; ass, Rs. 15. 8 as.
26. Jadu, 12 yrs. 7 mo. ; Gopal, 11 yrs. 6 mo. ; Haradhan 10 yrs. 5 mo.
27. 32. 28. 4 mi. 1320 yds. 29. 41. 30. 7 yds.
31. 15 yds. 1 ft. 8 in. 32. A, Rs. 4. 4 as. 5 g. ; B, Rs. 2. 2 as. ; C, Rs. 4. 4 as. 5 g. 33. 365. 34. 5.
35. Rs. 5 4 as. 8 p. ; Rs. 26. 7 as. 4 p. 36. 16. 37. 8 as.
38. Rs. 1125. 5 as. 4 p.
39. Jadu, Rs. 78. 2 as. 6 p. ; Keshab, Rs. 35. 6 as. 6 p.
40. Rs. 264. 41. 36.
42. 20. sec. 43. 26 half-rupees, 45 pice.
44. 50 half-rupees, 80 pice. 45. Rs. 100.

46. 1 yr. 9 mo. 4 days.

47. 34 yrs. 7 mo. 5 days.

48. 1 yard.

49. *A*, Rs. 225, *B*, Rs. 100.

50. $4\frac{1}{2}$ miles.

51. $1\frac{1}{2}$ min.

52. *A*'s age 25 years, *B*'s age 30 years.

53. *A*, Rs. 50 ; *B*, Rs. 25 ; *C*, Rs. 15.

54. $17\frac{4}{11}$ days.

55. $5\frac{1}{3}$ days.

56. $1\frac{1}{2}$ hours.

57. $21\frac{3}{8}$ days.

58. Rs 1829. 5 as. 4 p.

59. Rs 25. 9 as 6 p.

60. $2\frac{1}{2}$ hours.

61. *A*, 6 as. 6 p. ; *B*, 9 as. 6 p.

62. $90\frac{3}{8}$ galls.

63. *A*, 3·24*s.* ; *B*, 4.32*s.* ; *C*, 5·76*s.* ; *D*, 7·68*s.*

64. Rs. 90, 225 half-rupees, 7 four-anna pieces, 108 two-anna pieces.

65. 100, 180, 20, 40, 20, 40.

66. Rs. 1000.

67. *A*, Rs. 425, *B*, Rs. 575.

68. $1\frac{1}{2}$ md.

69. Rs. $14\frac{4}{7}$.

70. 22 oxen.

71. $2\frac{1}{2}d.$; 96 oz.

72. 26th September, 1820.

73. $14\frac{2}{27}$ min.

74. 4 hours.

75. 75 gallons ; 15 hours.

Examples XXXI. Pages. 45—47.

1. $192\frac{2}{5}$ yds. ; Rs. 108. 8 as.

2. Rs. 27. 6 as. 5 p.

3. Rs. 5803. 8 as.

4. 203 bghs.

5. $326\frac{11}{72}$ cub ft.

6. Rs. 5. 1 p.

7. 42 ft. by 7 ft.

8. Rs. 9936.

9. 6300.

10. Rs. 126, Rs. 64. 12 as. $9\frac{3}{8}$ p.

11. Rs. 521. 9 as. 4 p.

12. 16 ft. 8 in.

13. $29\frac{19}{81}$ cub. in.

14. 1000 oz.

15. Rs. 311. 0 a. $7\frac{17}{21}$ p.

16. £120. 15*s.*

17. $1666\frac{2}{3}$.

18. 16 ft

19. $12\frac{1}{2}$.

20. Rs. 25. 12 as. 8 p.

21. Rs. 1584. 3 as.

22. Rs. 19. 3 as.

23. 1036800.

24. 35625 lbs.

35. Rs. 683. 7 as.

Examples XXXII. Page 47.

1. £52. 1*s*. $9\frac{1}{18}$*d*.	2. £100. 15*s*. $1\frac{88}{160}$*d*.
3. Rs. 2970. 10 as. $3\frac{3}{4}$ p.	4. Rs. 80. 11 as. $2\frac{7}{10}$ p.
5. £1103. 15*s*. $6\frac{3}{8}$*d*.	6. £15. 6*s*. $7\frac{3}{8}$*d*.
7. Rs. 4199. 11 as. 7 p.	8. Rs. 1160. 9 as. $8\frac{1}{4}$ p.
9. Rs. 47. 14 as. 3 p.	10. Rs. 1463. 4 as. 4 p.

Examples XXXIII. Pages 48—61.

1. 1 : 2027520.	2. 1 : 6400.	3. 1 : 13824.
4. (1). 144 : 175.	(2). 7 : 576.	(3) 40 : 121.
5. 16 : 9.	6. 5 : 4.	7. 4 : 9.
8. 188 : 187.	9. 2 : 3.	10. No, no, yes.
11. 6859 : 125.	12. 125 : 8.	14. 5 : 6.
15. 8 : 9.	16. 1 : 1.	17. Rs. 1234.
18. 7920 shillings.	19. 10 gallons.	20. 25 : 24.
21. (1). $1\cdot9\frac{1}{4}$.	(2). Rs. 1136. 15 as. 3 p.	
(3). 1234.	(4). 261 kathas.	(5). $32666\frac{2}{3}$ hrs.
(6). $7\frac{9}{16}$ kanchas.	(7). 2 dandas 13 pals.	(8). 18′.
22. (1). 20·5.	(2). $34\frac{2}{7}$*s*.	(3). 12 lbs. Avoir.
(4). 320 sq. yds.	(5). 1000 kilog.	(6). $\frac{1}{39370\cdot8}$ in.
23. (1). $\frac{7}{8}$.	(2). 4 as.	(3). 18 chts.
(4). 11 sq. kths.	(5). 12″.	(6). $\frac{1}{10}$ cub. metre.
24. 97 yds.	25. Rs. 126. 10 as. 8 p.	
26. 36 days.	27. 18 yds.	28. Rs. 336. 1 a. $8\frac{1}{4}$ p.
29 14*s*. $5\frac{1}{4}$*d*.	30. Rs. 2590000.	31. 62 ft. 6 in.
32. Rs. 324.	33. Rs. 3750.	34. $8\frac{1}{4}$ hrs.
35. 220 : 107.	36. 6 days.	37. 8 oz.
38. 3·12*s*.	39. Rs. 216.	40. 187.
41. 124*ll*.	42. 16 annas.	43. $11\frac{1}{4}$ oz.

44. 231. 45. 5 hrs. 46. $6\frac{3}{4}$ days.

47. 19 days. 48. Rs. 200. 49. 20 men.

50. 4 p. 51. Rs. 15726. 52. £23689.

53. 65 men. 54. 145 yds. 55. ·5236 cub. yds.

56. 4′ 47″ to 1 P.M.; at 8 P.M. on Tuesday.

57. At 1 A.M. on Monday week; 1h. 13′ A.M., 12h. 55′ A.M.

58 48″. 59. 11 h. $28\frac{1}{2}$′ A.M.

60. At 4 A.M. on Tuesday.

61. 3 h. $49\frac{28}{33}$ min. A.M.

62. At 1 P.M. on April 24, 1896.

63. 20 days ago; 880 days.

64. The faster must be put back $16\frac{16}{143}$ min. or the slower put forward 16 min.

65. 12-30 P.M. on June 6. 66. 10′ past 9 P.M.

67. $8\frac{8}{59}$′ past 9 P.M. 68. After 36 days.

69. $2\frac{1124}{1309}$ min.

70. (1). $54\frac{6}{11}$′ past 10; (2). $21\frac{9}{11}$′ past 10;
(3). $5\frac{5}{11}$′ and $38\frac{2}{11}$′ past 10.

71. (1). $17\frac{5}{11}$′ and $58\frac{10}{11}$′ past 7; (2). $13\frac{1}{11}$′ past 7.

72. $30\frac{70}{143}$ min. past 3. 73. $53\frac{227}{341}$′ past 4.

74. 1′ $59\frac{1}{3}$″ to 1 P.M. 75. 1 min. division put back.

76. It gains $1\frac{1}{2}$ min. 77. 176 yds.

78. 9 min. 79. $8\frac{1}{4}$ min.

80. 320 yds. 81. 57.

82. *C*; 492. 83 *A* by $508\frac{1}{3}$ yds. 84. 175 yds.

85. 93 secs. 86. 88.

87. *A* in 3′ $10\frac{8}{13}$″, *B* in 3′ 37″, *C* in 2′ 20″.

88. *A* by 160 yds. 89. $1\frac{1}{2}$ Rupee. 90. 19.

91. $2\frac{8}{9}$ days. 92. 1500 93. 564.

94. $10\frac{1}{2}$ mi. and $4\frac{1}{2}$ mi. per hour. 95. $9\frac{1}{2}$ mi. and $7\frac{1}{2}$ mi per hour.

96. (1). $4\frac{1}{8}''$ and $12\frac{7}{8}''$; (2). $5\frac{5}{8}''$ and $16\frac{7}{8}''$.
97. 4 mi.
98. $48\frac{1}{2}$ min.
99. 30 mi. an hour.
100. $6\frac{8}{11}$ mi. per hour
101. $16\frac{1}{2}$ mi.
102. 10 hrs. 42′.
103. 10 hrs. a day.
104. 26 ft.
105. 36.
106. 3008000.
107. 18 oz.
108. Rs. 5. 2 as.
109. 10·73 lbs.
110. $9\frac{1}{4}$ hrs.
111. 372.
112. 7 hrs.
113. 42.
114. 101 ft. 4 in.
115. 6 as.
116. Rs. 4036. 8 as.
117. 1035.
118. Rs. 1540.
119. 64.
120. Rs. 33 12 as.
121. 7 hrs.
122. 255.
123. 792 yds.; 7′.
124. 464.
125. $6\frac{1}{4}$ hrs.
126. £16800.
127. Rs. 322560.
126. Rs. 2240.
129. Rs. 3400.
130. £2447.
131. Rs. 5000.
132. Rs 3588.
133. £25000.
134. 11as. $6\frac{2}{3}$ p.
135. £21600; £15300.
136. Rs. 28800.
137. 28·8 in.
138. $\frac{1}{16}$ seer.
139 1610 ft.; 305·9 ft.
140. 11·9 years nearly.

Examples XXXIV. Pages 61, 62.

1. £47. 16*s*. 3*d*., £77. 0*s*. $7\frac{1}{2}$*d*., £125. 14*s*. 7*d*., £173. 19*s*. $8\frac{1}{4}$*d*., £2. 1*s*. $5\frac{1}{4}$*d*.
2. 56 lbs., 1700 lbs.
3. 137.
4. 8 mds. 9 seers.
5. Oxygen 1110 lbs., hydrogen 138 lbs. 12 oz.
6. Rs. 750.
7. The radius is divided into 3 parts whose lengths are 4 in., 6 in. and 8 in.
8. 35 moidores, 42 sovereigns, 70 half-guineas, 252 half-crowns.
9. 1564.
10. 120 moidores, 340 guineas, 560 sovereigns, 780 crowns.

Examples XXXV. Pages 62—66.

1. 10456783750. 2. 49·856 per cent. of oxygen, 43·265 per cent. of carbon, and 6·879 per cent. of hydrogen. 3. 42 : 31.
4. $11\frac{1}{9}$ per cent. 5. 55 per cent. 6. Rs. 13440.
7. $30\frac{5}{8}$ per cent. 8. Rs. 37. 8 as. 9. 10 : 1.
10. $47\frac{13}{17}$ per cent. 11. Rs. 400. 12. 4 : 7.
13. 80 weeks. 14. 16*s*. 15. *A* 1309, *B* 1316.
16. $6\frac{1}{3}$ per cent. gain. 17. Rs. 5. 5 as. 4 p.
18. Rs. 592. 8 as. 19. 12 per cent. 20. 26 per cent.
21. 30. 22. $15\frac{15}{16}$ per cent. 23. Rs. 400.
24. $81\frac{1}{4}$ per cent. 25. Rs. 52. 1 a. 4 p. 26. He loses $\frac{3}{4}$ per cent
27. Rs. 17. 8 as , Rs. 20. 28. Rs. 500.
29. 95 : 123. 30. 48 : 129 ; $27\frac{31}{43}$ per cent.
31. He gains $35\frac{5}{16}$ per cent. 32. 5 for 3as. ; 1840.
33. 51 : 11.
34. $35\frac{5}{17}$ per cent. 35. 2 : 3. 36. 14 years.
37. 9 yrs. 6 mo. 38. $12\frac{1}{2}$ yrs. 39. Rs. 24.
40. 96°.

Examples XXXVI. Pages 66, 67.

1. Rs. 68000. 2. Rs. 6000, Rs. 5000, Rs. 5000.
3. Rs. 1500. 4. £1517. 5*s*., £664. 13*s*.
5. Rs. 1680, Rs. 2025. 6. 40.
7. Rs. 2784, Rs. 2280. 8. Rs. 3520, Rs. 3080.

Examples XXXVII. Pages 67—69.

1. 7th September. 2. $2\frac{1}{2}$ years ; $9\frac{3}{8}$ per cent. per annum.
3. 6. 4. $3\frac{1}{2}$. 5. 10 years.
6. Rs. 800. 7. Rs. 800. 8. Rs. 1000.
9. 5 years. 10. 11 per cent. 11. 3 years

12. 3 years. 13. Rs. 7300. 14. £5000; $3\frac{1}{3}$ per cent
15. 5 years. 16. £5000. 17. Rs. 10000.
18. Rs. 25000. 20. 25 years.

Examples XXXVIII. Pages 69, 70.

1. Rs. 99. 5 as. 6 p. 2. £70. 11 *s.* 8*d.*
3. Rs. 183. 9 as. 11·808 p. 4. £254. 14*s.* 6*d.*
5. £6325. 6. Rs. 438. 0 a. 5 p.
7. Rs. 390625. 8. Rs. 31250.
9. Rs. 6492. 14 as. 10·176 p. 10. £409600.
11. Rs. 32000. 12. Rs. 1000.

Examples XXXIX. Pages 70—72.

1. Rs. 2500000. 2. £829. 7*s.* 9*d.*
3. 5 years hence. 4. $2\frac{3}{4}$ per cent.
5. Rs. 807. 5 as. 4 p. 6. 7 years. 7. £6250.
8. Rs. 142100, $4\frac{1}{2}$ per cent. 9. Rs. 700.
10. £350. 11. Rs. 16000. 12. £700.
13. *B*'s offer. 14. 200 : 203 ; Rs. 25.
15. Rs. 1295. 16. $6\frac{1}{4}$ per cent. 17. $9\frac{77}{312}$ per cent.
18. £731. 5*s.* 19. Rs. 2. 4 as. 20. £5115.

Examples XL. Page 72.

1. $170\frac{1}{3}$ days. 2. After 8 months.
3. After 5 months. 4. After 22 months
5. 27th of May. 6. 5th of March 1892.

Examples XLI. Pages 72—76.

1. $106\frac{7}{8}$; Rs. 25000. 2. Rs 17200.
3. Rs. 954. 7 as. 8 p.

4 The Municipal Debentures ; Rs. 61285, Rs. 45864.

5. Rs. 1600.

6. Rs 122.

7. Rs 108.

8. Decrease of Rs. 10.

9. Rs. 1079.

10. Rs. 109. 7 as.

11. He loses ; Rs. 21182.

12 Rs. 3180, Rs 6420.

13. Rs. 105.

14 Rs. 31065.

15. Rs. 3840, Rs. 3000.

16. Rs. $9\frac{1}{10}$.

17. Rs. 45 ; increase of Rs. 100.

18. Rs. 15000, Rs. 13000.

19. Rs. 28602.

20. Rs. 1680.

21. $109\frac{4}{13}$.

22. Rs. 10000000.

23. Rs. 2891933.

24. Rs. 1600.

25. 4 : 3.

26. 6 per cent.

27. Rs. 40000 stock

28. Increase of Rs. $75\frac{24}{101}$

29. Rs. 32000 stock ; Rs 85.

30. Rs. 10920.

Examples XLII. Pages 76, 77.

1. 101 : 16.

2 36 seers of each kind.

3. 40 mds. and 24 mds.

4. 52 seers of cow's milk and 26 srs. of buffalo's milk ; 2 as. a seer.

5. 1, 3, 1, 3.

6. 10 seers.

7. 3, 4, 23.

8. 20, 25, 6, 7.

Examples XLIII. Pages 77, 79.

1. Rs. 12473. 3 as. 8 p.

2. Rs. 13. 9 as 4 p.

3. 7*s.* 6*d.*

4 $12\frac{1}{2}$*d.* a rupee.

5. Rs. 10. 2 as. 11 p.

6 Rs 11. 4 as 8 p

7. $13\frac{1}{4}$*d.* a rupee.

8. Rs. 105 12 as.

9. Rs. 18.

10. He gains £32. 13*s.* 4*d.*

11. Directly from St. Petersburg.

12. The circuitous remittance.

Examples XLIV. Pages 79, 80.

1. ·000001. 2. 35. 3. 6.

4. 705600. 5. (1). ·25 ; (2). $1\frac{1}{4}$.

6. 3600 ; 60. 7. 5 ft. 3 in. 8. 3500 ft.

9. 357. 10. 27 in., 18 in., 9in.

11. 65 ft., 26 ft., 30 ft. 12. Rs. 4. 13 as. per maund.

13. $12\frac{1}{3}$ in. by $6\frac{1}{4}$ in. by $4\frac{1}{3}$ in. 14. $9\frac{3}{8}$ per cent.

15. £1825 ; 5 per cent. 16. $6\frac{3}{4}$ per cent.

17. 311 yds. 4·5 in. 18. 51 ft. 3 in.

19. 3 ft. 6·43 in. 20. 160 miles.

Examples XLV. Pages 80, 81.

1. ·000000003. 2. 11. 7.

4. ·07 ; $1\frac{1}{2}$. 5. 84 ft. 14 in.

7. $5\frac{1}{3}$ ft. 8. $4\frac{1}{4}$ ft.

9. £78125 ; 4 per cent. 10. 9 ft. 2·9 in. ; 170 sq. ft. 96 sq. in.

APPENDIX B.

Examples I Page 1.

1. (1). 16009. (2). 1900705. (3) 6704371

(4). 5400000709809. (5). 2,345,678,912,345,678,912,345,678

2 (1). Nine hundred and eighty seven millions, six hundred and fifty-four thousand, three hundred and twenty-one

(2). Ten thousand two hundred millions, forty-thousand five hundred and six.

(3). Six thousand and six millions, six thousand and six

(4). Five hundred and fifty-five millions, five hundred and fifty five

(5). Five hundred and seventy eight billions, nine hundred and twelve thousand three hundred and forty-five millions, six hundred and seventy-eight thousand nine hundred and twelve

3. (1). CCII. (2) $\overline{V}$CCCCXCII (3) XXVIILX

(4). MMMDCCCLXXIV (5). CCCIↃↃↃ

4. (1). 24. (2). 70. (3) 96 (4). 1002.

(5). 937.

Examples II. Page 2.

1. 44. 2 165. 3 5504.

4. 2777220. 5. 4890335544 6. 84.

7. 102. 8. 17912. 9. 966786368 Rs.

10. 115 ; 175 ; 108 ; 93 ; 491

Examples III. Pages 2, 3

1. 1714. 2. 333333. 3. 909090909.

4. 2992. 5. 19594 trees. 6. 141238.

7. 3680 Rs. 8. 4800 Rs. 9. (1). 7863. (2). 17773.

10. 1793 ; 27 years old.

Examples IV. Pages 3, 4.

1. 834339635498O7. 2. 6603400400.

3. 60480. 4. 24981.

5. 9018009. 6. 2580 miles.

7. 205,158,958 feet. 8. 30,542,400 seers.

9. 3780 mangoes. 10. 17,280 boys ; 69120 Rs.

Examples V. Pages 4, 5.

1. 61728394, rem. 1 ; 41152263 ; 30864197, rem. 1 ; 24691357, rem. 4 ; 20576131, rem. 3.

2. 3350969, rem. 6 ; 2932098, rem. 5 ; 2606309, rem. 8 ; 2345698, rem. 9 ; 2132435, rem 4.

3. 288065, rem. 9 ; 265906, rem. 11 ; 246913, rem. 7 ; 230452, rem. 9 ; 216049, rem. 5.

4. 58097313 ; 54869684, rem. 9 ; 51981806, rem. 7 ; 49382716, rem 1.

5. 300066755, rem. 222611994.

6. 18182427747, rem. 98673. 7. 66554433. 8. 7947.

9. 70. 10. 47746148 seconds. 11. 400 men.

12. Rs. 59850.

Examples VI. Page 5.

1. (1). 8. (2). 8. (3). 16. (4). 18.
(5). 16. (6). 18. (7). 252. (8). 72.
(9). 6. (10). 32. (11). 4. (12). 6.

2. (1). 48. (2). 2520. (3). 2520.
(4). 1680. (5). 300. (6). 128.
(7). 2940. (8). 1080. (9). 72.
(10). 1260. (11). 1890. (12). 7560.

3. After 1680 min. or 28 hrs.; 840, 336, 240, 168, 140, 120, and 105 times.

4. 120 days. 5. 7.

Examples VII. Page 6.

1. (1). $\frac{15}{3}$; $\frac{20}{4}$; (2). $\frac{24}{4}$; $\frac{48}{8}$; $\frac{72}{12}$.

(3). $\frac{135}{15}$; $\frac{153}{17}$; $\frac{171}{19}$; $\frac{216}{24}$.

(4). $\frac{88}{8}$; $\frac{55}{5}$; $\frac{121}{11}$; $\frac{132}{12}$; $\frac{143}{13}$; $\frac{154}{14}$; $\frac{209}{19}$.

(5). $\frac{144}{12}$; $\frac{156}{13}$; $\frac{180}{15}$; $\frac{240}{20}$.

(6). $\frac{150}{5}$; $\frac{180}{6}$; $\frac{210}{7}$; $\frac{240}{8}$; $\frac{270}{9}$; $\frac{300}{10}$; $\frac{330}{11}$; $\frac{360}{12}$.

(7). $\frac{90}{2}$; $\frac{135}{3}$; $\frac{180}{4}$; $\frac{225}{5}$; $\frac{270}{6}$; $\frac{315}{7}$; $\frac{360}{8}$.

(8). $\frac{1100}{20}$; $\frac{1650}{30}$; $\frac{2200}{40}$; $\frac{2750}{50}$; $\frac{3300}{60}$; $\frac{3850}{70}$; $\frac{4400}{80}$; $\frac{4950}{90}$.

2. (1). 3. (2). 5. (3). 3. (4). 3. (5) 5.
(6). 3. (7). 15. (8). 19. (9). 16. (10). 30.
(11). 8. (12). 45.

Examples VIII. Pages 6–8.

(1). $2\frac{1}{2}$. (2). $2\frac{1}{3}$. (3) $2\frac{1}{4}$.
(4). $2\frac{1}{5}$. (5). $2\frac{3}{7}$. (6). $3\frac{1}{6}$.
(7). $2\frac{11}{12}$. (8). 5. (9). $7\frac{1}{7}$.
(10). 7. (11). $1\frac{1}{60}$. (12). $1\frac{33}{198}$ or $1\frac{1}{6}$.
(13). 7. (14). $10\frac{12}{24}$ or $10\frac{1}{2}$. (15). 5[illegible].
(16). 7[illegible]. (17). 10[illegible]. (18). 3.
(19). 4[illegible]. (20). 7[illegible].

(1). $\frac{5}{3}$. (2). $\frac{11}{4}$. (3). $\frac{19}{5}$.
(4). $\frac{29}{6}$. (5). $\frac{41}{7}$. (6). $\frac{55}{8}$.
(7). $\frac{71}{9}$. (8). $\frac{89}{10}$. (9). $\frac{109}{11}$.
(10). $\frac{131}{12}$. (11). $\frac{155}{13}$. (12). $\frac{181}{14}$.
(13). $\frac{209}{15}$. (14). $\frac{239}{16}$. (15). $\frac{271}{17}$.

(16). $\frac{062}{9}$. (17). $\frac{1181}{2}$. (18). $\frac{227109}{11}$.

(19). $\frac{10009}{11}$. (20). $\frac{100000000000001}{1000000}$.

(1). $\frac{8}{15}$. (2). $\frac{49}{83}$. (3). $\frac{27}{16}$.

(4). $\frac{27}{110}$. (5). $\frac{102}{35}$. (6). $\frac{83}{128}$.

(7). $\frac{72}{1}$. (8). $\frac{1072128}{125}$. (9). $\frac{2345939304}{128}$.

(10). $\frac{1780530871}{448}$. (11). $\frac{32361}{3000}$. (12). $\frac{544792027}{2268}$.

(1). $\frac{15}{32}$. (2). $\frac{2}{5}$. (3). $\frac{52}{63}$.

(4). $\frac{256}{133}$. (5). $\frac{144}{31}$. (6). $\frac{1}{2}$.

(7). $\frac{581}{138}$. (8). $\frac{19}{2}$. (9). $\frac{1}{78}$.

(10). $\frac{7}{15}$. (11). $\frac{8}{3}$. (12). $\frac{65}{9}$.

(13). $\frac{7}{8}$. (14). $\frac{1107}{1285}$.

(1). $\frac{7}{8}$. (2). $\frac{5}{6}$. (3). $\frac{110}{117}$.

(4). $\frac{103}{137}$. (5). $\frac{15}{23}$. (6). $\frac{40}{57}$.

(7). $\frac{60}{161}$. (8). $\frac{4}{7}$. (9). $\frac{21801}{2000000}$.

(10). $\frac{11}{18}$.

6. (1). $\frac{20}{30}$, $\frac{24}{30}$, $\frac{25}{30}$ and $\frac{27}{30}$. (2). $\frac{42}{105}$, $\frac{40}{105}$, $\frac{19}{105}$, and $\frac{25}{105}$.

(3) $\frac{90}{180}$, $\frac{45}{180}$, $\frac{36}{180}$, $\frac{30}{180}$ and $\frac{20}{180}$.

(4). $\frac{2520}{2520}$, $\frac{1680}{2520}$, $\frac{1512}{2520}$, $\frac{1260}{2520}$, $\frac{1080}{2520}$, $\frac{945}{2520}$ and $\frac{840}{2520}$.

(5). $\frac{504}{2520}$, $\frac{420}{2520}$, $\frac{360}{2520}$, $\frac{315}{2520}$ and $\frac{280}{2520}$.

(6). $\frac{420}{4200}$, $\frac{210}{4200}$, $\frac{140}{4200}$, $\frac{105}{4200}$, $\frac{84}{4200}$, $\frac{70}{4200}$ and $\frac{60}{4200}$.

(7). $\frac{195}{390}$, $\frac{208}{390}$, $\frac{156}{390}$ and $\frac{273}{390}$. (8). $\frac{140}{165}$, $\frac{121}{165}$ and $\frac{130}{165}$.

(9). $\frac{300}{405}$, $\frac{1260}{405}$ and $\frac{1575}{405}$. (10). $\frac{13}{35}$, $\frac{4}{35}$, $\frac{15}{35}$ and $\frac{14}{35}$.

(11). $\frac{180}{420}$, $\frac{378}{420}$, $\frac{300}{420}$ and $\frac{375}{420}$. (12). $\frac{18}{30}$, $\frac{237}{30}$, $\frac{490}{30}$ and $\frac{610}{30}$.

7. In order of value the fractions stand thus :—

(1). $\frac{3}{7}$, $\frac{2}{5}$ and $\frac{1}{3}$. (2). $\frac{3}{5}$, $\frac{7}{10}$ and $\frac{1}{2}$.

(3). $\frac{5}{6}$, $\frac{4}{5}$, $\frac{9}{12}$ and $\frac{4}{18}$.

(4). $\frac{9}{10}$, $\frac{5}{6}$, $\frac{7}{12}$ and $\frac{8}{15}$. (5). The fractions are equal in value.

(6). $\frac{6}{7}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{3}{4}$ and $\frac{2}{3}$. (7). $\frac{8}{15}$, $\frac{1}{2}$ of $\frac{1}{2}$ and $\frac{4}{15}$.

(8). $\frac{1}{10}$, $\frac{2}{100}$, $\frac{3}{1000}$, $\frac{4}{10000}$ and $\frac{5}{100000}$.

(9). $\frac{7}{10}$, [illegible], $\frac{5}{6}$ and [illegible]. (10). $\frac{1}{5}$ and $\frac{9}{11}$ of [illegible].

(11). $4\frac{1}{2}$, $\frac{4}{7}$ of $3\frac{1}{2}$, $\frac{3}{8}$ and [illegible] of [illegible].

(12). $4\frac{1}{3}$, $\frac{1}{4}$ of 5 and [illegible] of [illegible]. (13). [illegible], [illegible] and $\frac{9}{17}$ of [illegible].

(14). [illegible], [illegible], [illegible], [illegible], [illegible] and $\frac{4}{27}$.

(15). $\frac{7}{9}$ of $13\frac{8}{9}$, $9\frac{1}{2}$, [illegible], [illegible] of [illegible], $\frac{4}{17}$ and [illegible] of [illegible].

Examples IX. Pages 8, 9.

(1). [illegible]. (2). [illegible]. (3). $6\frac{84}{121}$. (4). $5\frac{8}{33}$.

(5). 1. (6). [illegible].

(1). [illegible]. (2). [illegible]. (3). [illegible]. (4). $2\frac{3}{7}$.

(5). [illegible].

(1). [illegible]. (2). [illegible]. (3). $5\frac{1}{3}$. (4). $19\frac{7}{38}$.

(5). [illegible]. (6). [illegible]. (7) [illegible].

Examples X. Pages 9, 10.

(1). [illegible]. (2). [illegible]. (3). $9\frac{1}{4}$. (4). $26\frac{1}{2}$.

(5). $26\frac{2}{3}$. (6). $\frac{8}{15}$. (7). [illegible].

2\. (1). By $2\frac{1}{70}$. (2). $\frac{4}{7}$. (3). $1\frac{8}{9}$. (4). 7.

(5). [illegible].

Examples XI. Page 10.

(1). [illegible]. (2). 26. (3). [illegible]. (4). [illegible].

(5). $\frac{1}{3}$. (6). $1\frac{7}{18}$.

(1). $\frac{2}{3}$. (2). 420. (3). [illegible]. (4). $1486\frac{1}{3}$.

(5). [illegible]. (6). [illegible].

3\. (1). $\frac{1}{3}$. (2). $\frac{5}{81}$. (3). $\frac{1}{3}$. (4). [illegible].

(5). $1\frac{2}{3}$. (6). 2500. (7). [illegible].

Examples XII. Page 11.

(1) [illegible]. (2). $1\frac{7}{30}$. (3). [illegible]. (4). [illegible].

(5). [illegible]. (6). $\frac{7}{3}$. (7). [illegible]. (8). [illegible].

(9). $12\frac{[illegible]}{178}$. (10). $13\frac{[illegible]}{[illegible]}$. (11). 29. (12). 116[illegible].

(13). $46\frac{[illegible]}{27300}$.

2. (1). 1. (2). 1. (3). $1\frac{9}{16}$. (4). 302[illegible].

(5). $\frac{1}{100}$. (6). [illegible]. (7) [illegible].

Examples XIII. Pages 12, 13.

1. (1). ·3. (2). ·9. (3). ·23. (4). ·57.

(5). ·007. (6). ·000065. (7). ·265. (8). ·000865.

2. (1). ·3. (2). 17. (3). ·32. (4). 1·045.

(5). ·045. (6). ·0065. (7). ·00057. (8). ·05.

(9). ·01. (10). ·00002. (11). ·0007.

(12). ·000000000009. (13). ·00001. (14). ·1.

3. (1). $\frac{4}{5}$. (2). $\frac{7}{20}$. (3). $\frac{2}{25}$. (4). $\frac{7}{200}$.

(5). $\frac{1}{50}$. (6). $\frac{1}{500}$. (7). $\frac{3}{2000}$. (8). $\frac{7}{1000}$.

(9). $2\frac{3}{20}$. (10). $2\frac{3}{200}$. (11). $8\frac{27}{100}$. (12). $\frac{11}{200}$.

(13). $1000\frac{7}{2000}$. (14). $\frac{133}{2000000}$.

(15). $\frac{87}{12500}$. (16). $4\frac{13}{25000}$.

4. (1). 4, 40, 400, 4000. (2). ·2, 2, 20, 200.

(3). 2, 20, 200, 2000. (4). 30000·3, 300003.

(5). 1230, 12300.

5. (1). ·002 ; ·00002. (2). ·02134 ; ·002134.

(3). 254·61 ; ·025461. (4). 123·45007 ; 12·345007.

(5). ·071314 ; ·00071314.

Examples XIV. Page 13.

1. (1). 2353·9849. (2). 1035·0227. (3). 855·288.

(4). 745·7961. (5). 400·877686.

2. (1). 456·7308. (2). ·135796. (3). 3200·00545.

(4). 1115·083. (5). 1666·665.

3\. (1). ·771056. (2). 784·51. (3). 100·777062.
(4). ·815088. (5). 3060200·090003.

Examples XV. Page 14.

1\. (1). 1·56. (2). 1·14. (3). 1·14.
(4). 1·111. (5). 599959·99621.

2\. (1). 2·7. (2). 3·96. (3). 6·993.
(4). 1·999998. (5). ·792.

3\. (1). 43·934. (2). 43·255. (3). 103·401.
(4). 567·745. (5). 401·6492.

Examples XVI. Page 14.

1\. (1). 400·824. (2). 1256·4035. (3). ·040755.
(4). 2672·3970. (5). 97·175344.

2\. (1). 1. (2). ·000000064. (3). 3·136.
(4). ·16. (5). ·000000081.

3\. (1). ·243000. (2). 20206816. (3). 552173760.
(4). 24·749569195. (5). 48·6658600943.

Examples XVII. Page 15.

1\. (1). 14. (2). 2200. (3). 20.
(4). 6006000. (5). 156250. (6). ·5388.
(7). 3000. (8). 41000. (9). 600600.
(10). 6·006. (11). ·0256256. (12). 5060000.

2\. (1). 0. (2). ·0006. (3). 1032000.
(4). 300000. (5). 44·046.

3\. (1). 1568·627450... (2). ·00015625.
(3). ·0006009...... (4). 16·7417...... (5). 4476·25.

4\. 32. 5. ·0032. 6. ·22.

Examples XVIII. Pages 15, 16.

1. (1). 1·075 ; ·625 ; ·21875. (2). ·64 ; ·3125 ; 3·16.
   (3). ·472 ; ·2375 ; 15·8. (4). 1·25 ; ·35.
   (5). ·6 ; 3·1171875 ; ·008125 ; 4·0064.

2. (1). ·30769 ; ·53333 ; ·90909. (2). ·09375 ; ·96875 ; 7·04288.
   (3). ·00664 ; ·03808 ; ·21428 ; ·45454.
   (4). ·81818 ; ·53846 ; ·42857 ; ·33333.
   (5). ·01666 ; ·83333 ; ·69696 ; ·77777.

3. (1). $\cdot\dot{3}$; $\cdot\dot{6}$; $\cdot\dot{7}$. (2). $\cdot\dot{5}$; $\cdot\dot{8}$; $\cdot\dot{1}7\dot{1}$; $\cdot0\dot{2}\dot{7}$.
   (3). $\cdot\dot{6}\dot{9}$; $\cdot\dot{7}\dot{2}$; $\cdot0\dot{3}\dot{6}$. (4). $\cdot\dot{3}0\dot{4}5$; $\cdot\dot{6}\dot{3}9$; $\cdot\dot{8}5714\dot{2}$.
   (5). $\cdot\dot{4}6153\dot{8}$; $\cdot\dot{0}0003\dot{6}$; $\cdot\dot{2}3809\dot{5}$.

4. (1). $\frac{2}{3}$, $\frac{7}{9}$, $\frac{8}{9}$. (2). $\frac{3}{11}$, [illegible], [illegible], [illegible].
   (3). [illegible], [illegible], [illegible].
   (4). [illegible], 36[illegible], 853[illegible].
   (5). 1234[illegible], 221[illegible], [illegible].

Examples XIX. Pages 16, 17.

1. (1). ·001235 ; ·095872. (2). ·983218 ; ·000088.
   (3). ·123457 ; ·102030. (4). ·765432 ; ·987654.
   (5). ·090807 ; ·554467.

2. (1). 54·74717. (2). 12·74271. (3). 3·35556.
   (4). 1·87370. (5). 3·81824. (6). 49·29832.
   (7). 156·51148. (8). 3·6363 ; ·04392.
   (9). ·86825 ; ·00586. (10). 2·29729 ; 9·66222.
   (11). 352·08564 ; 31·79179.
   (12) 49 ; $1\cdot\dot{1}\dot{4}5$. (13). ·05763. (14). 1

Examples XX. Pages 17, 18.

1. (1). 4935 p. ; Rs. 52. 1 a. 4 p. (2). 30714 p. ; Rs. 253. 2 as.
   (3). 7832*d.* ; £208. 6*s.* 8*d.* (4). 34963*d.* ; £16. 11*s.* 2*d.*

(5). 53400*d.* ; 89 half-guineas. 7s. 2*d*

(6). £198. 7*s.* 6*d.* ; 620 crowns.

(7). 19272 grains ; 5 oz. 2 dwts. 27 grains.

(8). 5974 dwts. (9). 85 lbs. 8 oz. 9 dwts.

(10). 16744 pounds. (11). 161124 kanchas.

(12). 1250 seers. (13). 339537 yards

(14). 113 miles 1120 yards. (15). 14217¼ sq. yds.

(16). 62 acres. (17). 266112 cub. inches.

(18). 20360 hats. (19). 7 kroses 839 dhanus

(20). 11488 chataks. (21). 210 bighas.

2 4520 pice. 3. 7862400 seconds. 4. 240000*d.* ; £40.

5 1638400 acres ; 293 bighas. 6. 128187 rupees

7 138519 pies. 8 Rs30541. 10 as 8p.

Examples XXI. Pages 18—20.

1. (1). £27. 6*s.* 3*d.* (2). £245. 5*s*

(3). Rs1908. 6as. 6p. (4). Rs 16032 4as 9p

(5). 164 tons. 8 cwts. 3 qrs. 14 lbs.

(6). 1494 mds. 33 seers. 7 chts.

(7). 116 days 16 hours 16 mins.

(8). 2337 bighas 10 kathas 3 chts.

(9). 238 ac. 2 ro. 25 po. 4 ft. 80 in.

(10). 42 weeks 1 day 18 hours 34 mins. 39 secs.

2. Rs9156. 4 as. 8 p. 3. Rs123. 14 as. 7 p

4. Rs9091. 12 as. 3 p. 5. Rs11982. 8 as. 10 p.

6. 78 mds. 2 seers.

Examples XXII. Pages 20—22.

1. (1). Rs10. 3 as. 3 p. (2). Rs33. 7 as. 2½ p.

(3). Rs60. 8 as. 2½ p. (4). Rs15. 3 as. 10 p.

(5). £249. 17*s.* 10*d.* (6). £70. 2*s.* 11*d.*

(7). £15. 5*s.* 1*d.* (8). £20. 3*s.* 11*d.*

(9). 208 mds. 10 seers. 7½ chts.

(10). 230 tons. 18 cwts. 1 qr. 4 lbs.

(11). 685 bighas. 3 kathas. 2⅓ chts.

(12). 6 ac. 1 ro. 3 po. 14⅜ sq. yds.

(13). 52 hrs. 51 mins. 10⅛ secs.

(14). 17 weeks 1 day 14 hours 59 mins.

(15). 8911 miles 1 fur. 3½ po.

(16). 3 lbs. 2 oz. 3 dwts. 9 grs.

2. Rs2086. 2 as. 6 p. 3. 1 year 3 months 10 days.

4. Rs31343. 12 as. 8 p.

5. 480 guineas is the greater and by £14.

6. Satis has more than Prahbash by R. 1. 2 as. 8 p.

Examples XXIII. Pages 22, 23.

1. (1). Rs. 84. 13 as. 1½ p., Rs. 141. 5 as. 10½ p., Rs. 254. 7 as. 4½ p.

(2). Rs. 607. 5 as. 4 p., Rs. 1062. 13 as. 4 p., Rs. 1214. 10 as. 8 p.

(3). £113. 1*s.*; £169. 11*s.* 6*d.*; £197. 16*s.* 9*d.*; £310. 17*s.* 9*d.*

(4). £243. 3*s.* 11½*d.*; £389. 2*s.* 4*d.*; £486. 7*s.* 11*d.*, £583. 13*s.* 6*d.*

(5). 69 mds. 18 srs. 6 chts., 115 mds. 30 srs. 10 chts. 162 mds. 2 srs. 14 chts., 185 mds. 9 srs.

(6). 13 tons 16 cwt. 3 qrs. 4 lbs., 19 tons 7 cwt. 2 qrs. 30 tons 8 cwt. 3 qrs. 20 lbs., 33 tons 4 cwt. 1 qr. 4 lbs.

(7). 218 bighas 19 kathas 10 chts., 243 bighas 6 kathas 4 chts., 291 bighas 19 kathas 8 chts., 389 bighas 6 kathas.

(8). Rs568. 7 as. 10 p.; Rs636. 11 as. 4 p.

(9). £541. 2*s.* 10½*d.*; £1236. 18*s.*

(10). 2758 ac. 3 ro. 8 po.; 3950 ac. 0 ro. 16 po.

(11). 1893 yds. 2 ft. 3 in.; 7272 yds.

(12). Rs. 295. 6 as. 3 p.; Rs. 406. 3 as. 6 p.

(13). £1914. 4 s. $7\frac{1}{4}$d., £2147. 13s. $5\frac{1}{2}$d.

(14). £26367. (15). £545695. 1s. $10\frac{1}{2}$d.

2. Rs. 85. 15 as. 3. Rs. 1857. 3 as. 4. Rs. 125.

5. Rs. 2323. 3 as. 6. Rs. 1977. 1 a. 4 p. 7. Rs. 10125.

8. Rs. 258940.

Examples XXIV. Pages 23, 24.

1. (1). Rs. 76. 2 as. $7\frac{1}{2}$ p., Rs. 57. 1 a. $11\frac{3}{8}$ p.

(2). Rs. 1061. 13 as. $4\frac{3}{4}$ p., Rs. 646. 5 as. $4\frac{1}{23}$ p.

(3). £65. 4s. $8\frac{3}{28}$d.; £60. 17s. $8\frac{11}{36}$d.

(4). £1276. 5s. $2\frac{13}{28}$d.; £389. 2s. $0\frac{117}{128}$d.

(5). 352 mds. 0 srs. $5\frac{44}{75}$ chts., 74 mds. 19 srs. $5\frac{80}{345}$ chts.

(6). 132 miles 6 ft. $13\frac{7}{11}$ po., 55 miles 5 ft. $6\frac{80}{165}$ po.

(7). 138 days 8 hrs. 21 mins. $31\frac{1}{3}$ secs., 23 days 3 hrs. 58 mins. $1\frac{73}{215}$ secs.

(8). 31 tons 5 cwts. 2 qrs. $3\frac{3}{8}$ lbs.; 5 tons 8 cwts. 1 qr. 11 lbs.

(9). Rs. 2374. 9 as. $3\frac{1}{6}$ p.; Rs. 237. 7 as. $3\frac{9}{23}$ p.

(10). Rs. 236. 11 as. $11\frac{81}{100}$ p.; Rs. 23. 10 as. $9\frac{881}{1000}$ p.

(11). $103\frac{357}{1702}$. (12). $218\frac{88}{285}$.

2. Rs 111. 8 as. $4\frac{55}{73}$ p. 3. 120 bighas 13 chts., 26 chts. rem.

4. Rs 19. 10 as. $3\frac{2}{7}$ p. 5. Rs 3. 14 as. 10 p.

6. $252\frac{4}{17}$ maunds. 7. Rs. 33. 4 as. $10\frac{22}{23}$ p.

8. Rs 1532. 0 as. $4\frac{4}{15}$ p. 9. 18496 maunds.

10. Rs 2. 8 as. $1\frac{7}{17}$ p. 11. Rs. 5. 12 as.

12. 156 pieces

Examples XXV. Pages 24, 25.

1. (1). Rs. 3. 11 as. 0 p. (2). Rs. 6. 9 as. 6 p.

(3). Rs. 10. 12 as. 1 p. (4). Rs. 1. 2 as. 0 p.

(5). Rs. 2. 9 as. 3 p. (6). Rs. 4. 4 as. 4 p.
(7). Rs. 7. 15 as. 11 p. (8). Rs. 12. 6 as. 3 p.
(9). £14. 14*s*. 7*d*. (10). £13. 0*s*. 0*d*.
(11). £18. 17*s*. $\frac{3}{4}d$. (12). £23. 9*s*. 2*d*.
(13). £3. 14*s*. 1*d*. (14). £5. 17*s*. 4*d*.
(15). £1. 19*s*. 9*d*. (16). £1. 2*s*. 1*d*.
(17). 11 cwt. 0 qr. 13 lbs. (18). 8 mds. 6 seers 12 chts.
(19). 22 days 19 hrs. 36 min. (20). 46 yds. 2 feet 4 inches.

2. (1). $\frac{5}{8}$. (2). ·7. (3). $\frac{5}{8}$. (4). $\frac{13}{32}$. (5). $\frac{33}{38}$.

3. $\frac{9}{37}$; ·31$\dot{6}$. 4. ·0$\dot{8}$92857$\dot{1}$4 ; $\frac{11}{120}$. 5. $\frac{9}{40}$; $\frac{241}{4000}$.

6. In order of value the quantities will stand thus :—

(1). $\frac{1}{3}$ of 14 as. ; $\frac{1}{5}$ of R. 1. ; $\frac{1}{12}$ of R. 1. 4 as.
(2). $\frac{3}{5}$ of 15*s*. ; $\frac{1}{4}$ of £1. ; $\frac{2}{3}$ of a crown.
(3). $\frac{1}{5}$ of a maund ; $\frac{5}{8}$ of 3 seers 6 chts. ; $\frac{1}{7}$ of 14 seers.
(4). $\frac{3}{7}$ of 21 yards ; $\frac{2}{3}$ of 7 ft. ; $\frac{3}{5}$ of 11 ft.
(5). $\frac{3}{5}$ of 5 days ; $\frac{3}{7}$ of 20 hrs. ; $\frac{1}{3}$ of 59 min.

Examples XXVI. Pages 25, 26.

1. 13*s*. $6\frac{1}{18}d$. 2. Rs. 8. 10 as. 4p.
3. £1. $6\frac{2}{3}s$. 4. 2 cwt. 3 qrs. $8\frac{1}{18}$ lbs.
5. £3. 16*s*. 11*d*. 6. 8 months 22 days 12 hrs. 40 min.
7. 15 bighas $13\frac{13}{18}$ kathas. 8. 10*s*.
9. 36205 mins. 10. 9*s*. 9*d*.

Examples XXVII. Page 26.

1. 8 annas. 2. Re. 1. 8 as. 3. 2 as. 6 p.
4. £1. 10*s*. 5. £5. 2*s*. $10\frac{3}{4}d$. 6. £27. 1*s*. $0\frac{3}{4}d$.
7. 17 hrs. 5 mins. 8. 21 seers $14\frac{2}{5}$ chts. 9. 3 as. $11\frac{1}{3}$ p.
10. 2 cwt. 0 qr. $20\frac{4}{11}$ lbs.

Examples XXVIII. Page 26.

1. Rs. 1 0 a. $2\frac{1}{4}$ p. 2. Rs. 13. 3 as. 3p. 3. Rs. 4. 4 as. $\frac{1}{30}$ p.
4. £1. 6s. $2\frac{2}{3}d$. 5. £50 10s. $5\frac{1}{3}d$. 6. £11. 12s. $1\frac{3}{20}d$.
7. 12 seers. $3\frac{8}{13}$ chts. 8. 3 qrs. 9. $1\frac{67}{80}d$.
10. $1\frac{2}{27}$.

Examples XXIX. Page 27.

1. £31. 3s. 9d. 2. £113. 13s. 4d. 3. Rs. 2. 7 ans. $2\frac{2}{3}$ p.
4. 6. 5. Rs. 205. 6. 4 feet. $7\frac{1}{2}$ in.
7. 11 mds. $3\frac{8}{11}$ seers. 8. $4\frac{9}{10}$. 9. 41 bighas. $9\frac{9}{20}$ kths.
10. $\frac{80}{81}$.

Examples XXX. Page 27.

1. Rs. 1680. 2. Rs. 2688.
3. Rs. 5333. 5 as. 4p. 4. $283\frac{8}{9}$ Sicca Rs.
5. £12 10s. $7\frac{1}{2}d$. 6. Rs. 488. 4 as.
7. $4\frac{19}{38}$ Fcy. mds.; 2 cwt. 3 qrs. 16 lbs. 8. $72\frac{90}{121}$ acres.
9. $114\frac{2}{3}$ dandas. 10. 81312 bghs.

Examples XXXI. Pages 27, 28.

1. (1). 55 sq. yds. $3\frac{1}{9}$ sq. ft. (2). 20 sq. yds. $2\frac{1}{4}$ sq. ft.
(3). 22 sq. yds. $2\frac{20}{27}$ sq. ft. (4). 23 sq. yds. $5\frac{7}{11}$ sq. ft.
(5). 7 sq. yds. $6\frac{2}{3}$ sq. ft.
2. (1). $1\frac{7}{8}$ cub. ft. (2). $23\frac{11}{14}$ cub. ft. (3). $2\frac{2}{3}$ cub. ft.
(4). 21 cub. ft. (5). $75\frac{1}{32}$ cub. ft.
3. 19200 sq. cubits. 4. $60\frac{1}{3}$ sq. yds. 5. Rs. 121.
6. 3000 cub. ft. 7. 2250 ft. 8. 36·3 ft.

Examples XXXII. Page 28.

1. (1). 23 sq. ft. 54 sq. in.
(2). 106 sq. ft. 132 sq. in. 0'', 7''', 8''''.

(3). 44 sq. ft. 112 sq. in. (4). 45 sq. ft. 132 sq. in.

(5). 395 sq. ft. 36 sq. in. (6). 2100 sq. ft. 100 sq. in.

(7). 744 sq. ft. 130 sq. in. (8). $1046\frac{1}{4}$ sq. ft.

(9). $3568\frac{2}{3}$ sq. ft.

106 yds. $1\frac{4}{9}$ ft.

Examples XXXIII. Page 29

1. (1). £8. 10*s*. (2). £189. (3). £802. 10*s*.
(4). £2314. (5). £117. 3*d*. (6). Rs. 615. 1a. 6p.
(7). Rs. 773. 8as. 6 p. (8). Rs. 36493. 0 a. 6 p.
(9). Rs. 221. 0 a. $8\frac{a}{11}$ p. (10). £467. 1*s*. $6\frac{6}{7}d$.

2. £7. 9*s*. $7\frac{1}{2}d$. 3. Rs. 1239. 13 as. 4 p.

4. Rs. 175. 4 as. $5\frac{49}{100}$ p. 5. £19994. 10*s*. $5\frac{5}{8}d$.

6. £2. 0*s*. $8\frac{1}{4}d$. $\frac{7}{16}q$. 7. 14 days 21 hours 45 minutes.

Examples XXXIV. Pages 29—31.

1. (1). 9. (2). 21. (3). 8. (4). Rs. 3.
(5). 8 yds. 1 ft.

2. (1). 27. (2). 80. (3). $\frac{3}{4}$. (4). $2\frac{1}{2}s$. (5). $\frac{6}{13}$ as.

3. £10260. 4. Rs. 40. 5. Rs. 68. 1 a. 6. 25 days.

7. 126 ft. 8. Rs. 40. 9. $4\frac{4}{8}\frac{?}{?}d$. 10. $6'\ 43\frac{1}{3}''$.

11. £2000. 12. 5 min. $17\frac{1}{1}\frac{?}{3}$ sec. to 11 A.M.

13. $16\frac{1}{1}\frac{?}{?}$ days. 14. 240 men. 15. 11 months.

16. 60 masons. 17. 5 as. $7\frac{1}{2}$ p. 18. $12\frac{1}{1}\frac{?}{?}$ days.

19. 45 men. 20. 72000 galls. 21. $6\frac{2}{3}$ hours.

Examples XXXV. Pages 31—33.

(1). Rs. 13. 12 as. 2 pice, Rs. 289. 6 as. 2 pice.

(2). Rs. 576. 15 as. $1\frac{1}{2}$ cowries, Rs. 6071. 9 as. 10*g*. $1\frac{1}{2}$ cowries.

(3). Rs. 152. 15 as. 18 gandas, Rs. 535. 7 as. 13 gandas.

(4). Rs. 332. 4 as. $16\frac{1}{2}$ gandas, Rs. 2227. 8 as. 11 gandas 2 cowries.

2. (1). £5. 13*s*. 11*d*. $0\frac{3}{8}$*q*. (2). £8. 8*s*. $4\frac{1}{8}$*d*.
   (3). £20. 15*s*. $0\frac{3}{5}$*d*. (4). Rs. 1483. 15 as. $\frac{9}{20}$ p.
   (5). Rs. 1101. 15 as. 8 p.

3. (1). 3 years 8 months $13\frac{1}{3}$ds. (2). 6 years.
   (3). 8 years 4 months. (4). 5 yrs. 6 months. 19 days.
   (5). 2 years 8 months $4\frac{17}{27}$ds. (6). 3 years 17 days.

4. (1). $16\frac{2}{3}$ per cent. per annum. (2). $13\frac{1}{3}$ per cent. per annum.
   (3). $3\frac{1}{3}$ per cent. per annum. (4). $1\frac{7}{8}$ per cent. per annum.
   (5). 10 per cent. per annum.

5. (1). Rs 1300. 6 as. 8 p. (2). Rs 2877. 4 as. 9 p.
   (3). ~~Rs~~ 3031. 15 as. 8p. (4). £666. 10*s*. $8\frac{1}{4}$*d*. $\frac{22}{27}$*q*.
   (5). ~~Rs~~ 68899. 1 a. $4\frac{2}{7}$p.

6. £1118. 15*s*. 7. £402. 10*s*.

8. £776. 9. $3\frac{1}{2}$ per cent.

10. $5\frac{1}{2}$ years.

Examples XXXVI. Pages 33, 34.

1. (1). Rs. 259. 6 as. $11\frac{7}{8}$ p. (2). Rs. 1153. 13 as. $6\frac{4}{13}$ p
   (3). Rs. 4909. 1a. $6\frac{96}{205}$ p. (4). Rs. 5911. 9 as. $1\frac{16015}{18128}$ p.
   (5). £24. 4*s*. $6\frac{7}{9}$*d*. (6). £558. 0*s*. $11\frac{1}{2}$*d*.
   (7). £1216. 13*s*. 4*d*. (8). £141. 19*s*. 1*d*.
   (9). £329. 0*s*. $7\frac{23}{31}$*d*. (10). £85793. 10*s*. $9\frac{9}{10}$*d*.

2. £43. 9*s*. $4\frac{8}{9}$*d*. 3. 4 per cent.

4. (1). Rs. 122. 10 as. 8 p. (2). Rs. 156. 15 as. $3\frac{122}{276}$ p.
   (3). Rs. 355. 0 a. 8 p. (4). Rs. 47. 10 as. $4\frac{2}{7}$ p.

5. £16. 19*s*. $11\frac{1}{4}$*d*. $\frac{88}{103}$ *q*. 6. Rs. 44. 6 as. 1 p.

7. £15. 3*s*. $7\frac{31}{35}$*d*. 8 £3. 10*s*. 9. £140.

10. £49. 10*s*. $4\frac{2}{7}$*d*. 11. 4 per cent.

12. The interest of £100 for 1 year at 5 per cent. is £5.

The discount on £100 due 1 year hence at the same rate of simple interest is £4. 15*s*. $2\frac{6}{7}$*d*.

Their difference is 4*s*. $9\frac{1}{7}$*d*.; and this is the interest on £4. 15*s*. $2\frac{6}{7}$*d*. at 5 per cent. per annum.

Examples XXXVII. Pages 34, 35.

1. (1). Rs. 2500. (2) Rs. 5555. 8 as. $10\frac{2}{3}$ p.

(3). Rs. 63353. 12 as. $4\frac{29}{53}$ p. (4). Rs. 9614. 0 a. $6\frac{1}{8}$ p.

(5). Rs. 8976. 2 as. $0\frac{108}{377}$ p.

2. $104\frac{3}{8}$. 3. Increase in income Rs. 6. 4 as.

4. Increase in income Rs. 100. 5. Gain £260.

6. Increase in income £32. 5*s*. 7. Money invested £1350.

Increase in income £9. 5*s*.

Examples XXXVIII. Pages 35, 36.

1. (1). 65. (2). 165. (3). 316. (4). 9876. (5). 123456789.

2. (1). 16·1116. (2). ·001. (3). 1·2649. (4). ·4. (5). ·1264.

3. (1). ·5775. (2). 6·0369. (3). ·8819.

(4). ·2886. (5). 16·388.

4. ·00010201 ; ·1008. 5. 999. 6. 216·3330...ft.

7. 70·7493...ft.

APPENDIX C.

ANSWERS TO ENTRANCE EXAMINATION PAPERS.—CALCUTTA.

1858.

1. $33\frac{1}{3}$.   2. [illegible].   3. 1·7320508 ; ·5477225.
4. $1\frac{1}{2}$[illegible] oz.

1859, A.

1. 5 : 22.   2. 407 yds.   3. The former ; ·2236.
4. $857\frac{1}{7}$ ac. ; $\frac{81}{640}$.   5. £2400. 15*s*. $0\frac{3}{4}$*d*. $\frac{1}{18}$*q*.
6. 13·427 poles ; ·17325.   7. £1350.

1859, B.

1. 8333 hrs. 20 min.   2. Rs. 6.   3. $1\frac{1}{8}$; ·0079.
4. $10\frac{1}{7}$.   5. ·00064 ; ·009 ; 400000.

1860.

1. Rs. 9963.   2. 7564 ; ·70716.   3. 29 ; 2.
6. Rs. 6.

1861.

1. $2243\cdot\dot{1}\dot{8}$.   2. ·035 ; [illegible].   3. £2142. 5*s*. $4\frac{1}{2}$*d*.
4. 1103[illegible] ac.   5. ·0316.

1862.

1. ·54.
2. $\frac{7}{10000}$; 1*s.* 9*d.*
3. $4\frac{4}{13}$ as
4. $25\frac{15}{22}$ minutes.
5. ·03162.

1863.

1. $11\frac{16}{67}$; 11·2388....
2. $3\frac{28779}{210100}$.
3. £143. 7*s.* $11\frac{5}{14}d.$
4. $14\frac{27}{35}$ days.
5. 31·052.
6. £529. 4*s.* $7\frac{1}{4}d$ $\frac{7}{13}q.$

1864.

1. 540.
2. 1 ; 2.
3. £77. 14*s.* $7\frac{8}{19}d.$
4. $4\frac{1}{4}d.$; $\cdot 588541\dot{6}$.
5. ·014 ; ·0001.
6. 6800 : 7221.

1865.

1. $79\frac{853}{2108}$; 79·4048 : ·3415.
2. ·001764 : 10.
3. $329\frac{7}{11}$ yds. ; Rs. 1023. 9 as. $7\frac{1}{3}$ p.
4. 45 men.
5. Rs. 84. 1 a. 10 p. ; Rs. 16. 8 as.

1866.

1. ·10444637 ; 1.
2. £21. 3*s.* $6\frac{1}{2}d.$ $\frac{4}{5}q.$
3. ·00041616 ; 9·042 ; 21·7272...
4. 256·256 ; ·0256256.
5. Rs. 210.
6. $99\frac{5}{18}$; £176. 4*s.* $2\frac{117}{217}d.$

1867.

1. 19 miles 836 yds. 2 ft.
2. 102960 ; 320·87.
3. £1. 3*s.* 4*d.* loss.
4. 1 ; $9\frac{648}{875}d.$
5. $\frac{51}{40000}$; ·001275 ; $\frac{491}{289000}$; ·001699.
6. $9\frac{1}{4}$.

1868.

1. 11*s*. 3*d*. ; 5.
2. 12·375 ; 1·816.
3. 440 miles.
4. 401 : 544.
5. £12. 18*s*. 10[illegible]*d*.
6. 58[illegible] yds.

1869.

1. 4 ; $\cdot 02392609\dot{1}2698\dot{4}$.
2. £10. 10*s*. ; $\frac{11}{12}$
3. ·02 ; ·0000002. ; ·1414 ; ·0004.
4. £14. 7*s*. 11[illegible]*d*.
5. 16 years.

1870.

1. Rs. 15. 11$\frac{1}{19}$ annas ; 8091 cub ft.
2. 998999[illegible]. (1). ·001353 ; (2). 290 ; $2\cdot 522\dot{7}$.
3. 140½ ; 2·0025.
4. 10$\frac{2}{7}$ days.
5. The second.
6. 2070$\frac{30}{371}$.

1871.

1. Rs. 2732. 13 as.
2. [illegible] is the greatest, $\frac{29}{100}$ the least ; £7. 0*s* 3[illegible]*d*. ; 1.
3. ·001875 ; 67952·25 ; Rs. 68. 3 as. 1½p. ; $\cdot 1\dot{5}\dot{4}$.
4. 55 miles.
5. Rs. 3250.

1872.

1. Rs. 1597. 10 as. 3 p.
2. $\frac{3}{8}$; Rs. 15 2 as 4p. ; 2$\frac{5}{8}$.
3. 5050 ; (i). $\cdot 0\dot{7}\dot{5}$; (ii). $\cdot 6\dot{7}71662\dot{5}$; 30·84.
4. Rs. 197. 11 as. 7[illegible]p.
5. Rs. 262. 8 as.

1873.

1. (i). $\frac{7}{8}$; (ii). Rs. 2569. 7 as. 7 p. ; Rs. 48.

2. $\dot{0}03\dot{3}$; $\frac{1}{10}$; ·5.

3. Rs. 20. 11 as. $2\frac{8}{11}$ p.

4. 19 years.

5. 5 as. $7\frac{1}{3}$ p. ; Rs. 5498. 7 as.

1874.

1. $\frac{89}{81}$; 1 : 161 ; 3328·226... ; $\cdot\dot{2}3076\dot{9}$.

2. 63 days.

3. $3\frac{65}{144}$ cub. ft. ; $15\frac{1}{4}\frac{1}{4}\frac{2}{4}$ cub. ft.

4. 120000.

5. Rs 66666. 10 as. 8 p. ; Rs 108.

1875.

1. 2 ; Rs 50 ; ·2213.

2. $\frac{1}{17}$.

3. Rs 35. 1 a. 4 p.

4. Rs 16540.

5. Rs. 58. 2 as. ; $3\frac{16}{63}$.

1876.

1. $1\frac{3741}{10724}$; Rs 13. 13 annas $6\frac{2}{10}$ p ; ·44419...

2. 9 ; 23·04484.

3. $12\frac{1}{6}$ yds. ; Rs. 1. 12 annas ; £215. 16*s*. $8\frac{2}{3}$*d*.

4. 200 days.

5. $4\frac{1}{2}$ per cent.

1877.

1. $\frac{1}{4}$; £3. 9*s*.

2. Rs. 9105. 1 a. 6 p.

3. £78. 15*s*.

4. 125.

5. 39 days.

6. Rs. 3312 ; Rs. 219.

1878.

1. 2062·649.

2. 1·00015.

3. $\cdot 37\dot{5}$.

4. Rs. 24. 14 as. $6\frac{18}{19}$ p.

5. $\cdot 009945\dot{4}365079\dot{9}$

6. £512. 9*s*. $1\frac{1}{5}$*d*.

1879.

1. 400, 50, 6, $\frac{9}{10}$, $\frac{8}{100}$, $\frac{4}{1000}$. 2. 104.

3. (a) 4 ; (b) $1\frac{47}{210}$; (c) $\cdot 0270\dot{4}\dot{5}$; (d) ·001.

4. $23\frac{7}{18}$. 5. 68 men.

6. Income decreased by £11. 4*s.* 3*d.* 7. 18.

1880.

1. 100, 20, 3, $\frac{4}{10}$, $\frac{8}{100}$, $\frac{9}{1000}$; $\frac{1}{3}$.

2. (a) $\frac{70}{333}$; (b) £40 ; (c) 2·65.

3. Each boy £4. 11*s.* ; each woman £13. 13*s.*, and each man £27. 6*s.* 4. 65 gallons ; 13 hours.

5. *C* wins by $\frac{880}{3741}$ yds. 6. Rs 25.

1881.

1. $4\frac{67}{140}$; 3. 2. ·78125 ; Rs. 380 6 as.

3. $\frac{41871}{11000}$; 1·8548. 4. £15400.

1882.

1. 4321. 2. ·57. 3. After 8 hours 20 min. 30 sec.

The bells will have struck 15015, 10010, 6006, 2002, 1430, 462, and 390 times respectively.

4. (*i*) 1600. (*ii*) 27·96424. 5. 18 days.

1883.

1. $\frac{1}{3}$. 2. ·75. 3. $\cdot 0069\dot{4}$; Rs. 6 ; £10. 10*s.* 10*d.*

4. £21. 11*s.* $2\frac{3}{4}$*d.* 5. Rs. 96. 9 as. $7\frac{1}{3}$ p.

6. $28\frac{1}{2}$ years ; Rs. 567. 8 as. ; 75.

1885.

1. $\frac{13}{6}$; $\frac{37}{660}$.   2. ·12 ; $\frac{1}{6}$; ·3047.   3. $3\cdot\dot{4}6153\dot{8}$; 30*s*.

4. £513. 6*s*. 6¼*d*. ; 3·1225 ; ·2828.   5. 18 ; $8\frac{8}{13}$. per cent.

6. The former ; £1342. 10*s*. ; $3\frac{63}{175}$ per cent.

1886.

1. $\frac{878}{975}$.   2. $\cdot 0348\dot{3}$.   3. $\frac{151}{701}$; $\frac{978}{800}$.

4. ·5 ; $\cdot 0011\dot{3}\dot{6}$.   5. £36. 17*s*. 6*d*.   6. Rs. 28659. 6 as.

7. Rs. 12⅜ ; Rs. 133⅛.

1887.

1. (1). $\frac{1}{6}$;   (2). 350.

2. ·0203125.

3. (*a*). £17. 12*s*. 2¼*d*.   (*b*). Rs. 2000.

4. 10 hrs. a day.   5. Rs. 510.   6. 13·31 ; ·471

1888.

1. $\frac{1}{14}$.   2. 11200 ; 37·96.

3. $138\cdot 449\dot{7}\dot{1}$; £20. 16*s*. $9\frac{7}{10}$*d*.

4. £1034. 14*s*. $4\frac{11}{16}$*d*.   5. 15⅝ days.

6. 6¼ per cent. per annum ; £100.

1889.

1. 59·59159412.   2. 8·62126.   3. £5247. 2*s*. $6\frac{3}{4}$*d*.

4. 1·000122.   5. £6705. 14*s*. 7*d*.

1890.

1. 3 ; Rs. 23931. 7 as. 7 pie.
2. 7305·405 ; [illegible].
3. Rs. 1771.
4. 51 days.
5. Rs. 104. 4 as.

1891.

1. (α) $\frac{8}{11}$; (β) [illegible].
2. —3·577158.
3. Rs. 408. 3 as. $4\frac{11}{12}$ p.
4. 9 hrs. 20 minutes.
5. Rs. 20800.
6. $8\frac{1}{3}$ yards.

1892

1. [illegible].
2. 26219.
3. ·312 ; ·098 ; ·998.
4. Rs. 1250.
5. £2500.

1893.

1. (1). [illegible] ; (2). 3.
2. ·0789 ; [illegible] ; [illegible] ; 1·1.
3. £345. 7s. [illegible]d.
5. Rs. 90,000 in the 4 per cent. stock and 73000 in the 5 per cent. Municipal debenture stock.

1894.

1. £37. 0s. $8\frac{1}{4}d$.
2. £491. 8s.
3. 16s. 0·375013d.
4. ·9998.
5. 6 Rupees per head.

1895.

1. 1·80001.
2. Rs. 12345.
3. 3 francs 84 centimes.
4. 1.
5. Increase of Rs. 47 ; ·6832876712.

ANSWERS TO ENTRANCE EXAMINATION PAPERS—BOMBAY.

1859.

1. Rs. 542. 2. Rs. 58. 7 as. 9 p. 3. $8\frac{3}{8}$.

4. $1\frac{1}{14}$. 5. $2\frac{1}{17}$. 6. 4200 men.

7. 108 days. 8. .00972. 9. 1.581 ; ·25 ; 1010101.

10. 15·404 feet. 11. Rs. 12550. 14 as. 1·152 p.

12. Each man Rs. $83\frac{1}{3}$; each woman Rs. $55\frac{5}{9}$; and each child Rs. $27\frac{7}{9}$.

1860.

1. $622\frac{7}{8}$ac. 2. 21 ; $\frac{3}{8}$. 4. 11·8125 ; ·559375.

5. ·000018 ; 1800000 ; ·01536. 6. £720.

7. Increase of Rs. 500. 8. Rs. 1852. 14 as. $10\frac{1}{3}$ p.

9. Rs. 4169. 9 as. $10\frac{3}{8}$ p. ; $29\frac{11}{17}$ years.

10. 3 tons. 4 cwt. 3 qrs. 4 lbs. 13 oz. 11. 7·6 feet.

1861.

1. Eleven thousand six hundred and three millions, seven hundred thousand, one hundred and sixty ; MMMMCMLX ; $\overline{\text{X}}$DCLXXXIV.

2. Rs. 50648. 1 a. ; $2\frac{2}{11}$. 3. R. 1. 7 as. $7\frac{811}{1187}$ p.

4. Rs. 233. 15 as. 10 p. 5. $\frac{13}{17}$; Rs. 4. 6 as. $4\frac{8}{17}$ p.

6. (1). ·84765625. (2) ·00084765625.

(3). 84765·625, 1250, ·0125, ·0000000125.

8. Rs. $11111\frac{1}{9}$. 9. Simp. Int. £50, Comp. Int. £51.

1862.

1. $3x^6+4x^5+5x^4+2x^3+x^2+6x+7$; MCMXVIII, MCCXXXI, MCCLXII, MDCCCLXII.

2. 248·2……, ·004028…… 3. 33[illegible] years.

4. Rs. 240, 360 two-anna pieces, 960 pice.

5. $2\frac{9}{18}$. 6. $\cdot 2\dot{6}$. 7. $\cdot 0\dot{8}9\dot{1}$, ·2986.

8. Rs. 2164. 6 as. $2\frac{1}{3}$ p. 9. ·579. 10. Rs. $93\frac{1}{13}$, Rs. $86\frac{8}{13}$.

11. R. 1. 3 as. per 100. 12. 8·2265625 metres.

1863.

1. Four millions ten thousand and ten ; 7369.

2. Rs 45. 3 as. 4 p. 3. 2*s.* 4*d.*

4. *A*, *B*, *C* Rs. 615, *D* Rs 410.

5. 91 days 21 hrs. 14 min. 15 sec 6. ·0246, ·940625.

7. 215·484. 8. Rs. 333. 5 as. 4 p. 9. £11. 11*s.* $5\frac{11}{13}$*d.*

10. Rs 18.

1864.

1. 37000000, 69000000, 494000000 ; nine hundred millions, three hundred thousand eight hundred and four ; sixty thousand six hundred and sixty millions, six hundred and eight thousand and eight.

2. 3 ; 3 is the common measure.

3. 5656567742. 4. [illegible].

5. 4261·7415 ; 3888·8985 ; 759727·26738 ; 21·86……

6. 840 lbs. saltpetre ; 112 lbs. sulphur , 168 lbs. charcoal.

7. 10·737……

8. Rs. $3451\frac{17}{18}$ *A*'s, Rs. $2876\frac{2}{3}$ *B*'s, Rs. $10462\frac{11}{18}$ *C*'s.

9. Rs. 16043[illegible] ; but taking a year = 366 days we get the answer Rs. 16,000.

10. 1584 lbs.

1865.

1. 1,234,567,654,321 ; one billion, two hundred and thirty-four thousand five hundred and sixty-seven millions, six hundred and fifty-four thousand three hundred and twenty-one.

12,34,56,76,54,321 ; one mahapadma two nikharvas three kharvas four padmas five arbudas six crores seventy-six lacs fifty-four thousand three hundred and twenty-one.

5,020,040,003,060 ; five billions, twenty thousand forty millions three thousand and sixty.

50,20,04,00,03,060 five mahapadmas two kharvas four crores three thousand and sixty.

2. Rs. 40457. 11 as. 9 p. 3. Rs. 52.

4. 405756 ; Rs. 12065. 12 as. 5. $5\frac{3}{40}$.

6. ·03 , ·0322580645161290 , ·03125.

7. Increased by Rs. 3. 7 as. $4\frac{18}{18}$ p.

8. *A* gets Rs. $22222\frac{2}{9}$, *B*, Rs. $33333\frac{1}{3}$, *C*, Rs. $44444\frac{4}{9}$

D, Rs. $46153\frac{11}{13}$, *E*, Rs. $30769\frac{3}{13}$, *F*, Rs. $23076\frac{12}{13}$.

9. 12 shares ; Rs. 1460. 10. 53 hours. 11. 416·4 , 12·3.

1866.

1. 99099099 ; 8000000207005 ; three nikharvas one kharva nine padmas six arbudas eight crores two lacs nine thousand and seventy-eight ; twenty thousand ninety millions sixty thousand and two.

2. $1\frac{3}{8}$. (*a*). $\frac{1}{18}$.

3. Rs. 5000 ; Rs. 2000 value of his share.

5. ·149553571428.

6. (i). 44·153157. (ii). 11·569328. (iii). ·499500.

(iv). 50000. (v). 46·04.

7. Rs. $1636363\frac{7}{11}$; increase of Rs. 10000.

8. £515. 16*s*. $7\frac{1}{2}$*d*. 9. 14 as. 3 p. 10. $6\frac{7}{17}$ months.

1867.

2. 2,2,5,3,7,43.

3. $14\frac{8}{35}$.

4. $\cdot\dot{8}5714\dot{2}$, $1\frac{3478}{4000}$.

5. $788\cdot423$.

6. $4\frac{4}{5}$ as. ; R. $\frac{1}{18}$.

8. 90 men.

9. 3 years 8 months 24 days.

10. 110 are learning Geography, 90 Grammar, 30 can not read and 10 advanced as far as Algebra.

11. 78.0064, ·0158, ·3902.

12. Rs. 41. 10 as. 8 p. ; 8 cwt. 1 qr. $20\frac{4}{5}$ lbs.

1868.

1. $52\frac{1}{2}$ yds.

2. Rs. 14556. 12 as. $9\frac{3}{5}$ p.

3. $\frac{1}{1260}$.

4. $\frac{7}{300}$ p.

5. £25[illegible].

6. $45\cdot\dot{6}9230\dot{7}$.

7. Rs. $76363\cdot\dot{6}\dot{3}$.

8. $1\frac{9}{16}$ per cent.

9. 466.

10. 9*s*. $4\frac{838}{1293}$*d*.

11. 23 men.

12. £30. 4*s*. $8\frac{1}{4}$*d*.

1869.

1. 97, 1008.

2. 1.

3. 2.

4. $\frac{1}{14}$, $\frac{1}{84}$.

5. ·6489583.

6. Rs. 393. 13 as., Rs. 656. 5 as. 8p., Rs. 1050. 2 as. 8 p., Rs. 1181. 7 as.

7. 4 per cent.

8. £1340. 1*s*. 10·95375*d*.

9. 9 days.

10 Increased by Rs. 428.

11. 1769, $20\cdot\dot{8}\dot{3}$.

1870.

1. 654,323,004,021,050,301 ; one billion, three hundred and twenty-seven thousand eight hundred and seventy-five millions, four hundred and

thirty thousand and twenty-nine ; one mahapadma three nikharvas two kharvas seven padmas eight arbudas seven crores fifty-four lacs thirty thousand and twenty-nine.

2. $11\frac{87}{106}$, 11·8208.
3. $\frac{5}{88}$, $1\frac{24}{85}$.
4. £81.
5. Rs. 25600.
6. 401 : 544.
7. 5 dwts. $3\frac{17}{28}$ grs., 3 dwts. $15\frac{8}{11}$ grs.
8. £35. 16*s*. $10\frac{1}{13}$*d*.
9. Rs. 900, Rs. 600, Rs. 2100.
10. ·314642, 1·816590.
11. £585. 2*s*. $1\frac{3}{7}$*d*.

1871.

1. 192000 miles.
2. 1287 ; 9009.
3. $\frac{7}{1800}$.
4. $\frac{1}{32}$, $\frac{3}{37}$; Rs. 5. 11 as.
5. £606.
6. $4\frac{10}{21}$.
7. $9\frac{3}{8}$; Rs. 2956. 4 as.
8. 83149.
9. Decreased by Rs. $5\frac{15}{16}$.
10. 999.

1873.

1. $2\frac{1}{35}$.
2. $\frac{87}{54}$.
3. $\frac{9}{16}$.
4. Rs. 1500.
5. $7\frac{1}{2}$ hours.
6. £2376. 5*s*.
7. 30780.
8. 76 rows.
9. $55\frac{1}{3}''$.
10. 1234.

1874.

1. 1.
2. 4·8.
3. $\frac{9}{37}$.
4. £213. 12*s*.
5. Rs. 14586.
6. 13*s*. 4*d*.
7. Rs. 26.
8. $7\frac{1}{33}$; Rs. 1840 ; $4\frac{8}{15}$.
9. 36 miles and 24 miles per hour.
10. They are in order of magnitude.

1875.

1. (*a*) Eight millions two hundred and seventy-one thousand and ninety-six.

(b) Nine millions thirty-two thousand eight hundred and four.

(c) Three hundred nineteen thousand and eighty millions, two hundred and fifty-nine thousand four hundred and seventeen.

(d) Eight millions four thousand six hundred and forty.

2. 21 miles 6 fur. 33 po. 3 yds. 2 ft. 7 in. 4. 528093440.

5. $1\frac{78}{157}$. 6. $\frac{1}{2}$, $\frac{8}{13}$, $\frac{48}{98}$. 7. 10·017. 8. $6792\frac{8}{9}$.

9. 27·3 ; 32. 10. $3\frac{9}{25}$*s.*, $4\frac{8}{25}$*s.*, $5\frac{13}{25}$ *s.*, $7\frac{17}{25}$*s.*

11. Rs. 18750 ; increased by Rs. 56. 4 as. 12. $3\frac{127}{288}$ sq. ft.

1877.

2. *B* is $\frac{1}{24}$ of a mile in advance of *A*. 3. $\frac{1}{18}$.

4. Tea Rs. 2. 8 as., sugar 2 as. 8 p. 5. $217\frac{1}{2}$ ft. ; 242 times.

6. *A*, Rs. 850, *B*, Rs. 846, *C*, Rs. 1182. 7. £650.

8. £818. 8*s.* 9. ·0061. 10. $9\frac{93}{220}$ miles.

1878.

1. £90. 18*s.* $11\frac{1}{4}$*d.* 2. 1. 3. £4. 0*s.* 9*d.*

4. 83 ft. 5 in. 5. 25 per cent. 6. Rs. 7678. 2 as. ; 10 as. 2·85 p.

7. £1500. 8. £215. 8*s.* $9\frac{15}{817}$*d.* 9. £20. 10. £2890. 10*s.*

1879—80.

1. 48023601, 45942521. 3. £5. 0*s.* $0\frac{3}{4}$*d.*

4. 1, $\frac{4}{5}$. 5. Rs. 10. 6. $3\frac{1443}{11881}$ per cent.

1880—81.

1. $2\frac{3}{17}$, $4\frac{1}{2}$, $6\frac{11}{12}$. 2. $30\frac{11}{12}$". 3. 26 coolies.

4. 2 years. 5. Rs. 2625.

1881—82.

1. £1508. 15*s*. $7\frac{11}{18}$*d*. 2. 1 minute. 3. 10*s*. $10\frac{2}{3}$*d*.

4. Rs. 2646. 5. 4 per cent. 6. ·08$\dot{3}$, 75·1.

1882—83.

1. 11*s*. 11·25*d*., ·03671875. 2. 24 posts.

3. $9\frac{3}{4}$ weeks, £341. 5*s*. 4. £4328. 2*s*. 6*d*.

5. $77\frac{9}{16}$ yds.

1883—84.

1. (*a*) 16000075040002. (*b*) 1. (*c*) £24. 19*s*. $5\frac{1}{3}$*d*.

2. 36 in the 6th standard, 40 in the 5th, 45 in the 4th, 60 in the 3rd and 72 in the 2nd; 360 boys.

3. $\frac{1}{16}$ of a furlong. 4. £86. 11*s*. 5. 13312; 93·05; 9·1.

1884—85.

1. $\frac{3}{7}$, ·725, £1. 14*s*. 2. £2. $9\frac{3}{7}$*s*. 3. £15. 16*s*. 2*d*.

4. Income increased by £7. 5. 4 per cent.

1885—86.

1. ·$\dot{4}85714\dot{2}$. 2. 113 boys. 3. $5\frac{1}{3}$ qrs, £$1\frac{8}{40}$. 4. £72. 6*s*. 8*d*.

5. The latter investment is more profitable; £$457\frac{1}{187}$.

1886—87.

1. 5 × 7 × 11 × 13, 22·9999891208453...

2. $192\frac{1}{2}$ ft. 3. $\frac{1}{6}$.

4. (*i*) 7 ft. 2 inches, (*ii*) 3·5752. 5. £5103.

6. $3\frac{11}{17}$ per cent.

1887—88.

1. 6. 2. 19*s.* 3*d.* 3. 20 months.

4. 20 ; 7 ; 5*s.* $1\frac{1}{3}$*d.* 5. 5 as. 4 p.

1889—90.

1. 2. 2. $5\frac{5}{8}$ ft. long by $5\frac{5}{8}$ ft. broad by $5\frac{5}{7}$ ft. deep.

3. 5-15 o'clock. 4. Rs. 32000.

5. 3 parts of the one to 13 parts of the other.

1891—92.

1. (*i*) $\frac{1}{8}$, (*ii*) $\cdot 8\dot{3}$.

2. Weight allowed is 100 lbs. ; they had 2 cwts and 3 cwts.

3. They last agreed at 10 hrs. 30 min. P.M. when they both indicated 10 hrs. 30′ 50″. 4. Rs. 640.

1892—93.

1. $\cdot 0050208\dot{3}$; 15 annas. $7\frac{11}{16}$ pie ; $\frac{1}{7}$. 2. 10 days.

3. 3 hrs. 30 min. 4. £259 - 3*s.* - $5\frac{1}{4}$*d.* 5. 401 : 544.

1893—94.

(Set in the Mofussil).

1. 20577 ; 39690 ; ·518442932728647014361 30007...

2. £11. 11*s.* $6\frac{3}{4}$*d.* 3. 2 cwt. 2 qrs. 20 lbs.

4. £83. 6*s.* $\dot{8}$*d.* 5. $12\frac{1}{2}$ hrs.

6. 4 lbs. of the inferior to 5 lbs. of the superior quality.

1893—94.

(SET AT BOMBAY.)

1. (i). 24 ; (ii) $\frac{58}{115}$. 2. £32. 14s. $3\frac{6}{7}d$.

3. $3\frac{9}{11}$ months. 4. £3. $2\frac{2}{25}s$. 5. $\frac{19}{24}$ ths.

6. At the same time on the afternoon of the 23rd August when the first clock will show 1 – 46′ and the second 2 – 16′.

1894—95.

1. 146097 days. 2. 156. 3. 30.

4. $1\frac{1}{3}$ days. 5. Rs 2160. 6. 7·72 ; $15\frac{1}{4}$ annas.

ANSWERS TO ENTRANCE EXAMINATION PAPERS—MADRAS.

1857.

1. $\frac{4}{7}$. 2. ·019. 3. Rs. 381. 4. 13·71.

5. 4 ft. in. 6. £39. 7s. 6d.

1858.

1. Rs. 49. 11 as. $11\frac{7}{8}$ p. 2. 152 tons 15 cwt. 3 qrs. 6 lbs. ; 47 hours 32 minutes 5 seconds.

3. ·23 mile nearly. 4. 57°17′44[illegible]″.

5. Rs. 774. 0 as. $6\frac{18}{31}$ p. 6. Gain Rs. 25. 11 as. nearly.

1859.

2. Receipts per mile per week in 1858 were Rs. 6. 1 a. 4[illegible] p. more.

4. ·096 ; 6·6 ; ·620. 5. 13 ac. 3 ro. 16 po. 18 sq. yds. 7 sq. ft.

6. 11960 sq. yds. 4 sq. ft. 20·41 sq. in.

7. 79·8 ; 81·1 ; 74·6 ; 82·1.

1860.

1. 56831327.　　2. 26292.　　3. ·20045.

4. [illegible].　　5. 34·004.　　6. Rs 1087. 8 as. 11 [illegible] p.

1861.

1. Rs 22840 ; Rs 11420 ; Rs 3806 10 as. 8p. : Rs 7613 5 as 4p

2. 1 [illegible].　　3. [illegible] ; 343.　　4. Rs 68. 2 as. 5·825536 p

5. Rs. 6714. 12 as. 11 [illegible] p.　　6. 1091495·428571.

7. ·065.　　8. $26\frac{6}{15}$ p c.　　9 Rs. 202. 2 [illegible] as.

1862.

1. Fourteen thousand and six ; three thousand one hundred and seventy-nine millions forty thousand six hundred and one , seventeen, and four hundredths, six thousandths and one ten-thousandth.

3. $\frac{1}{108}$.　　4. 17.　　5. £47. 18s. 8 [illegible] d

6. ·571428.　　7. 8·039 ....　　8. 9 days

9. 45 miles per hour.

1863.

1. 1962 rem. 123.

2. (1). £206. 12s. $5\frac{1}{2}d$.　　(2). Rs 2066. 3 as. 8 p.

3. (1). [illegible].　　(2). 8 [illegible].　　(3). Rs. 3. 14 as.

4. 4·4137 ; 411·1.　　5. £56. 2s. 6d.　　6. 357 ; 3·57 ; 1·414.

8. Rs. 716. 10 as. 8 p.; Rs. 358. 5 as. 4 p.　　9. The steamer ; 16 hrs.

1864.

1. 9 hrs. $37\frac{8}{11}$ min.
2. $\frac{113}{300}$ is greater and by $\frac{1}{7810}$.
3. 696lbs. $9\frac{3}{8}$ oz.
4. $1\frac{193}{385}$.
5. Rs. 14. 9 as. 4 p.
6. $4\frac{1}{7}$; ·123.
7. £115. 18*s*. 9*d*.
8. $3{\cdot}2743\dot{6}$.
9. 25 miles.

1865, A.

1. £22. 4*s*. $10\frac{1}{8}$*d*.
2. Rs. 120.
3. (1). 10 ; (2). 20.
4. 11 ft. $6\frac{1}{8}$ in.
5. 2·3804 ; ·0670.
6. $12\frac{1}{2}$.
7. $\cdot 208\dot{3}$.
8. £5888.

1865, B.

1. Rs. 16666. 10 as. 8 p.
2. 7 days 10 hrs. 52′ 30″.
3. Rs. 241. 0 as. 3 p.
4. $\frac{8}{21}$.
5. 20.
6. (*a*). $\frac{113}{108}$. (*b*). 10 as. 6 p.
7. (*a*). 7·31. (*b*). $\cdot\dot{4}3$.
8. 8 hours.

1866.

1. 226875 lbs.
2. Rs. 32. 7 as.
3. $\frac{189}{1000}$; ·189.
4. 57 men.
5. $\frac{3}{16}$.
6. 14·003.
7. 25 p.c.
8. $\frac{3}{4}$.
9. The first Rs. 5184 ; the second Rs. 2592 ; the third Rs. 1728.

1867.

1. 240 men.
2. 64 yds. 1 ft. 4 in.
3. 5.
4. ·09091.
5. Rs. 3.
6. 32 days.
7. 60 stones.
8. 25 chapters.

1868.

1. $1\frac{5}{18}$. 2. £2428. 15*s*., £1238. 13*s*. 3*d*.; £1190. 1*s*. 9*d*.
4. £900. 4. $156\frac{1}{4}$ ft. 5. $16\frac{64}{121}$ ft. 6. 6 hrs. 59′ 15″.
7. Rs. 9. 7 as. 3 p. 8. $8\frac{1}{4}$. 9. 3 as. $6\frac{6}{25}$ p.

1869.

1. The second is greater by $\frac{17}{240}$; the difference expressed as a decimal is $\cdot 0708\dot{3}$.
2. $\frac{1}{1412256}$; 1587·49.........
3. 220 yds. and 165 yds.; and the area $= 7\frac{1}{2}$ ac.
4. Rs. 15000 legacy; Rs. 4500 to each charity.
5. Rs. 2790. 10 as.; $\frac{407}{1152}$. 6. *A* receives 6*s*. $8\frac{15}{40}$*d*. more than *B*.
7. 12 lakhs; Rs. 5. 8. $36\frac{3}{4}$ miles; 37′ past 8 A. M.
9. $266\cdot\dot{6}$. 10. $14\frac{2}{7}$ days.

1871.

1. Rs. 5. 2. (*a*) 1. (*b*) $\frac{1}{3}$. 3. £419. 19*s*. 3*d*.
4. 6 pies per Rupee. 5. £233. 17*s*. 10*d*.; $5\frac{1}{2}$*d*. 6. 2 years.
7. 12*s*. 8. 4 sq. in. 9. 70 oz. 10. 22 miles.

1873.

1. 76 acres. 2. 1; ·25. 3. £1000; £4000.
4. 35 measures. 5. 18 miles. 6 Rs. 770; 1 per cent.
7. £1000; 2 and $2\frac{1}{2}$ years. 8. Rs. 13680.
9. 8 yds.; 3 hours. 10. 99 yds.; 77 yds.

1874.

1. 39; 319. 2. 17*s*. 6*d*. 3. (*a*) $\frac{4}{7}$; (*b*) ·125.
4. 9*s*. 7*d*.; $\cdot 0\dot{1}$. 5. 30 seers. 6. £250; 4 per cent.

7. £690. 8. $277\frac{1}{2}$ cub. in. 9. 2 : 1.

10. $2\frac{1}{2}$ miles.

1875.

1. Rs. 441. 7 as.
2. $1\dot{6}\dot{1}$; $11344\frac{3}{5}$.
3. $\cdot02291\dot{6}$.
4. $10\frac{1}{8}$ days ; $42\frac{7}{18}$ cub. ft.
5. Rs. 46. 8 as.
6. 3, 3, 5, 7, 7, 11, 11, 13, 13 ; 5
7. 15 ft.
8. £1600.
9. 67 yds. ; 125.
10. 9 per cent.

1877.

1. Rs. 603. 13 as. 9p.
2. $13\frac{1}{2}$ days.
3. ·017 ; 35·0622.
4. Rs. 17 1 a. 8p.
5. 13 per cent.
6. Rs103. 1 a. 3p
7. £49. 16s.
8. Rs4200.
9. $4\frac{1}{2}$ measures.

1878.

1. (*a*) Rs1239. 13 as. 4 p. ; (*b*) Rs51738. 3 as. 3 p.
2. 52 days.
3. (*a*) ·00032 ; 3·2 ; (*b*) 8.
4. Rs9180.
5. R. $1\frac{97}{175}$.
6. Rs276. 5 as. 3 p. ; $314\dot{4}0$.
7. 27 days.
8. ·0047 ; 27·6568.
9. 25 shares.

1879.

1. Rs606. 11 as. $3\frac{1}{4}$ p.
2. 10 days.
3. Rs825 ; 8 p. c.
4. Rs13. 2 as. $10\frac{2}{3}$ p.
5. 2·0918...... ; ·5773......
6. Rs68. 13 as. $\frac{4}{6}$ p.
7. £1780. 19s. $8\frac{4}{11}d$.
8. Rs$11111\frac{43}{441}$.
9. 3 hrs. 40 minutes.

1880.

1. $6\frac{3}{8}$ feet. 2. 3 miles. 3. $\cdot\dot{5}9\dot{2}$.

4. 23 as. 5. $18\frac{1}{4}$ feet wide, $14\frac{1}{2}$ feet high.

6. 2469 ; ·0788... 7. $7\frac{1}{2}$ hours. 8. $3\frac{1}{2}$.

9. Each child Rs 960 ; each brother Rs. 495.

1881.

1. Rs. 241. 8 as. 8 p. ; Rs. 5267. 11 as. $7\frac{1}{2}$ p. 2. Rs. 666. 12 as.

3. Rs. 37350. 4. $48\frac{1}{16}$; ·2070. 5. [illegible] ; [illegible] ; [illegible] ; [illegible].

6. 14 years. 7. Rs 10020. 5 as. 8. Rs 28500.

9. slower $15\frac{8}{22}$ miles ; faster [illegible] miles.

1882.

1. ·387. 2. 9. 3. $13\frac{1}{4}$ ft. 4. 294·151.

5. (*a*) 210 ; (*b*) 179.

6. (a) Percentage gained by *A*, 52, *B*, 60, *C*, 64·1$\dot{6}$, *D*, 57·1$\dot{6}$. *E*, 68·$\dot{6}$, *F*, 49·1$\dot{6}$.

   (*b*) 64·3 in Arithmetic , 55·4 in Algebra , 46·$\dot{2}$ in Euclid ; 67·1 in English , 65·2 in History , 62 in Geography , 41·9 in Hand-writing.

   (*c*) 60 per cent.

7. ·$\dot{4}2857\dot{1}$. 8. Rs 1711. 11 as. 11[illegible] p. 9. 675 lbs.

1883.

1. 27 gallons. 2. 4*d*. 3. 27 days.

4. 4 hours. 5. £1345. 16*s*. 8*d*. , £107. 13*s*. 4*d*. ; 8 p. c.

6. $4\frac{1}{2}$ hrs., $7\frac{7}{11}$ hrs. 7. £190. 8. 0.

9. 1000 yds.

1884.

1. 1. 2. Rs. 1455. 4 as. 4 p. 3. 1·00904 ; 107·916.

4. Rs. 962. 3 as. 5 p. 5. 343 cub. in. . 7776.

6. *A* Rs. 480, *B* Rs $533\frac{1}{3}$, *C* Rs. $466\frac{2}{3}$; $1\frac{1}{3}$ per cent.

7. Rs. 3125 8. £8.

9 $4\frac{1}{8}$ p c. 10. Rs. 211. 9 as.

1885.

1. $\frac{1}{4}$ 2 $\frac{96}{192}$, 64 Rupees. 3. £1. 6*s*. $0\frac{1}{2}d$.

4 £19 3*s* $10\frac{7}{9}d$. 5. 11*s*. $10\frac{1}{2}d$. 6. Rs. 1920.

7. 3*s*. 9*d* 8. 4 years. 9. £5000.

10. ·9196 , £4 2*s* 6*d* 11. 350000 men.

1886.

1. 1. 2. ·9705. 3. £2. 11*s*. $5\frac{3}{4}d$.

4. Rs 3955. 3 as. 11 p. 5. 1-13 P. M. 2nd July.

6. 60 men 7. £1000. 8. £180.

9 Rs 17. 8 as 10. 520344000 cub. ft. ; $1\frac{17}{18}$ in.

1888.

1. (1). £116566. 8*s*. $7\frac{1}{2}d$, (2). Rs 142076. 4 as. 9 p.

2. $\frac{3}{4}$. 3 1 a. 4 p. ; ·114583. 4. £721. 15*s*. $6\frac{9}{11}d$.

5. £335000 6. £416. 13*s*. 4*d*. 7. Rs 7. 2 as.

8. Increase of Rs 502. 8 as. 9. Rs 500000.

10. 500400.

1889.

1. £14004. 12*s*. $4\frac{1}{2}d$. , Rs. 103992. 12 as. 7 p.

2. $\frac{3}{8}$. 3. ·08273029 ; 6*s*. $9\frac{3}{8}d$.

4. Rs. 1730. 13 as. $6\frac{1}{4}$ p. Rs. 48. 2 as.

6. £1694. 13*s*. 9*d*. Rs. 280000.

8. 10 days. 9. 7500274.

10. £39. 3*s*. 9*d*. 11. 2·08008.

1890.

1. 342 ac. 2 ro. 39 sq. po. 2 sq. ft. 36 sq. in. ; 160 yds.

2. 1·5. 3. Rs. 975358. 9 as. $2\frac{1}{8}$ p. 4. 30 weeks.

5 Rs. 6744273. 6. 4 months. 7. Increase of £397.

8. 12 cwt. 1qr. 19lbs. 4 oz. ; £33. 2*s*. 6*d*.

9. 343 : 169. 10. 19487.171.

1891.

1. £42900. 2. R. 1. 11 as. 8 p. 3. R 1. 10 as. 2 [illegible] p.

4. 9 ; 46·94718. 5. 12′. 6. Rs. 6. 6 as. 4 p ; Rs. 158.

7. £291. 9*s*. $5\frac{1}{4}$*d*. nearly. 8. 20′ afternoon.

9. 10*d*. 10. Rs. 9180.

1892.

1. 311993 tons. 10 cwt. 12 lbs. ; 219505 kandis 7 mds. 1 v. 3 7 palams.

2. $\frac{3}{4}$. 3. 5*s*. 3*d*. ; ·0037115625.

4. Rs. 67567. 9 as. $7\frac{1}{3}$ p. 5. £416. 13*s*. 4*d*.

6. 3·700965. 7. 3221625 tons.

8. Rs. 355. 13 as. 4 p. 9. Rs. 55. 8 as. 4 p.

10. $3\frac{3}{16}$ per cent. 11. 25640000.

1894.

1. Rs. 2130333. 11 as. 9 p. ; 755 mi. 4 fur. 2 po. 4 yds.

2. $\frac{1}{3}$. 3. 5*s*. 2[illegible]*d*. 4. Rs. 1593. 4 as. $5\frac{5}{8}$ p.

5. Rs. 93333. 5 as. 4 p. 6. £976. 11s. 3d.

7. Rs. 23. 12 as. 4 p. 8. $4\frac{1}{2}$ miles. 9. Rs. 1062.

10. 14·1625 per cent. 11. 9·0073210.

ANSWERS TO THE ENTRANCE EXAMINATION PAPERS—PUNJAB.

1875.

1. 1010001 ; 766.

3. (1). $6\frac{1}{3}$. (2). [illegible].

4. $1633\frac{1}{3}$ sq. yds ; $\cdot 7\dot{2}$. 5. 999 ; 1·772. 6. 15 pumps.

1876.

1. 2881·16... 2. Rs. 3281. 1 a. 6 p.

3. Rs. 800. 1 a. 9 p. 4. $22\frac{2}{7}$ seers per rupee.

5. Rs. 3992. 11 as. 8[illegible] p.

1891.

1. 1s. 10[illegible]d.

2. (a). 18 minutes ; 2. ·0064453125.

3. 34·3168 ; 5·858.

4. (a). 4838[illegible] Rs. (b). 531[illegible] ; 31[illegible] Rs.

1878.

1. Rs. 5. 10 as. 2 p. 2. $1\frac{6}{13}$. ; $\cdot\dot{1}9780\dot{2}$.

3. (a). $\cdot 00\dot{0}\dot{3}$; ·0029644268. 4. 338 sq. ft.

5. 5 per cent. per annum. 6. ·316 ; ·0001.

1879.

1. (*b*). [illegible] and [illegible] ; ·28472. 2. [illegible]. 3. 2·115 ..cub. in.

4. (*a*). [illegible] ; (*b*). ·0316. 5. 316·227..,yds.

1881.

2. 9·45 ; 2·2371. 3. ·03168567807351077З. .

4. His income is increased by 14[illegible] Rs.

1883.

1. (*a*). ·6848. 2. 12. 3. (*b*). [illegible].

4. 3[illegible] hours. 5 140 ; 170 ; 190.

6. His income is diminished by Rs. 507. 8 as.

1884.

1. 25. 2. ·02688, .002688 ; 25·6, 2·56.

3. 1. 4. 1·0001.

5. Rs. 62. 10 as. 5[illegible] p. 6. 8 as. 11[illegible] p.

7. The first is greater [illegible]

PAPERS—A

18

1. 1 ; 123 times. [illegible] 2. [illegible] ·08125 ; ·0003 ; ·038961.

3. Rs. 884. 15 as. 3 p. 4. £9. 15*s*.

5. 7056 ; 2420 sq. yds. 6. The latter ; Rs. 49000.

1886.

1. ·375 ; ·612. 2. ·7895. 3. [illegible]. 4. She loses.

5. Rs. 195. 13 as. 0·96 p. , Rs. 172. 8 as. 5[illegible] p.

1887.

1. (*a*). 3000005409062. (*c*). 3·025.
2. £133. 6*s*. 8*d*. 3. Rs. 9. 6 as. $8\frac{4}{13}$ p. per maund.
4. Rs. 480, Rs. 405, Rs. 336, Rs. 297.
5. Rs. 11250. 6. ·04.

1888.

1. $1\frac{4}{25}$. 2. 1·09375. 3. 16 days. 4. 17 per cent. 5. 32·867.

1889.

1. ·322083 ; 799. 2. $2\frac{3}{4}$ miles. 3. 3·7651 ; 126 seconds.
4. Rs. 14. 8 as. $6\frac{6}{7}$ p. 5. $\sqrt[3]{3}$ is greater ; the latter.

1890.

1. (*a*). 1 ; (*b*). ·03. 2. $\frac{9}{73}$; ·390625. 3. £1274.
4. 160 men. 5. Rs. $86\frac{21}{64}$. 6. 2 p.

1891.

1. (1). $1\frac{1}{24}$; (2). 11. 2. ·0064453125. 3. [illegible] years.
4. £200 ; 5 years. 5. 45 gallons.

1892.

1. $\frac{1}{101}$; the latter comes nearest. 2. 5·90625.
3. 218972·16 gallons. 4. £$10166\frac{2}{3}$; £6000.
5. 17*s*. 3*d*. per gallon.

1893.

1. Rs. 1000600. 2. 13·713729902. 3. ·41421.

4. Rs. 17. 5 as. $9\frac{1}{4}$ p. 5. 32 miles.

1894.

1. ·571428 ; ·428571. 2. Length 44 ft., breadth 33 ft. 3. £511.

4. Present value by common calculation *i. e.*, by deducting interest is Rs. 987. 8 as. ; present value by deducting discount is Rs. 987. 10 as. $5\frac{5}{7}$ p.

5. 10·04987.

1895.

1. 1·0714285. 2. 1·25 lbs. 3. £391, £529, £1311.

4. 2·2360679. 5. Length 76·2 yds., breadth 38·1 yds.

ANSWERS TO THE ENTRANCE EXAMINATION PAPERS—ALLAHABAD.

1889.

1. $1\frac{5}{13}$, 1·384615. 2. (*a*) $3\frac{8}{11}$. (*b*) ·0003, ·00296...

3. ·0316..., ·01. 4. Rs. 1300. 5. 12 min. $40\frac{8}{9}$ sec.

6. 8 days. 7. $39\frac{3}{8}$ miles from starting place.

1890.

1. ·697684787, 36·623873. 2. $\frac{1}{24}$.

3. Rs. 8600, 13 as. $10\frac{1}{4}$ p. 4. ·99999.. 5. 2·115.

6 $11\frac{8}{21}$ in.

1891.

2. 116 [illegible].   3. [illegible].   4. 5 per cent.   5. 9999. $7\frac{1}{8}$.

1892.

2. (*a*) 12.   (*b*) [illegible], [illegible], ·84375.   3. $56\frac{1}{4}$ days.

4. £3.   5. 1 0001...

1893.

2 2 fur. $12\frac{9}{11}$ poles.   3 Rs. 3. $7\frac{11}{13}$ as.   4. £14 1*s*. $3\frac{3}{4}$*d*.

5. £350. 11*s*. 8*d*   6. 1869.   7. 79·032, $8\frac{7}{16}$.

1894.

1. (*a*) $999 \times 807 = 806193$.   (*b*) -1.

2 (*a*) $\cdot 000\dot{9}$.   (*b*) 5·059.

3 444 miles.   4. Rs 555.